LI. Remarks on a paper by Professor Challis, "On the analytical condition of the rectilinear motion of fluids."

G.G. Stokes B.A.

To cite this article: G.G. Stokes B.A. (1842) LI. Remarks on a paper by Professor Challis, "On the analytical condition of the rectilinear motion of fluids.", Philosophical Magazine Series 3, 21:138, 297-300, DOI: [10.1080/14786444208621550](https://doi.org/10.1080/14786444208621550)

To link to this article: <http://dx.doi.org/10.1080/14786444208621550>

Published online: 01 Jun 2009.

Submit your article to this journal [↗](#)

Article views: 4

View related articles [↗](#)

Citing articles: 1 View citing articles [↗](#)

Mr. G. G. Stokes on the Rectilinear Motion of Fluids. 297

in which *each* of the six coefficients, the first, for example, expresses a series of the form

$$A_0 + \frac{A_1}{\lambda^2} + \frac{A_2}{\lambda^4} + \frac{A_3}{\lambda^6} + \&c.,$$

where λ denotes the wave-length, and all the other quantities are constant. The ellipsoid itself is the reciprocal of that ellipsoid by which the wave-surface is constructed, and its semiaxes are the three principal indices of refraction. As λ is supposed to vary, not only the length but the direction of the principal axes vary, and thus we have a different wave-surface for every different wave-length *within* the crystal.

The optic axes are perpendicular to the circular sections of the above ellipsoid, and describe, in general, two fragments of a cone, the equation of which may be found by supposing λ to be variable in the equation of the ellipsoid. But only very particular cases have been hitherto observed, and I shall not stop to discuss them.

Trinity College, Dublin,
September 1842.

J. MACCULLAGH.

LI. *Remarks on a paper by Professor Challis, "On the analytical Condition of the Rectilinear Motion of Fluids."*

By G. G. STOKES, B.A., Fellow of Pembroke College, Cambridge*.

IN the August Number of this Magazine (p. 101), Professor Challis has written an article, of which the object is to prove that, in all cases of fluid motion in which $u dx + v dy + w dz$ is an exact differential, the motion is rectilinear. The importance of this question may apologize for these remarks, since, if the reasoning in that article be correct, it will affect the validity of much that has been written on the subject. It appears to me however that Professor Challis has made an assumption which is not allowable, and consequently the conclusion founded on it is not allowable either. In what follows, I shall call the path of a particle of fluid in space a *line of motion*, and a line traced at a given instant from point to point in the direction of the motion a *line of direction*.

As the basis of his reasoning Professor Challis assumes, that in every case where the continuity of the fluid is maintained, the most general supposition that can be made respecting the directions of motion in each indefinitely small element of the fluid is, that they are normals to a surface of continuous curvature, and as such pass ultimately through

* Communicated by the Author.

two focal lines; that is to say, that the above is true neglecting quantities of the order Pp^2 , P and p being any two points in the element; that this is the meaning is shown by the fact that the whole investigation depends on quantities of the order Pp . Now, not only in the case where $u dx + v dy + w dz$ is an exact differential, but also in the case where it is integrable by a factor, there exists a surface of displacement passing through P , and the above statement will be true for an element of this surface. But it will not generally be true for an element of three dimensions; for, let p be taken in the line of direction passing through P ; then, if ϖ be the radius of absolute curvature of this line at the point P , and $Pp = \delta s$, the angle between the tangents at P and p will be ultimately $\frac{\delta s}{\varpi}$. Neglecting quantities of the order δs^3 , a line PT' drawn

through P parallel to the tangent at P may be taken instead of the tangent at p . Now, even if we suppose the line PT' to pass through the focal line which is at a distance r from p , the least distance between it and the other focal line, which is at a distance r' from p , will be ultimately $r' \frac{\delta s}{\varpi}$. Hence,

it cannot ultimately pass through both focal lines, unless ϖ be at every point infinite, i. e. unless all the lines of motion be right lines, which is evidently a very limited case. Consequently, it is only in this case that it is proved that surfaces of displacement are surfaces of equal velocity.

There is another part of Professor Challis's reasoning with which I cannot agree. It is that $d \left(\frac{d\phi}{dt} \right)$ or $\frac{d^2\phi}{dt dx} dx + \frac{d^2\phi}{dt dy} dy + \frac{d^2\phi}{dt dz} dz = 0$, in passing from one point to another of a surface of displacement. For, $d \left(\frac{d\phi}{dt} \right) = 0$ is the differential equation to a family of surfaces whose general equation is $\frac{d\phi}{dt} = C$, which family of surfaces is in general quite different from that whose equation is $\phi = C'$. Now the proof requires that the variations dx, dy, dz should be taken along that surface of the second family which passes through the point (x, y, z) , whereas the variations for which $d \left(\frac{d\phi}{dt} \right) = 0$, must be taken along that surface of the first family which passes through the same point. If $\phi = \psi(t)(x^2 - y^2) + \chi(t)xy$, for instance, these two surfaces will be different.

In any possible case of fluid motion, the motion, which would result by supposing the whole mass of fluid so in motion to be besides moving forward in space with a uniform velocity, would also be possible. But if the components u, v, w of the velocity in the first case be such that $u dx + v dy + w dz$ is an exact differential, it will be easily seen that the components u', v', w' of the velocity in the second case will also be such that $u' dx + v' dy + w' dz$ will be an exact differential. But if the lines of motion in the first case be right lines, they will not be so in the second, unless the velocity at each point of the same line be the same. If, for instance, $u = \frac{ax}{x^2 + y^2}$,

$v = \frac{ay}{x^2 + y^2}$, $w = 0$, and if we now suppose the whole mass of fluid to be moving besides with a uniform velocity parallel to x , the lines of motion and of direction will both be right lines in the first case, but neither of them will be right lines in the second.

Professor Challis objects to the case of motion to which he alludes, where $u = ax$, $v = -ay$, $w = 0$, by saying that the arbitrary quantities introduced in the process of integration cannot be satisfied, unless the fluid be in confined spaces or narrow canals; that is in *indefinitely* narrow canals, as his reasoning which follows shows to be the meaning. It will appear however from the following reasoning that the canals need not be narrow.

Conceive a mass of incompressible fluid to be at rest, bounded by material parallel planes, and by cylindrical surfaces whose bases are part of a branch of a rectangular hyperbola, its asymptotes (which I shall take for the axes of x and y), and two lines perpendicular to them. Of the two planes whose bases are the two latter lines, conceive one, whose equation is $y = y_1$, to be made to move with a velocity $-f(t)y_1$ parallel to y , and the other, whose equation is $x = x_1$, with a velocity $f(t)x_1$ parallel to x , and conceive the planes to contract or expand, so as always to reach from the hyperbola to an asymptote. Then the motion is determined by the equations of motion, the equation of continuity, and the condition that the particles in contact with a surface, whether fixed or moveable, neither penetrate into, nor separate from it. Since the motion is determinate, and these are the only conditions to be satisfied, any values of u, v, w and p which satisfy them, will be the true values. Such values will be found to be

$$u = f(t)x, \quad v = -f(t)y, \quad w = 0, \quad \frac{p}{\rho} = \psi(t) - \frac{1}{2}(u^2 + v^2).$$

The function f is arbitrary, and may be discontinuous. It is supposed to be nothing at first. If it suddenly acquires a finite value, the motion will begin with impact. It will be easily seen that the equations of impulsive motion, and the conditions with respect to the surfaces, will be satisfied by the above values of u and v , and the value of the impulsive pressure $C - \frac{\rho}{2} (u^2 + v^2)$.

LII. *On Conchylometry.* By the Rev. H. MOSELEY, M.A., F.R.S., Professor of Natural Philosophy and Astronomy in King's College, London.

To the Editors of the *Philosophical Magazine and Journal*.

GENTLEMEN,

IN a paper printed in the Transactions of the Royal Society (1838, part ii. *) I have investigated certain properties of the spiral curves traced upon the surfaces of shells (concho-spirals†) common to them and to the well-known logarithmic spiral.

The results deduced from my admeasurements have since been confirmed by those of Professor Naumann of Freiberg (Poggendorff's Journal, 1840), who has developed, by an independent investigation, several new properties of these curves, and determined with his accustomed accuracy, in respect to an extensive series of Conchylia, the particular value of the constant angle according to which each traces its concho-spiral.

With a view to a further development of the geometrical properties of shells, I have in my paper, above referred to, investigated certain formulæ representing the equation to the concho-spiral, the volume of a conchoidal solid, the position of its centre of gravity, and the area of a conchoidal surface. In the inclosed paper I have continued these researches in respect to concho-spirals and conchoidal surfaces, and in some particulars corrected them.

King's College, London,
July 20, 1842.

Yours, &c.,
HENRY MOSELEY.

I. *The Polar Equation to a Concho-spiral.*

Let a logarithmic spiral, whose polar equation is $R = R_0 e^{\theta \cot A}$, be conceived to be wrapped upon a cone the angle at

[* An abstract of Prof. Moseley's paper here referred to was given in Phil. Mag. S. 3. vol. xiii. p. 464.]

† I have adopted the nomenclature of Prof. Naumann.