XIX.—Temperature Observations in Loch Earn. Part II. By **E. M. Wedderburn**, D.Sc., and **A. W. Young**, M.A., B.Sc.

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INTRODUCTORY.

The observations which were made in Loch Earn during August 1911 and described in the *Trans. Roy. Soc. Edin.*, vol. xlviii, p. 629, demonstrated that in that loch there was during autumn a fairly definite temperature or density discontinuity, and suggested that the loch was a peculiarly suitable one in which to observe in greater detail the boundary waves occurring at the discontinuity. The previous investigation had for its main purpose the demonstration of the existence of standing oscillations at the discontinuity, and in that was entirely successful. The observations showed, however, that in addition to the primary standing oscillations there were other less regular movements about which the observations gave little information, as they were not made at sufficiently close intervals of time.

The current observations which were made in 1911 gave a disappointingly small amount of information, and it was hoped that with improved methods of observation the nature of the boundary changes which occur would be more evident.

PLAN OF THE INVESTIGATION.

A considerable grant from the Sir John Jackson Tait Memorial Fund of the University of Edinburgh made possible an extensive investigation and the employment of new instruments. We were fortunate in securing the collaboration of a number of those engaged in the previous investigation in Loch Earn, who were therefore experienced in the work required of them. In addition to the authors, Messrs T. G. IRONSIDE, M.A., B.Sc., W. G. M'EWEN, M.A., and A. J. ROSS, M.A., were engaged in observing during the whole of the period of observation, and assistance was given for part of the time by the following, viz.: Messrs JAMES CHUMLEY, WM. GILLON, M.A., B.Sc., W. JARDINE, M.A., B.Sc., R. M'CALLUM, J. MACKIE, M.A., B.Sc., W. R. M'LENNAN, A. S. TENNANT, M.A., I. E. DE WATTEVILLE, M.A. To all of these gentlemen cordial thanks are due for enthusiastic assistance ungrudgingly given.

It was decided to concentrate the work at the Lochearnhead end of the loch. To make observations at several points on the scale attempted at Lochearnhead would have been impossible, and it was thought that with the instruments which were available numerous observations at two points about half a mile apart at the end of the loch would yield most information.

A new type of thermograph and a current-meter designed to measure vertical as well as horizontal currents were employed.

Thermograph.—Two forms of recording thermometers have hitherto been used for measuring temperatures at considerable depths below the surface of water. (1) A platinum* thermometer with Callendar recorder was used in Loch Ness with some success, but it was out of the question to use so elaborate a means of measurement for observations limited in duration to one month, partly on account of expense, and partly on account of the time which would have been required to put the instrument in proper adjustment. (2) Dr F. M. Exner used an ordinary Richard thermograph in the Wolfgangsee.† The whole instrument was protected from the action of water by being enclosed in a heavy water-tight metal box, and was accordingly very sluggish and quite unsuitable for the observation of rapid temperature changes.

The new thermograph consisted of the usual form of metallic thermometer supplied by Messrs Short & Mason for use in air, with a special clock and recording gear supplied and largely designed by Messrs J. Ritchie & Sons, Edinburgh. The registering portion of the thermograph was not protected from the water in any way, and therefore the lag of the instrument was negligible. The arm of the thermograph was made to press against the recording drum at intervals of ½ min., 1 min., 2 min., or 4 min., according as the drum was made to revolve in 3, 6, 12, or 24 hours. A prick-mark on the drum registered the temperature at the moment of contact, and the distance between successive prick-marks was ·05 inch or 1·27 mm. On the drum was placed a sheet of fine millboard, as Mr Ritchie found this to be most suitable after numerous experiments with waxed papers and other substances. Interposed between the metal drum and the millboard was a rubber sheet, and when the instrument was in proper adjustment an excellent record was obtained. The records were taken on plain unruled paper, and for the purpose of reading them a scale was drawn on tracing paper, after the style of an ordinary printed thermograph record form, with parallel lines for each degree Cent., and parallel arcs of circles marking intervals of time. With this scale the temperature could be read to within ·1° C.

The recording drum was an ingenious arrangement designed by Mr James Ritchie. The clockwork which drove the drum and pressed the arm of the thermograph against it was encased in a water-tight metal box, and water-tight metal screw plugs gave access for winding the clockwork and for altering the rate of the drum. The drum was driven by a rocking movement which, while pressing the thermograph arm against it, gave the drum a forward impulse, so that the drum moved round in a series of jerks given periodically. The junction between the clockwork and the mechanism which gave the impulse to the thermograph arm and drum was made

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* "The Temperature of the Fresh-water Lochs of Scotland," *Trans. Roy. Soc. Edin.*, vol. xlv, p. 410.

† *Sitzber. der K. Akad. d. Wiss. in Wien, math.-nat. Kl.*, Abt. ii *a*, January and December 1908.

water-tight by means of a rubber sleeve connecting the projecting part of the clockwork to the water-tight case. The arrangement may be understood from a study of the photograph in fig. 1.

At the commencement some difficulty was experienced in making the clock case water-tight. This difficulty was overcome by cementing the large joints with marine glue, but the admission of water had allowed some of the clock parts to rust, which caused an unfortunate stoppage at a rather critical time. On the whole, however, the instrument worked well, and, with the experience gained in its manufacture, it will be possible to design a reliable thermograph for the measurement of temperature under water.

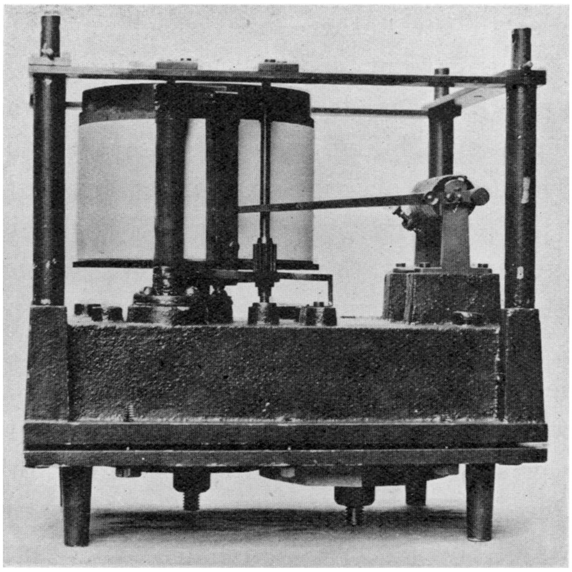


Fig. 1.—Thermograph.

The instrument was calibrated in the Physical Laboratory of the University of Edinburgh by Mr Young and Mr M'Ewen by immersion in a water bath. A difference in temperature of 1° C. caused a scale deflection of 3·4 mm. An instrument with a more open scale would have been an advantage, but the thermograph used had been designed for ordinary atmospheric work. As it was, some extremely interesting records were obtained and will be referred to later.

Current-Meter.—The current-meter referred to was of a completely new design, and was constructed by Mr A. H. Baird from drawings prepared by him on suggestions and directions given by Mr Wedderburn. Current-meters hitherto in use have only been capable of recording the horizontal component of currents, and for most purposes, *e.g.* the measurement of the flow of rivers and the strength of tidal currents in the ocean, this is sufficient. The currents in inland lakes are so slow that the usual type of current-meter is not suitable. Moreover, it was suspected that at least towards the end of lakes currents would have appreciable vertical components, and it was desired to have an instrument to measure these. The requirements of such an instrument were that it should measure (1) the horizontal component, (2) the vertical component, (3) the direction, and (4) the strength of the current. As previous experience had shown that currents in lakes varied rapidly in direction and in strength, it was thought best to have an instrument which would record these four factors at any point of time, in preference to an instrument which would give an average over a considerable time. It was accordingly decided to measure the strength of the current by the angle through which it caused a propeller to revolve, the propeller being kept in tension by a spring similar to the balance spring of a watch. The propeller was mounted in gymbals and was therefore free to move in any direction, and was kept directed towards the current by means of two sets of vanes at right angles to each other—one set of vanes being acted on by the vertical component of the current and the other by the horizontal component. When an observation was to be made, the current-meter was suspended from the boat at the desired depth by a bifilar suspension, so that the zero position of the vanes was known. By a series of trigger arrangements released by a messenger weight, the vanes and the propeller were clamped in the position which the current caused them to take up, and when the instrument was brought to the surface the directions and strength of the current could be read off from scales attached to the instrument.

Numerous experiments were made to determine the most suitable form of vanes, and ultimately the type adopted consisted of two vanes at right angles to each other, forming a cross, with a third vane at right angles to these at some distance from the propeller. One of the vanes was attached to the propeller so as to be in the same plane as the axis of the propeller, and the centre portion of this vane was cut away. Subsidiary vanes were also attached to that portion of the gymbals which was free to move about a vertical axis, and these made the instrument more sensitive to the horizontal component of the currents. A fixed propeller with its vanes inclined in the opposite direction to those of the pivoted propeller was placed alongside the latter to counterbalance the torsion which would otherwise have been exerted on the instrument by currents.

The instrument was carefully adjusted by means of counterpoise weights so that when immersed in water the vanes would remain in whatever position they were placed. Wooden floats were attached to the movable parts of the instruments to reduce their weight in water and the consequent friction on the bearings. The gymbals were set in phosphor-bronze ball bearings. The bearings of the propeller were phosphor-bronze points on agates. The spring which kept the propeller in tension consisted of a steel clock mainspring heavily silvered to prevent rust. A general idea of the instrument may be gathered from the photograph forming fig. 2.

The calibration of the instrument was, through the courtesy of Professor HUDSON BEARE, carried out in the Engineering Laboratory of the University of Edinburgh by Mr Young, Mr Ironside, and Mr M'Ewen. The propeller was towed at as uniform a rate as possible across a water tank, and the deflection corresponding to a definite rate was noted. The calibration was attended with considerable difficulty, as no means were available for towing the propeller through the water at a rate which would remain constant for any length of time. Fig. 3 gives the result of the calibration, and it will be seen that even for currents of 1 cm. per sec. considerable accuracy may be expected. This compares very favourably with the accuracy of the Ekman meters at our disposal, which could not be relied on to record satisfactorily currents of less than 3 cms. per sec.

The experimental tank was not sufficiently large to admit of the determination of the directing power of the vanes, but from repeated observation of the instrument in use it is thought that when in proper adjustment the directions registered for a current of the strength of 1 cm. per second were accurate to within ±10°. The wooden floats which have been already referred to were a temporary expedient, and, though heavily varnished, gradually altered in density during their immersion in water. This altered the disposition of density in the instrument, and necessitated constant adjustment of the counterpoises. This defect could easily be remedied in any subsequent instrument.

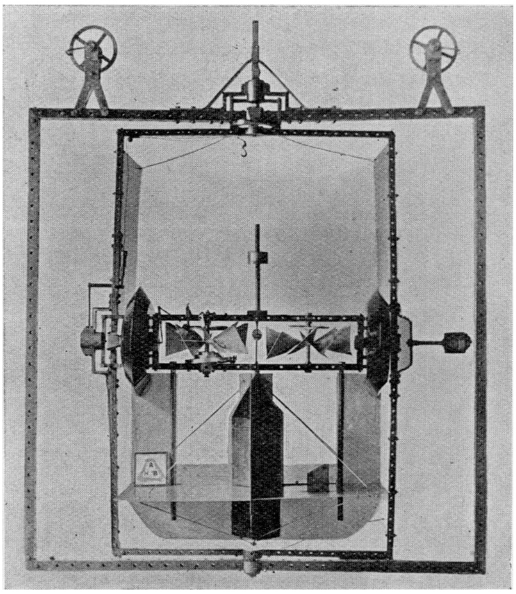


Fig. 2.—Current-Meter.

Experience, however, showed that the current-meter, with its gymbal bearings and large vanes, was rather unwieldy and sluggish. The adjustment of the vanes for registering vertical currents was a constant difficulty, and the instrument was not always certain in its action. The measurement of the strength of the current by the deflection of a spring propeller was quite successful, and this method would certainly be used if further measurements were to be made. It would be possible to dispense with directing vanes altogether by having three sets of spring propellers set in three planes in directions at right angles to one another. The movable parts of such an instrument could be made quite light and of small momentum. The whole instrument would be compact and easy to handle, and probably more reliable in its action than an instrument depending on the movement of heavy vanes.

The thermograph and the current-meter each weighed about 50 lbs. in water. Owing to their weight and size it was not practicable to use them from a single rowing boat. A raft was constructed by mooring two boats side by side, but four feet apart, and fixing over them a framework from which the instruments could be suspended. For ease in handling, both the large instruments were balanced by counterpoise weights. For measuring the depths to which the instruments were lowered measuring pulleys of ½-metre circumference, made by the late Mr A. Frazer, were used.

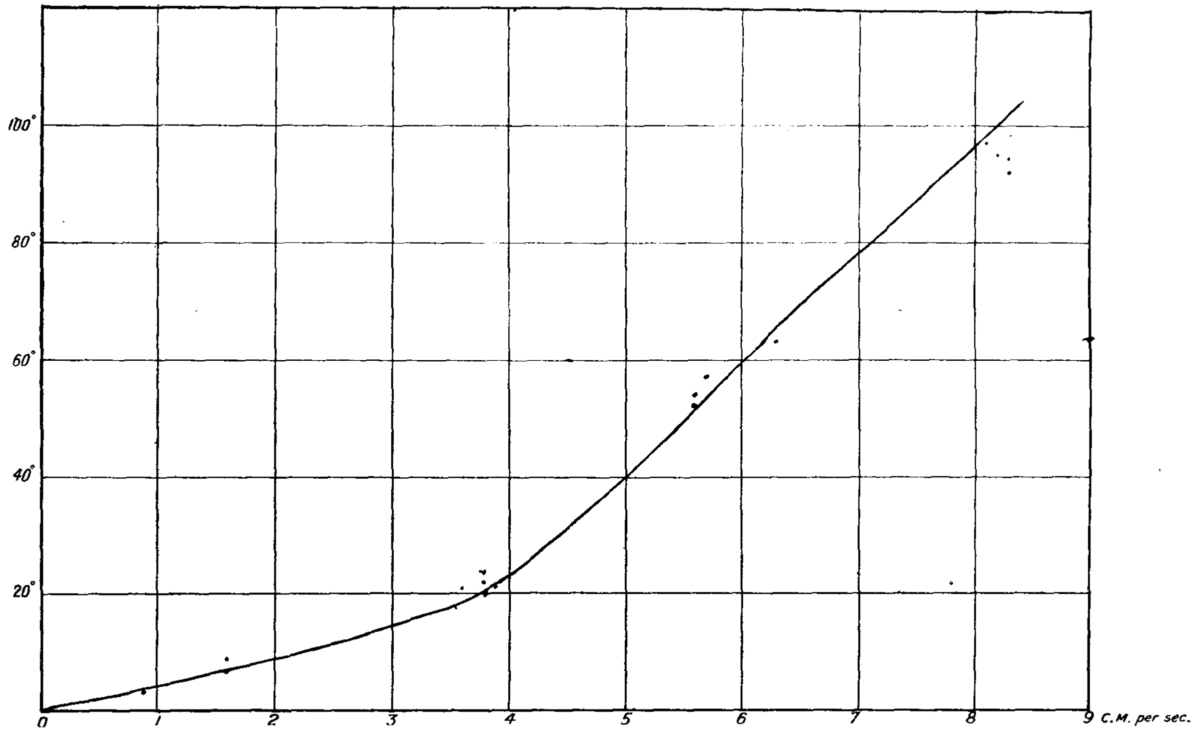
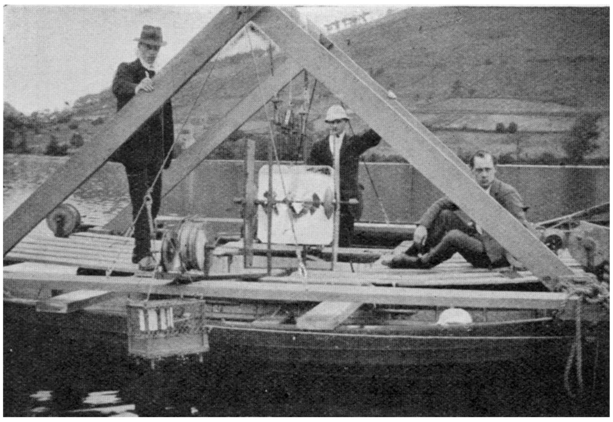


FIG. 3.—Calibration of Current-Meter.

A general idea of the whole arrangement may be gathered from the photographs forming fig. 4, the upper of which shows the thermograph and current-meter in position for lowering to any desired depth.

Six ordinary reversing deep-sea thermometers were used, two of which were lent by the late Sir John Murray, K.C.B., and one by Dr W. S. Bruce. All the instruments were carefully compared with a thermometer for which a Kew certificate had been obtained, and corrections were applied. The observations were made to the nearest twentieth of a degree C.

The raft was moored at the head of the loch by means of four anchors (one at each corner) in 30 metres of water, and another boat was moored at a distance of 650 metres down the loch in 45 metres of water, for making simultaneous observations. The positions of these two stations are shown on the map of the loch in fig. 5. Their positions were obtained by means of cross bearings.



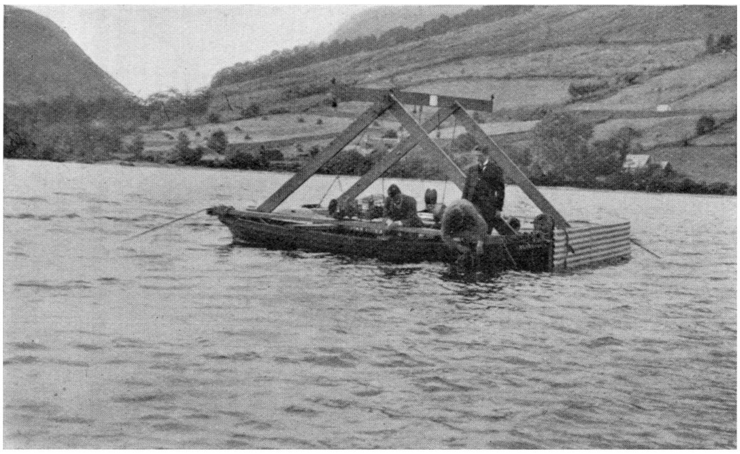


Fig. 4.

Thermograph Observations.

As only one thermograph was available it was impossible by means of it alone to follow the different temperature changes, and for this purpose observations with mercury thermometers were relied on. The thermograph was used principally to record in the neighbourhood of the discontinuity where it was anticipated rapid variations of temperature would occur—for apart from the standing oscillations at the surface of discontinuity there were certain to be at times slow-travelling waves caused by local depressions in the upper layer.

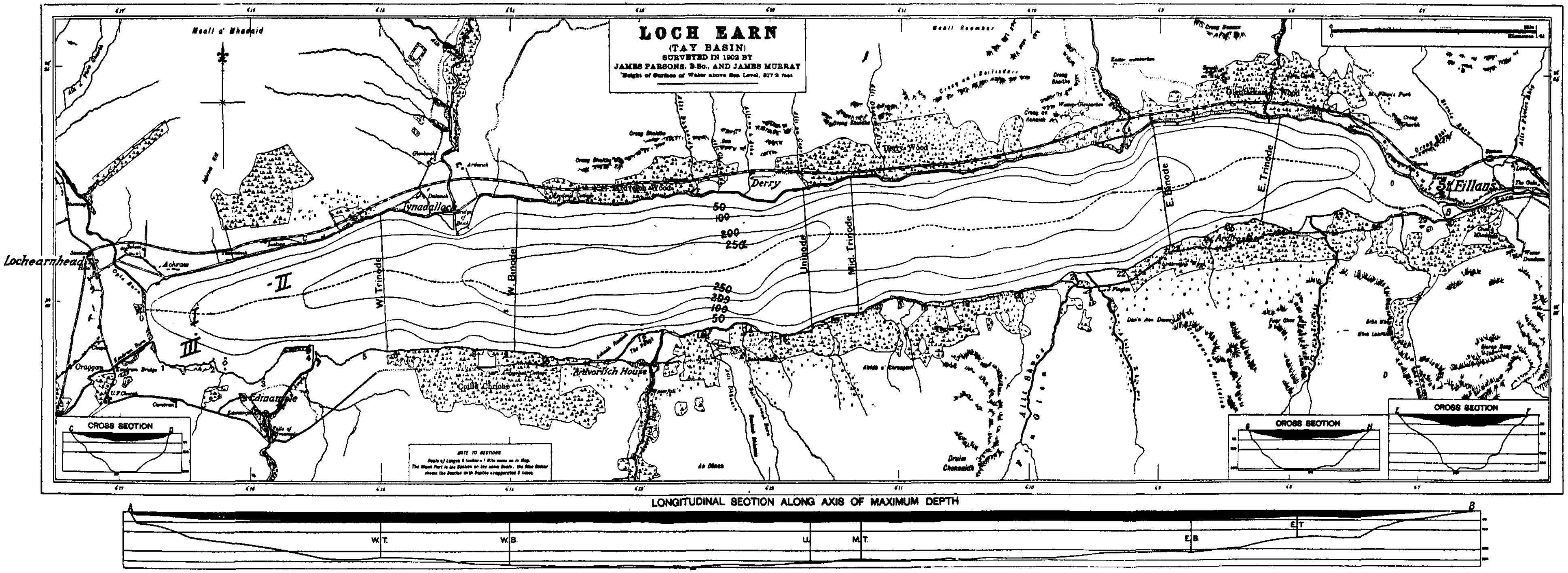

LOCH EARN
(TAY BASIN)
SURVEYED IN 1902 BY
JAMES PARSONS, B.Sc., AND JAMES MURRAY
Height of Surface of Water above Sea Level, 317·2 feet
Lochearnhead
Achraw
Tynadalloch
Derry
St. Fillans
Ardvorlich House
Coille Chriche
Edinample
Craggan
Meall a' Mhadaid
Meall Reamhar
Drum Chonnaidh
An Dinan
Allt Bhàlachain
W. Trinode
W. Binode
Uninode
Mid. Trinode
E. Binode
E. Trinode
50
100
200
250
II
III
CROSS SECTION
LONGITUDINAL SECTION ALONG AXIS OF MAXIMUM DEPTH
A
B
W.T.
W.B.
U.
M.T.
E.B.
E.T.

FIG. 5.

Fig. 6 contains examples of some thermograph records. For purposes of reproduction the prick marks of the record were inked over, then traced and photographed.

In examining the records it must be borne in mind that the records in themselves give no indication of the distance through which isotherms move, but simply record the variations in temperature at a given point which may be due to oscillations of the isotherms. If, however, the vertical temperature distribution during the period of the record is known approximately, some idea of the amplitude of the oscillation may be gained.

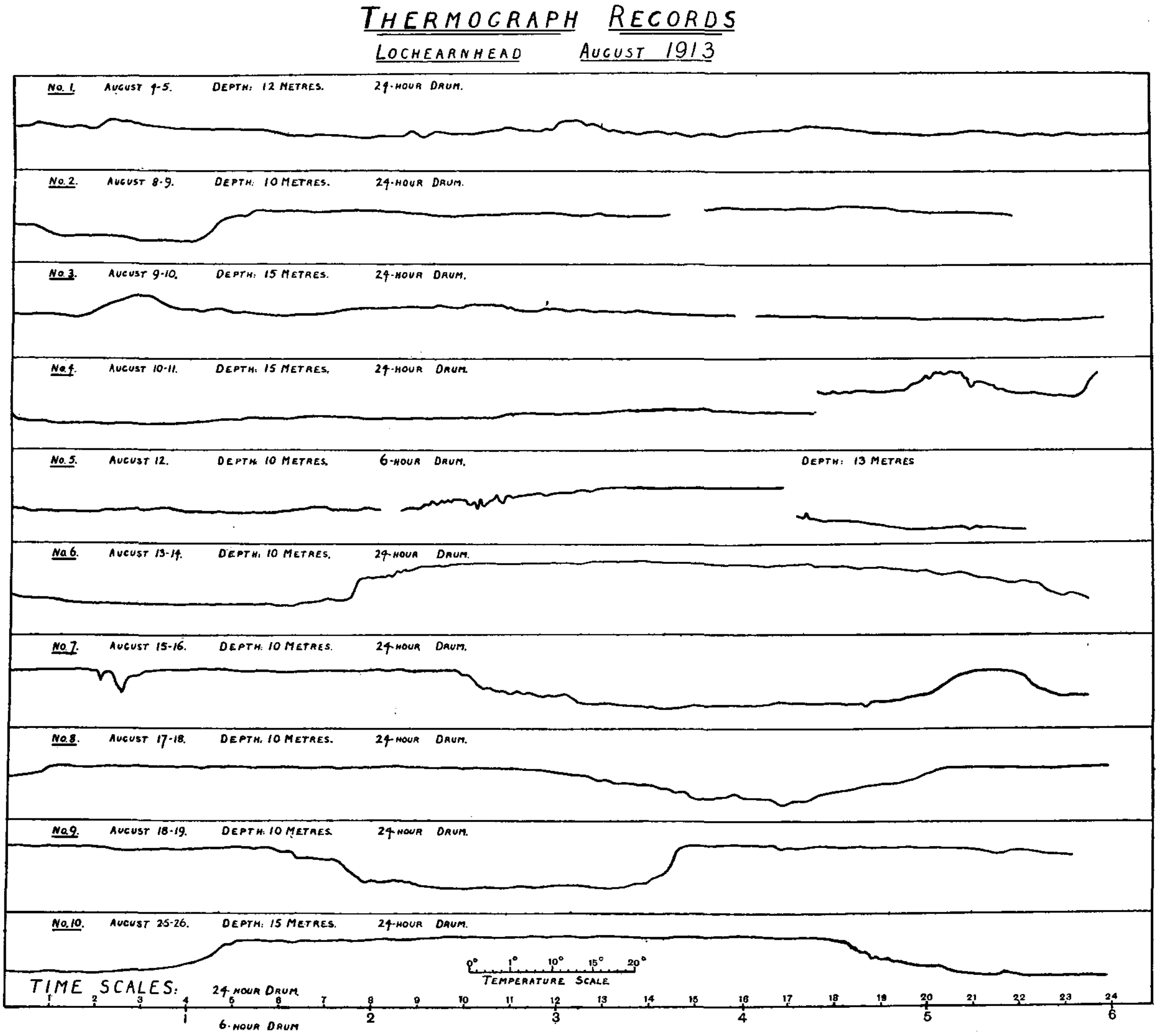


FIG. 6.

1. Record at 12 metres for 24 hours commencing 19 h. 31 m. on 4th August. This shows a slight oscillation, with a maximum about 23 h. on 4th August and a second maximum about 10 hours later. The total variation in temperature does not exceed 2° C., but there are now and again rapid changes, showing that the temperature has

changed by as much as a third of a degree in the interval between successive pricks, *i.e.* 4 minutes.

2. Record at 10 metres for 24 hours commencing 19 h. 8 m. on 8th August. About 4 hours after the beginning of this record there is a rapid rise in temperature caused by the discontinuity passing across the 10-metre depth. It is seen that there is a change of about 4° C. in an hour and a half. Although the change is not perfectly uniform, there is little evidence of rapid movements such as would result from boundary waves at the surface of discontinuity, and it must be assumed that none such occurred in this instance.

3. Record at 15 metres for 24 hours commencing 18 h. 21 m. on 9th August. The rise in temperature shown about three hours after the beginning of the record is probably due to an easterly wind depressing the isotherms at the west end of the loch.

4. Record for 24 hours commencing 18 h. 36 m. on 10th August. The first part of this record was made at a depth of 15 metres, and shows a uniformity which is markedly absent in the latter part of the record, when the instrument was raised to 10 metres, and was in the discontinuity layer. The presence of boundary waves is well shown in this record.

5. Record for 6 hours commencing at 12 h. 54 m. on 12th August. The first part of this record was made at a depth of 10 metres. A train of boundary waves has evidently occurred about 3 hours after the commencement of the observation, and it is clear that the interval of 1 minute between successive pricks has been too long to pick up all the variations. The last part of the record is taken at a depth of 13 metres.

6. Record at 10 metres for 24 hours commencing at 16 h. 38 m. on 13th August. This record shows very clearly the passage of the discontinuity past the instrument, and there is distinct evidence of boundary waves in the upper part of the layer of discontinuity with a period of about 11 minutes. The observations with mercury thermometers for the same period are shown in fig. 8, and will be explained later.

7. Record at 10 metres for 24 hours commencing at 16 h. 40 m. on 15th August. This record shows about 10 hours after the commencement the rise of the isotherms past the instrument, and about 10 hours later their much more gradual fall, and the usual embroideries are present. About 2 hours after the commencement there has been a sharp drop in temperature, which may have been due to a solitary boundary wave, or to a bore such as was described in the "Experimental Investigation of the Temperature Changes occurring in Fresh-water Lochs," *Proc. Roy. Soc. Edin.*, vol. xxviii, p. 2.

8. Record at 10 metres for 24 hours commencing at 17 h. 0 m. on 17th August. This record also shows an oscillation with a period of about 10 hours, and embroideries with a period of about 25 minutes.

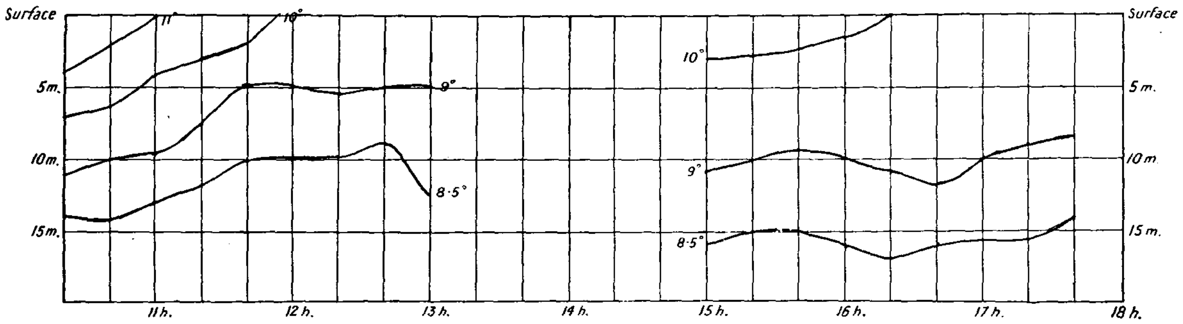
9. Record at 10 metres for 24 hours commencing at 18 h. 5 m. on 18th August. This record shows that a well-marked seiche is in progress, but the smaller embroideries are not so much in evidence as might have been expected. The variation of temperature in this record is about 5·6° C., and is approximately from 15·6° C. to 10° C. No observations with mercury thermometers are available either on 18th or 19th August at the hour when the large changes are shown on the thermograph.

10. Record at 15 metres for 24 hours commencing at 15 h. 5 m. on 25th August. The observations with mercury thermometers for the same period are shown in fig. 10, and will be explained later. Attention may be drawn to the uniformity of the curves above and below the discontinuity, and the small embroideries which appear while the discontinuity is passing the instrument.

CURRENT OBSERVATIONS.

In addition to the observations made with the new current-meter, a number of observations were made with Ekman current-meters, of which two were available.

With the exception of a solitary observation near the surface on 12th August, 22nd August was the only date on which currents of 2 cms. per sec. were recorded. On 22nd August a steady westerly wind was blowing all day, force 6–7 on the Beaufort scale, and, as the occasion seemed particularly favourable for the formation of currents, a large number of observations were made.



22nd August 1913.

FIG. 7.

Fig. 7 illustrates the temperature conditions on this date. It represents the depth at which the isotherms for 8·5°, 9°, 10°, and 11° C. occurred during the day. The surface temperature varied from about 10° to 11° C., which was very low, indicating that owing to the action of the westerly wind the warm surface waters had been accumulated at the other end of the loch. On 20th August the surface temperature at Lochearnhead was over 15° C.

The following table gives all the observations made by means of the Ekman meters on 22nd August. A number prefixed to a direction indicates that there were two determinations of that direction, *e.g.* 2 S 80 W means that the current-meter twice during the observation indicated a current of S. 80° W.

TABLE I.

Observations with Ekman Meters on 22nd August 1913.

Hour of Commencement.	Duration of Observation in Minutes.	Depth in Metres.	Direction Indications.	Velocity in cms. per sec.
h m				
10 26	27	10	2 S 80 E	1·7
11 6	23	5	...	·9
11 15	12	15	1 S 30 E; 2 S 40 E; 1 S 60 E; 1 N 10 E; 1 N 20 E; 1 N 20 W; 1 N 30 W	7·1
11 27	24	5	2 N 70 W; 4 N 60 W; 1 N 50 W; 2 S 60 W; 1 S 50 W	1·8
11 44	14	15	1 S 40 E; 1 S 70 E; 1 N 30 E; 1 N 40 E	3·9
11 57	19	4	1 S 40 W; 1 N 20 W	4·7
12 6	10	10	...	4·3
12 20	10	6	1 S 20 W	2·4
12 23	11	10	1 N 20 E; 2 S 50 E; 1 S 30 E; 1 S 20 E	6·8
12 24	23	8	1 S 20 W; 1 S 60 E; 1 N 80 E	2·9
12 40	9	20	...	·7
13 0	160	20	1 N 20 W; 2 N 40 W; 3 N 50 W; 1 S 70 W; 1 S 40 W	6·2
13 3	127	10	1 S 40 W; 1 S 30 E; 2 S 40 E; 2 S 50 E; 2 S 70 E; 1 N 50 E	3·7
15 21	33	20	1 N 30 W; 3 N 20 E; 1 E	2·5
15 51	24	20	1 N 30 E; 1 N 40 E; 1 N 40 E; 1 S 50 E	4·5
16 23	6	0	...	4·9
16 25	6	2	1 S 60 W; 1 S 80 W	4·6
16 34	6	10	...	1·9
...	5	5	...	1·6
16 43	5	20	...	2·7
...	5	15	...	·7
16 52	6	28	1 N 50 E; 1 N 80 E	5·5
...	6	25	1 S 50 E	3·9
17 4	35	20	1 S 30 W; 1 S 40 E; 1 S 50 E; 1 N 50 E; 1 N 60 E; 1 N 70 E	3·4
...	35	15	1 N 10 E; 1 N 20 E; 1 N 30 W	2·2

The new current-meter was difficult to use in stormy weather on account of its size, and unfortunately was slightly damaged by a wave on the morning of the 22nd. It was not in proper adjustment until the afternoon in consequence of this.

The observations made with it are given in the following table :—

TABLE II.

Observations with New Current-Meter on 22nd August 1913.

Hour.	Depth in Metres.	Horizontal Component.	Vertical * Component.	Velocity in cms. per sec.
h m				
15 2	15	N. 49° E.	– 14°	2·8
15 22	20	N. 17° E.	– 47°	2·4
16 10	5	N. 54° W.	3°	4·3
16 26	15	S. 73° W.	– 16°	·5
16 34	10	N. 18° W.	– 12°	2·5
17 0	4	S. 60° W.	– 26°	·5
17 45	·5	N. 70° W.	– 6°	5·2

* 0° indicates a horizontal current. The negative sign indicates a downward current and the positive sign an upward current.

For the purpose of comparing the two kinds of current-meters the observations at 20 metres about 15 h. 20 m. are useful. The Ekman meters indicate a current mainly from the north-east of 2·5 cms., and the new meter also gives a north-easterly current with a velocity of 2·8 cms. This observation indicates that the observations with the new meter are comparable to those made with the Ekman meters.

The most interesting of the observations is that at 20 metres commencing at 13 h., which gave a velocity of 6·2 cms. The directions of the currents were mainly westerly, *i.e.* in the same direction as the wind. No temperature observations are available during the period of this observation, but it is evident that there has been a disturbance of some sort in the level of the isotherms, and it is likely that the relatively large velocity is due to the passage of a boundary wave or bore. Another example of strong currents associated with considerable temperature changes will be referred to later.

TABLE III.

Observations with New Current-Meter.

Date.	Hour.	Depth in Metres.	Horizontal Component.	Vertical Component.	Velocity in cms. per sec.	Date.	Hour.	Depth in Metres.	Horizontal Component.	Vertical Component.	Velocity in cms. per sec.
	h m						h m				
5/8/13	18 0	8	N. 41 E.	6	2·2	14/8/13	20 5	3	N. 83 E.	9	3·5
6/8/13	10 15	10	S. 37 E.	0	2·2		20 25	5	E.	− 8	2·5
	10 40	15	N. 80 W.	0	2·3	19/8/13	11 45	·5	S. 75 E.	−27	3·0
	12 20	15	N. 12 E.	− 3	2·8		12 0	2	N. 20 W.	12	2·5
	17 20	5	S. 65 W.	90	2·2		14 38	·5	N. 88 W.	−12	4·1
7/8/13	12 30	15	N. 84 E.	86	2·9	20/8/13	13 2	4	S. 14 E.	2	2·2
11/8/13	18 50	10	N. 85 W.	−67	2·8		13 45	10	S. 18 E.	− 4	5·9
	19 30	10	N. 45 E.	−82	2·5		14 18	10	S. 38 E.	− 4	4·0
	22 5	3	N. 17 E.	−64	2·2		14 32	12	N. 78 E.	− 4	3·6
13/8/13	11 25	10	N. 52 E.	− 2	3·1		15 18	8	S. 66 E.	2	2·5
	13 40	20	S. 42 E.	− 3	5·2	25/8/13	15 20	10	N. 66 E.	− 4	4·4
14/8/13	1 7	10	S. 18 E.	− 8	2·8		16 10	12	S. 43 E.	− 4	2·2
	1 45	5	N. 76 E.	−10	5·5	26/8/13	15 45	10	N. 28 W.	−14	2·4
	2 10	3	S. 70 E.	−10	4·0		16 0	14·5	N. 55 W.	−16	3·3
	3 10	5	N. 48 E.	−22	3·1	29/8/13	12 55	15	S. 86 W.	−47	2·7
	4 25	5	N. 12 E.	2	2·8		13 5	10	N. 59 W.	−22	3·6
	4 35	5	N. 12 E.	6	3·1		13 25	10	N. 82 W.	−22	3·7
	5 0	3	N. 12 E.	− 8	3·1		18 10	18	N. 60 W.	−18	3·0
	5 5	2	N. 34 E.	− 2	2·4		18 50	10	N. 87 W.	−40	2·4
	19 55	7	N. 81 E.	64	4·8		19 0	5	N. 1 W.	−26	3·2
							19 10	2	S. 85 E.	−15	3·6

Above 8 metres the currents are entirely in a westerly direction, from which it may be deduced that they are caused by the action of the wind. Below this depth there is great variation of direction, but the currents are mainly easterly, indicating that they are of the nature of return currents.

The above table contains all the observations made with the new current-meter where the velocities of the current exceeded 2 cms. per second, except those on 22nd August. Of the 41 observations there given, 22 show currents in which the vertical component is ±10° or less, 28 in which the vertical component is less than ±20°, and 33 in which it is less than ±30°. No means were available of testing the zero of the scale for measuring the vertical components, and it may be that there was a considerable zero error; but it is thought that the error in the observations could not exceed ±30°.

Excluding observations which do not show a vertical component of more than ±30°, and including one observation on 22nd August, there are only nine occasions on which a distinct vertical component was observed, out of a total of 179 observations, viz.:—

	h	m		°	
6th August	17	20 –	5 metres	+ 90	2·2 cms.
7th ,,	12	30 –	15 ,,	+ 86	2·9 ,,
11th ,,	18	50 –	10 ,,	− 67	2·8 ,,
,, ,,	19	30 –	10 ,,	− 82	2·5 ,,
,, ,,	22	5 –	3 ,,	− 64	2·2 ,,
14th ,,	19	55 –	7 ,,	+ 64	4·8 ,,
22nd ,,	15	22 –	20 ,,	− 47	2·4 ,,
29th ,,	12	55 –	15 ,,	− 47	2·7 ,,
,, ,,	18	50 –	10 ,,	− 40	2·4 ,,

Considering, first, the observations which show an upward current, the temperature observations on 6th and 7th August are not such as to suggest an explanation, save that at about the time of the observation there was a slight upward movement of the isotherms. On 14th August the upward current is so well marked that it can hardly be supposed that the record is due to instrumental error; and yet the occasion is eminently one on which a downward current might have been predicted. For at the time of observation the discontinuity was in process of making a very rapid descent, as is seen in fig. 8. The observation was made at a depth of 7 metres, whereas the depth of the discontinuity was at least 10 metres, and the only suggestion one can offer is that the rapid descent of the discontinuity caused eddies in all directions.

Turning to the downward currents on 11th August. There had been easterly winds all day, and these became stronger about 18 h. The discontinuity was well below 10 metres, and the thermograph which was recording at this depth showed uniformity of temperature. There seems little reason to doubt but that the observations show a steady downward motion of the water at the lee end of the loch, prior to the formation of the return current.

The downward current on the 29th is also easily explained by the fact that a strong easterly wind was blowing on that date, and reference to fig. 10 will show that the isotherms were rapidly falling. The horizontal component of the currents was mainly westerly, indicating that the observation was made within the return current. The return current seems to have been very near the surface. The observation at 2 metres shows an easterly current directly due to the wind, but at 5 metres the current is from the north, as if the return current was beginning to form.

The most interesting current observations are those made early on the morning of 14th August. The first indication of a current was obtained from the manner in which the wire to which a thermometer was attached bellied out. Observations with the current-meter were at once commenced, and strong easterly currents which were practically horizontal were found at depths down to 10 metres. Observations below that depth were also made, and showed either slow currents or none at all. The currents were well marked for about four hours, after which they seemed suddenly to cease.

During this time the loch was dead calm, and a surface current could hardly be detected. About 20 h. on the evening of the same day strong currents were also found from the east near the surface, and again at this hour there was a sudden fall in the isotherms.

From the observations it may be concluded that at the windward end of a loch the currents produced by even moderately strong winds have not a large vertical component, but that at the lee end there is quite a considerable vertical component. We may also conclude that near the end of a loch, when owing to the presence of a temperature seiche or other causes the isotherms are rising or falling rapidly, the vertical component of the currents so produced may be considerable, and that rapid movements of the isotherms produce eddies or vortices. In general, however, it must be said that the vertical component of loch currents is not considerable.

Observations with Mercury Thermometers.

A large number of observations was made with ordinary reversing thermometers at each of the two anchored boats, and on various occasions observations were made from boats anchored at intermediate points. The observations made at the beginning of the month were of a preliminary character to determine the temperature distribution and the nature of the changes in progress, but it was found that to be of any service observations would require to be made simultaneously for a longer period than 12 hours. Accordingly, from 10 h. on 13th August until 21 h. on 15th August observations were carried on at each of the two boats. Two thermometers were employed on each boat, and about 20 observations were made each hour.

From the observations so obtained curves were drawn showing the time variation in temperature at various depths; from this curve a second curve showing the oscillations in the level of various isotherms. Figs. 8 and 9 show the isotherms for 10°, 12°, and 14° at each of the stations.

Prior to the commencement of the observations on 11th and 12th August there were variable winds and calms. On the morning of the 13th it was calm; a moderate westerly wind sprang up about 11 h. 30 m. and lasted until 22 h. 30 m., after which there were calms or light winds until 22 h. on the 14th. A steady east wind then sprang up which fell away about 9 h. on the 15th, and during the remainder of the series there was a calm.

A considerable oscillation with a period of about 18 hours appears to have been in progress when the observations were commenced, and to have been damped out by the east wind which sprang up on the evening of the 14th.

The most noticeable point about the oscillation is that the isotherms fall very rapidly and rise gradually. This is more noticeable in the observations at No. 1 than at No. 2 station. The bottom line in fig. 8 represents the difference in level between the two isotherms at the two stations, and from this diagram it is seen that while the rapid descent is first felt at No. 2 station, the fall at No. 1 station very soon exceeds it. To a considerable extent, as will be shown later, the rapid fall of the isotherms is due to the combination of oscillations of different nodality, but if this were all there would not have been so much difference between the curves for the two stations.

The theory of seiches in lakes (both ordinary seiches and temperature seiches) assumes that the amplitude of the oscillation is small compared with the depth of the lake. This is no longer true in the case of temperature oscillations. The oscillation under discussion has an amplitude of about 16 metres, and the maximum depth of the lake below the discontinuity is only about 80 metres. Towards the end of the lake, where it is much shallower, the ratio between amplitude and depth is by no means small.

It will be sufficient to consider the case of an ordinary seiche where this ratio is not small, though it may still be assumed that the vertical acceleration of particles is small and negligible; for we have seen that the same process may be applied to the discussion of temperature oscillations as to ordinary seiches.

Using the notation of CHRYSTAL'S Memoirs, but assuming for the sake of simplicity that the lake is of rectangular cross-section and uniform breadth, the depth at any point being h, the method described in LAMB'S *Hydrodynamics*, 3rd ed., p. 245, gives for the seiche equation

$$\frac{\partial^2\xi}{\partial t^2}=gh\frac{\partial^2\xi}{\partial x^2}\bigg/\left(1+\frac{\partial\xi}{\partial x}\right)^3.$$

Since this equation is not linear a simple harmonic type of oscillation is not possible, and as the oscillation progresses there will be a change of type. In the deeper parts of the loch the ratio between amplitude and depth is smaller, and a change of type will be the more easily averted by a slight adjustment of the velocities of the water particles. Towards the ends of the loch, however, when the amplitude is large, considerable distortion of the wave surface may be expected. The water at station 2 was much deeper than at station 1, and it is to be expected that there should be considerable and irregular difference between the level of the isotherms at these two points. The suddenness of the fall of the isotherms which has been noted is also probably largely due to the change in type of the wave in shallow water. Where the amplitude of the oscillations is large, it is probable that the isotherms will move very rapidly indeed, and possibly the wave may break, just as travelling surface waves break on a shelving beach. If there is such breaking of the density wave strong currents will probably occur, and the conjunction of strong currents with rapid movements of the isotherms has been noted on page 755.

The observations during the week following the series of 13th to 15th August do not call for remarks other than have been already made in the discussion of current observations, but during the week arrangements were made for the final series from 24th to 29th August. For the first three weeks the number of observers never exceeded 5, but for the last series 12 observers were fully employed. During practically the whole of the time 3 thermometers were kept in use on each boat, and about 36 temperature observations were made hourly in each boat by 2 observers. It may be mentioned, as indicating the strenuous nature of the work, that the observers who were on night duty observed for 12 hours without intermission. On 28th August for 12 hours observations were made at a third station marked 3 on the map.

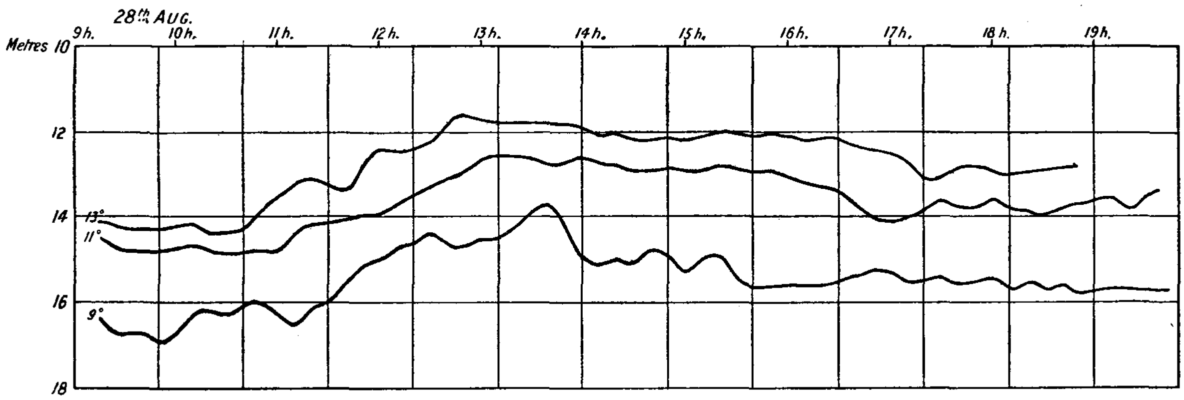


Fig. 12.—Observations at Station 3.

In all, during the week, about 9000 temperature observations were made, and these are shown on the lower half of figs. 10, 11, and 12, which represent the variation of temperature with time at various depths. Curves of the variation of temperature with depth at any moment, 3000 in number, were then drawn from these curves for intervals of 5 minutes during the series, and from these temperature-depth curves the curves showing the oscillations of isotherms were drawn, being the upper portions of the diagrams.

Notes showing the weather conditions during the series will be found on fig. 10, but during most of the time, with the exception of the last day, there was an absence of strong winds, and the conditions were as suitable for observing a temperature seiche as one could well hope for.

On 22nd August, as has been seen, there was a strong westerly wind, and the surface temperature at Lochearnhead fell from over 15° C. to 9·6° C., thus showing that a large quantity of warm water had been transferred to the east end of the loch.

On the 23rd the west wind still continued, but was not so strong. Unfortunately, no temperature observations were made on that date, as the observers were having a well-earned holiday in preparation for the strenuous week which was to follow. The thermograph, which had been out of gear, was, however, set to work again at 10 metres on the evening of the 23rd, and at 19 h. recorded a temperature at that depth of about 12·7° C. The temperature at this depth at 19 h. on 22nd August was 8·6° C. It is seen from examination of fig. 10 that the mean level of the 12·7° isotherm was about 10 metres, so that apparently the isotherms at 19 h. on the 23rd were approximately in their normal positions.

The thermograph record for the night 23rd to 24th August (fig. 13) shows that the temperature at 10 metres did not vary more than 1·7° C., so that apparently no large oscillation was in progress. In the first 24 hours after observations were commenced on 24th August the temperature at 10 metres varied 2·5° C., and as even in the record obtained by means of the thermograph 1·3° of the variation took place after 7 h. 50 m. on the morning of the 24th, it would appear that the observations were begun practically at the commencement of a series of oscillations.

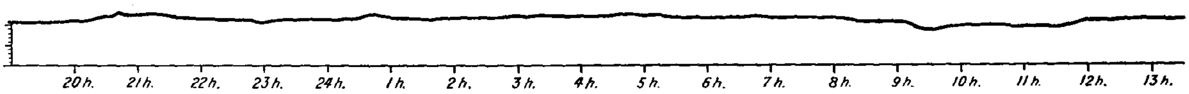


FIG. 13.—Thermograph Record at 10 metres—23rd-24th August 1913.

It is remarkable that this should be so, for one would have expected that the oscillation would have commenced immediately the isotherms began to swing back when the wind moderated. Apparently, however, this was not so, and the only explanation which can be suggested is that when the wind moderated a solitary wave was started at the Lochearnhead end which, after being reflected at the east end of the loch, gradually resolved itself into a standing oscillation. The experimental investigations made by Mr WEDDERBURN in 1905 support this.

CALCULATION OF THE PERIOD OF THE TEMPERATURE SEICHE.

The period of a temperature seiche can be approximated to by the method described in the communication on the temperature seiche, *Trans. Roy. Soc. Edin.*, vol. xlvii, part iv, p. 635, without the labour of finding the seiche equations for the loch.

If T is the period of the temperature seiche in August 1913 and t the period in 1911,

$$\mathrm{T}=t\,\frac{\text{area of surface of discontinuity in 1913}}{\text{area of surface of discontinuity in 1911}}\;\frac{(h\text{ of normal curve 1911})^{\frac{1}{2}}}{(h\text{ of normal curve 1913})^{\frac{1}{2}}}.$$

The average distribution of temperature on 28th August 1913 at the anchored boat was as follows :—

0 metres	15·0° C.	16 metres	10·0° C.
5 ,,	14·6° C.	18 ,,	9·4° C.
10 ,,	14·0° C.	20 ,,	9·0° C.
12 ,,	13·5° C.	25 ,,	8·1° C.
14 ,,	11·1° C.		

As this was after a prolonged period of calm it may be assumed that this distribution is typical for the whole loch. Applying the principles described in the previous Loch Earn calculation, the discontinuity must be taken as occurring at a depth of 12 metres. The area of the surface of discontinuity at this depth is 1830 × 48010 square feet as compared with an area of 1683 × 48010 square feet for a depth of 16 metres in 1911.

Assume that the minimum of the normal curves is proportional to $\sigma(v)$ at the deepest point in the loch, *i.e.* at sounding-line 10. In 1911 the value of $\sigma(v)$ for this line was ·04104 units, and a similar calculation for the distribution of 1913 gives ·02850 units. The calculated period for the temperature seiche in 1911 was 14·99 hours, hence, using the above formula, the corresponding period for 1913 is

$$
\begin{aligned}
T &= \frac{14{\cdot}99 \times 1830 \times \sqrt{4104}}{1683 \times \sqrt{2850}} \text{ hours} \\
&= 19{\cdot}6 \text{ hours.}
\end{aligned}
$$

This is the period of the uninodal seiche, and, assuming that the ratio between the periods of the uninodal and binodal seiches is the same for the temperature as for the ordinary seiche, the period of the binodal seiche would be 11·0 hours. It will be seen later that the agreement between the observed and the calculated period is good in the case of the uninodal period, though not so good in the case of the binodal period—probably because the assumptions on which the calculation is based do not hold in the case of the higher nodalities as the ratio between the wave-length and the depth of the loch becomes smaller.

Periodogram Analysis.

The observations at No. 2 boat during the last week were subjected to periodogram analysis to determine the main periods which were present. In the case of an oscillation which dies away so rapidly as do temperature seiches the ordinary periodogram method as used for the analysis of astronomical observations cannot be used, as there is not a sufficient number of periods to work with. A slight modification of the method gives a good idea of the component periods of the oscillation.

For the purpose of the analysis the variation in depth of the 11° isotherm was considered, as this isotherm seemed to be most nearly at the discontinuity on its lower side, and therefore free from surface disturbances. The average depths of this isotherm during successive intervals of 50 minutes were taken from the graph drawn from the observations, and since the determination of the period is not affected by the addition or subtraction of a constant from these depths, the constant 10 was subtracted to make the computation simpler.

The numbers obtained are given in the following table :—

TABLE IV.

No. of Interval.	No. Specifying Depth.	No. of Interval.	No. Specifying Depth.	No. of Interval.	No. Specifying Depth.	No. of Interval.	No. Specifying Depth.
1	168	36	70	71	619	106	319
2	166	37	96	72	611	107	349
3	217	38	173	73	670	108	353
4	423	39	330	74	612	109	349
5	588	40	431	75	635	110	359
6	552	41	609	76	661	111	372
7	535	42	834	77	641	112	395
8	475	43	866	78	595	113	449
9	207	44	894	79	585	114	493
10	137	45	962	80	578	115	522
11	88	46	941	81	483	116	469
12	108	47	922	82	447	117	431
13	246	48	871	83	432	118	379
14	243	49	851	84	411	119	381
15	344	50	842	85	392	120	336
16	474	51	791	86	379	121	330
17	581	52	748	87	398	122	356
18	673	53	715	88	408	123	364
19	729	54	667	89	545	124	355
20	750	55	607	90	606	125	377
21	791	56	542	91	600	126	363
22	742	57	479	92	599	127	335
23	701	58	409	93	550	128	333
24	578	59	356	94	430	129	299
25	550	60	296	95	453	130	318
26	557	61	227	96	454	131	359
27	569	62	204	97	440	132	382
28	534	63	290	98	472	133	466
29	582	64	502	99	500	134	554
30	534	65	558	100	494	135	572
31	448	66	687	101	512	136	624
32	400	67	808	102	478	137	610
33	346	68	727	103	403	138	586
34	167	69	689	104	389	139	585
35	86	70	679	105	359	140	620

It will be seen that there are only 140 observations in all, whereas in an astronomical investigation there may be a thousand or more. Hence if the largest trial period covers, say, 25 of these observations, there are only 5 complete periods available. If the smallest trial period covered 10 observations, 5 periods would only make use of 50 of the observations, but it was thought best to make use of all the available observations for each trial period. By this procedure the smaller periods will have greater weight given to their amplitudes than the longer periods, but it is easily seen that the increase of amplitude will be roughly inversely proportional to the length of the trial period. Hence, to reduce all the amplitudes to a uniform scale each must be divided by the number of "laps" used in determining the amplitude.

Examples are given of the 10 and 20 trial periods in the following tables :—

TABLE V.

Row 1	168	166	217	423	588	552	535	475	207	137
2	88	108	246	243	344	474	581	673	729	750
3	791	742	701	578	550	557	569	534	582	534
4	448	400	346	167	86	70	96	173	330	431
5	609	834	866	894	962	941	922	871	851	842
6	791	748	715	667	607	542	479	409	356	296
7	227	204	290	502	558	687	808	727	689	679
8	619	611	670	612	635	661	641	595	585	578
9	483	447	432	411	392	379	398	408	545	606
10	600	599	550	430	453	454	440	472	500	494
11	512	478	403	389	359	319	349	353	349	359
12	372	395	449	493	522	469	431	379	381	336
13	330	356	364	355	377	363	335	333	299	318
14	359	382	466	554	572	624	610	586	585	620
	6397	6560	6715	6718	7005	7092	**7194**	6988	6988	6980

The largest divergence is from 6397 to 7194, which gives an amplitude of 797. This, divided by the number of "laps" used—*i.e.* the number of rows—gives as "reduced amplitude" $\frac{797}{14}=57$.

TABLE VI.

Row 1	168	166	217	423	588	552	535	475	207	137	88	108	246	243	344	474	581	673	729	750
2	791	742	701	578	550	557	569	534	582	534	448	400	346	167	86	70	96	173	330	431
3	609	834	866	894	962	941	922	871	851	842	791	748	715	667	607	542	479	409	356	296
4	227	204	290	502	558	687	808	727	689	679	619	611	670	612	635	661	641	595	585	578
5	483	447	432	411	392	379	398	408	545	606	600	599	550	430	453	454	440	472	500	494
6	512	478	403	389	359	319	349	353	349	359	372	395	449	493	522	469	431	379	381	336
7	330	356	364	355	377	363	335	333	299	318	359	382	466	554	572	624	610	586	585	620
	3120	3227	3273	3552	3786	3797	**3916**	3701	3522	3475	3277	3543	3442	3166	3219	3294	3278	3287	3466	3505

The largest divergence is from 3120 to 3916, which gives an amplitude of 796. This, divided by the number of laps used, is $\frac{796}{7}=114$.

When the reduced amplitudes are plotted against the corresponding trial periods the result is as shown in fig. 14. This shows two distinct maxima at about 10 hours and at about 19½ hours—a good agreement with the period as calculated.

It will be noted that the maximum showing the binodal period is much sharper than the uninodal maximum. This is easily explained when it is remembered that the binodal period is defined by about 12 periods of observation, while the uninodal period is defined by only 5. To use the well-known analogy of the diffraction grating, it is as if there had been used for the determination of a wave-length a grating of double the number of lines. That this is the explanation is seen from the small dotted curve on fig. 14 at period 10. This was derived from the first half of the observations, when the amplitude was considerably greater than during the second half. The maximum period as far as the periodogram of these observations indicates is not much different: but from the point of view of the periodogram itself, it will be noticed that the use of only half the number of observations has lessened the "steepness" of the curve to a very great extent.

PERIODOGRAM ANALYSIS.

11° ISOTHERMAL. 1913 AUGUST 24-29.

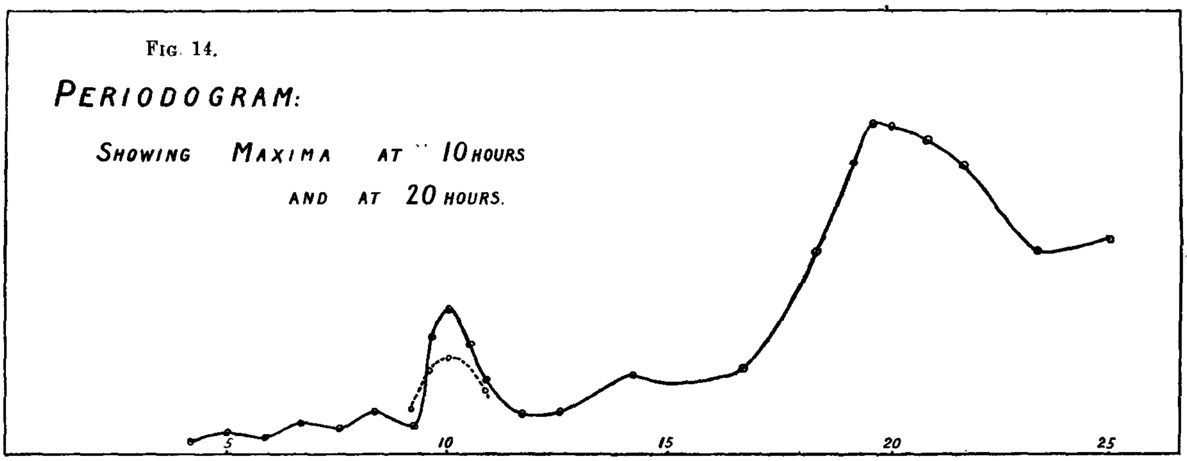


Fig. 14.

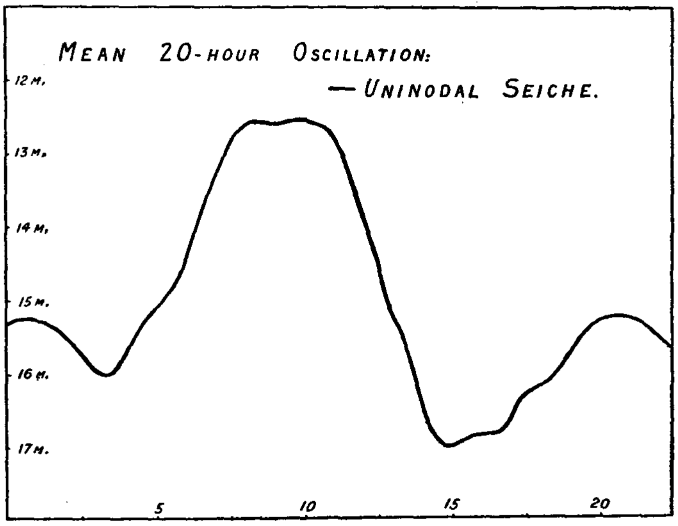


Fig. 15.

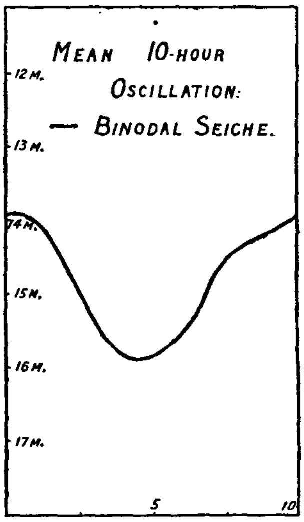


Fig. 16.

The average 20-hour (taken for simplicity as the maximum instead of 19·5 hours) and 10-hour oscillations are shown in figs. 15 and 16. It will be seen that the 10-hour curve is a fair approximation to a sine curve, but that the 20-hour curve is not. The reason for this is that the periodogram method is unable to separate the 10-hour oscillation from one that is so nearly of double the period, so that the average 20-hour oscillation as given by the periodogram is really a combination of the uninodal and binodal components.

When subjected to harmonic analysis the 20-hour oscillation is found to be :—

$$14{\cdot}99 + 1{\cdot}72\cos(\theta - 332^\circ) + 1{\cdot}10\cos(2\theta - 172^\circ) + 0{\cdot}10\cos(3\theta - 185^\circ)$$
$$+ 0{\cdot}14\cos(4\theta - 300^\circ) + 0{\cdot}14\cos(5\theta - 125^\circ).$$

It is thus shown that the uninodal and binodal constituents of this curve are very much more important than those of higher nodality. It will be seen from other considerations that the periods of the trinodal and quadrinodal seiches are about 8·5 hours and 6·0 hours respectively, and, as these are not approximate aliquot parts of 20 hours, the terms cos 3θ and cos 4θ cannot be taken to be the mathematical correspondents of the physical trinodal and quadrinodal seiches.

HARMONIC ANALYSIS.

In order to investigate the variations of the amplitudes and of the periods of the several component seiches during the series of observations which were subjected to periodogram analysis, the harmonic analysis of the 11° isothermal at both stations, and of the 10° isothermal at Station 2, was undertaken.

The period chosen was 20 hours. This period is exactly 24 of the "intervals" used above, and hence the method of 24 ordinates could be immediately applied to the numbers given in Table IV without any troublesome interpolation. All the calculations were made on the harmonic-analysis computation forms devised by Professor WHITTAKER for use in the Mathematical Laboratory of the University of Edinburgh.

The results for the 11° isothermal are given in Table VII (page 764). The numbers in italics refer to Station 1, and those in Roman type to Station 2. The results are also shown graphically in fig. 17, the dotted curve referring to Station 1 and the continuous line to Station 2.

The whole series of 140 intervals was divided into groups of 24 intervals each, viz., 1–24, 25–48, etc., and, in addition, intermediate groups were taken—13–36, 37–60, etc. The time of the middle point of these groups is shown in the column of time in the table.

We shall now examine the curves composing fig. 17.

1. *The Amplitude of the Uninodal Seiche.*—The amplitude is seen to be about 2·0 metres for the first period, and to rise through 2·5 metres to a maximum of about 3·6 metres at the group whose mid-time was 25th August 17 h. On the 24th and 25th there was an alternation of westerly winds of moderate strength with calms, and these may have been so timed as to build up the uninodal seiche in preference to the binodal. The increase of amplitude may also have been partly due to the gradual resolution of a travelling bore into a steady wave, as is suggested on page 758, where there is also a description of the weather conditions before the commencement of the series. During the next 36 hours there is a gradual fall in the amplitude of the

seiche. This was a period of calms and light winds, and hence from this part of the curve we can get an idea as to the decay of the uninodal temperature seiche in Loch Earn.

TABLE VII.

Time.	No.	0.	1.		2.		3.		4.	
Aug. 24, 21 h. . .	1–24	14·42 *14·90*	2·03 *1·93*	304° *305*	1·97 *3·15*	185° *172*	1·19 *2·12*	312° *260*	0·55 *0·64*	267° *329*
25, 17 h. . .	25–48	15·32 *15·58*	3·64 *3·74*	323 *322*	1·56 *1·90*	193 *181*	0·32 *0·54*	261 *276*	0·24 *0·39*	237 *150*
26, 13 h. . .	49–72	15·79 *16·02*	2·34 *2·37*	355 *347*	1·00 *1·45*	158 *168*	0·76 *1·08*	61 *81*	0·17 *0·78*	14 *330*
27, 9 h. . .	73–96	15·23 *15·42*	0·89 *0·95*	18 *12*	0·74 *1·15*	144 *149*	0·49 *0·72*	77 *69*	0·30 *0·37*	166 *0*
28, 5 h. . .	97–120	14·17 *14·26*	0·44 *0·97*	348 *334*	0·63 *0·77*	129 *137*	0·35 *0·14*	108 *135*	0·10 *0·14*	0 *15*
Aug. 25, 7 h. . .	13–36	14·87 *15·23*	2·58 *2·65*	146 *140*	1·25 *1·88*	167 *148*	0·26 *0·56*	59 *128*	0·15 *0·43*	106 *112*
26, 3 h. . .	37–60	16·35 *16·44*	3·34 *3·22*	161 *155*	1·20 *1·44*	198 *186*	0·44 *0·28*	234 *248*	0·22 *0·57*	240 *201*
23 h. . .	61–84	15·56 *15·75*	1·66 *1·80*	166 *164*	1·13 *1·69*	187 *185*	0·58 *0·97*	248 *260*	0·24 *0·45*	277 *299*
27, 19 h. . .	85–108	14·53 *15·09*	0·81 *1·10*	132 *109*	0·66 *1·04*	147 *144*	0·37 *0·43*	292 *271*	0·18 *0·71*	0 *348*
28, 15 h. . .	109–132	13·79 *14·07*	0·58 *0·95*	89 *117*	0·36 *0·37*	159 *160*	0·27 *0·26*	294 *289*	0·15 *0·30*	312 *303*

The dotted curve refers to the station nearer the end of the loch, and, as would be expected, the amplitude there is greater than at Station 2. If we were to assume that the isotherms lie in planes through the uninode as marked on the map, the amplitude at Station 1 would be 1·14 of that at Station 2. The actual ratio is about 1·04, indicating that the gradient of the isotherm diminishes towards the end of the loch. This may be due to the great amplitude of the oscillation and the damping effect of shallow water, or to the fact that the temperature normal curve is not parabolic, for it is only with parabolic normal curves that the isotherms can remain as planes.

2. *Variation of Uninodal Phase.*—This curve is drawn by taking as vertical scale the angle of the phase, and as the horizontal scale the mid-times of the groups. During the first 60 hours (or three groups) the phase is seen to be uniformly

mounting, but during the remainder of the series the variation is much more irregular. The irregularities are due to slight winds, which seem to have such a marked effect in changing the form of the oscillation, as has been noticed in previous investigations. The slow mounting during the calm gives a very accurate means of determining the true period of the oscillations. The rate of increase of phase is about 20° per group, and hence a better approximation to the true period of the uninodal seiche is $20 \times (1 + 20/360)$ hours $= 21{\cdot}1$ hours.

The curves for the two stations are very much alike, which is a strong corroboration of the standing-wave theory.

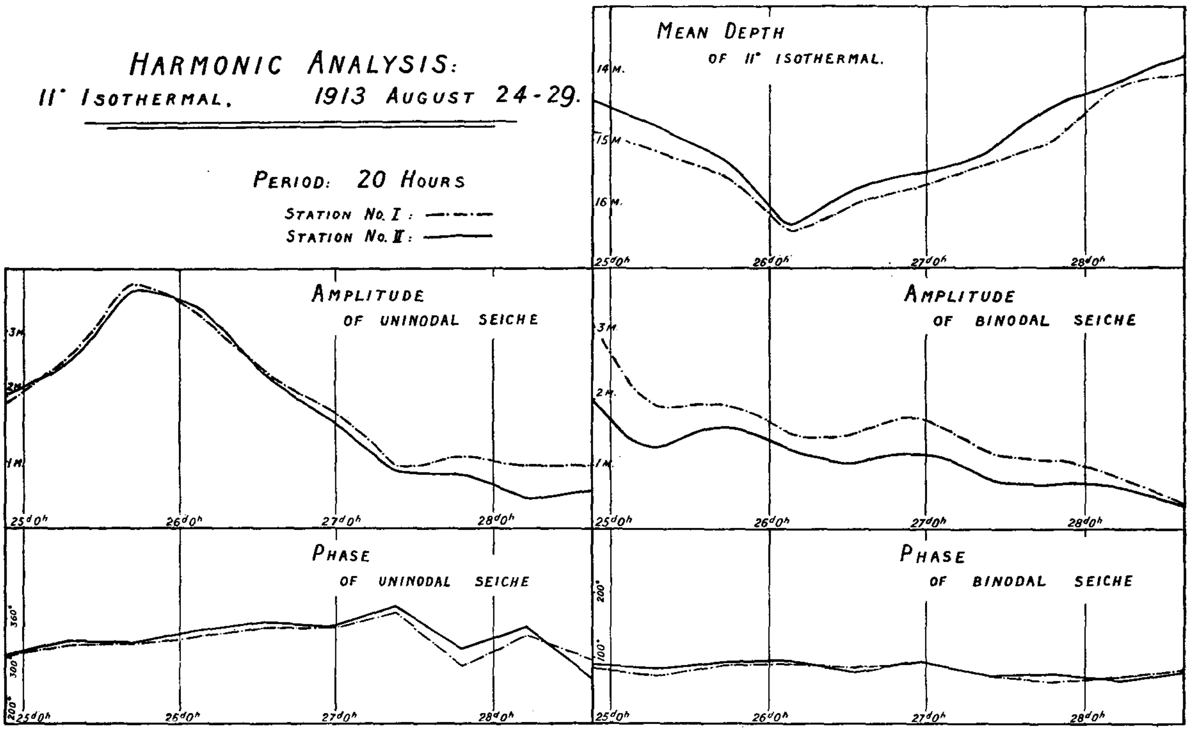


FIG. 17.

3. *Amplitude of the Binodal Seiche.*—In the case of the binodal seiche there is no strong maximum, but the whole is less regular than the curve of the amplitude of the uninodal seiche, and suggests that the former is more easily disturbed than the latter.

The difference between the curves for Stations 1 and 2 is much more apparent, as would be expected. A similar calculation to that given for the uninodal amplitude on the assumption of approximately plane isothermal surfaces through the binode gives the factor 1·28. The actual ratio is about 1·41, indicating that the gradient of the isotherm increases towards the end of the loch. The change of wave type discussed on page 756 gives a probable explanation of this, but at first sight it is rather remarkable that the gradient of the isotherms should be so different in the cases of the uninodal and binodal seiches. The wave-length of the binodal seiche being half of that of the uninodal seiche, however, irregularities may be expected to be greater in the case of the binodal seiche.

4. *Variation of Binodal Phase.*—The phase of the binodal seiche is seen to be very constant during the whole series, and suggests that the period of 10 hours is a very close approximation to the true period.

5. 10° *Isothermal at Station* 2.—The periodogram curves for this isothermal are shown in fig. 18, the time scales being as in fig. 17. As regards amplitude and phase they are seen to follow the 11° isothermal very closely.

6. *Mean Depth of the* 10° *and* 11° *Isothermals.*—A comparison of the curves showing the variations of the mean depth of the isothermals for 10° and 11° at Station 2 shows that there is a wide divergence between them. At the beginning of the series the distance between them was about 2·5 metres, but this decreases to less than a metre in about 30 hours, and thereafter each rises gradually about 2 metres in the next two days. With a variation such as this in calm weather, it is seen how impossible it is to find an accurate estimate of the period of the temperature seiche. It has been previously suggested that there may be extremely slow oscillations formed by rhythmical variations of the isothermal layers, but the observations are too limited to give much weight to this supposition.

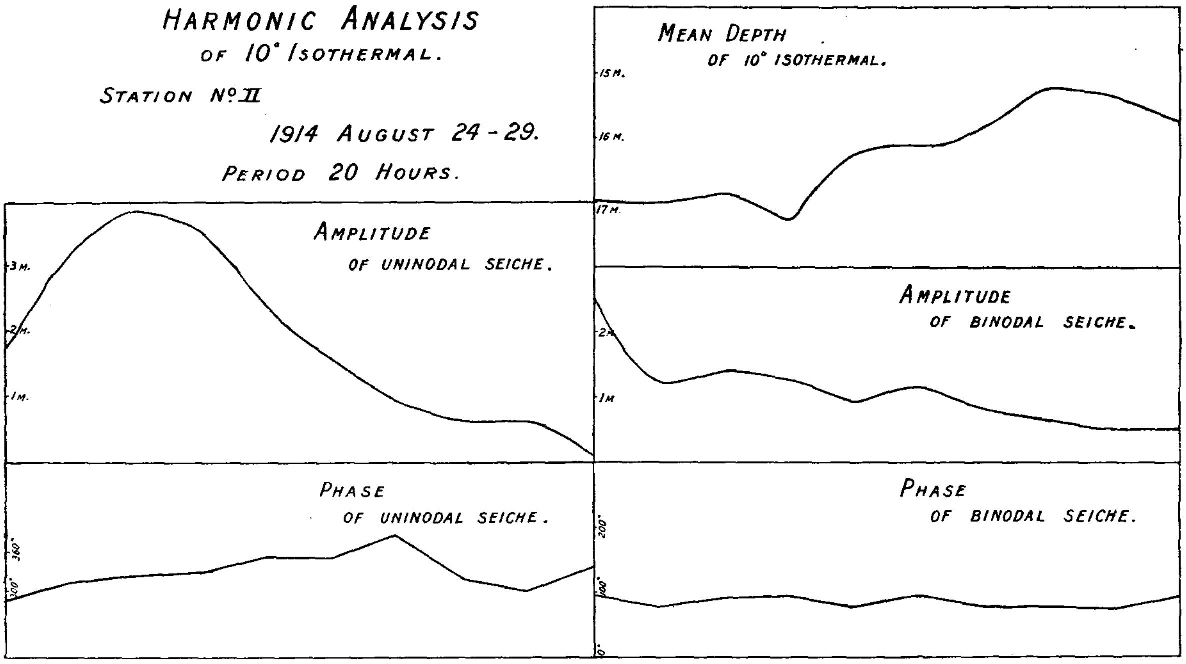


FIG. 18.

It will be noted that the average depth of the 11° isothermal at Station 2 is about one-third of a metre greater than at Station 1. This suggests that in their position of rest the isothermals are not plane surfaces but are slightly cup-shaped.

Periodogram Analysis for Short Periods.—On the periodogram curve shown in fig. 14 it is seen that there is no definite maximum in the curve below that corresponding to the 10-hour binodal seiche. This is not conclusive evidence against the existence of plurinodal seiches of appreciable amplitude, for it is probable that seiches of high nodality are more easily disturbed by surface winds than the uninodal and binodal seiches. If, then, the phase of the seiche be altered at various times during the period discussed in the periodogram, it is evident that the periodogram method will tend to obscure the periodicity rather than make it evident.

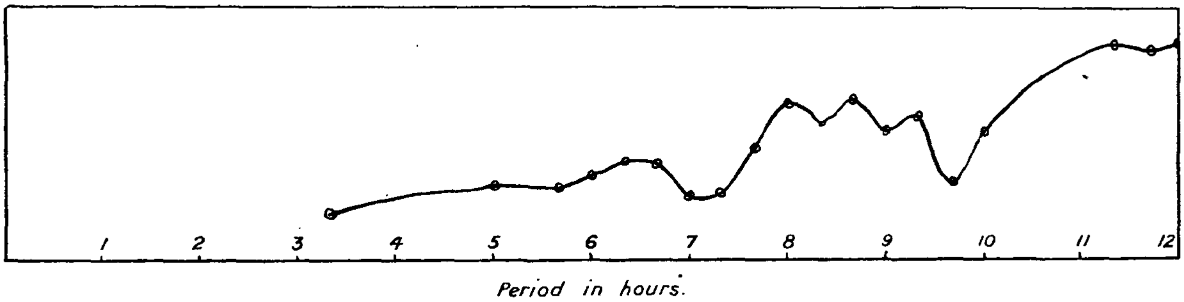


FIG. 19.—Periodogram Analysis for Short Periods.

In order to see whether evidence of plurinodal seiches could be found, a shorter period from 16 h. on 25th August to 14 h. on 27th August, which was a time of great calm, was chosen for examination. Readings of the 11° isothermal at Station 2 (fig. 11) were taken at shorter intervals of time than formerly, viz. 20 minutes, and to this series of readings the method of periodogram analysis was applied as described above. The curve of periods is shown in fig. 19. It could not be expected that there should be strongly marked maxima, but there is distinct evidence of the trinodal and quadrinodal seiches, the periods of which may be taken to be about $8\frac{1}{2}$ hours and $6\frac{1}{2}$ hours respectively.

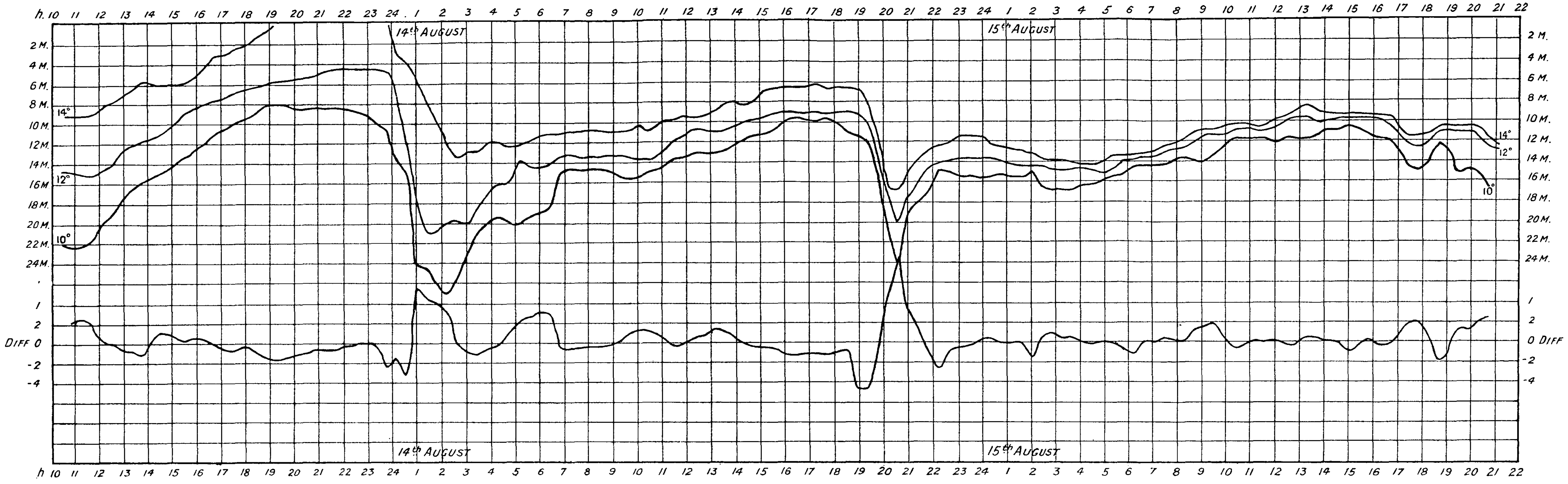
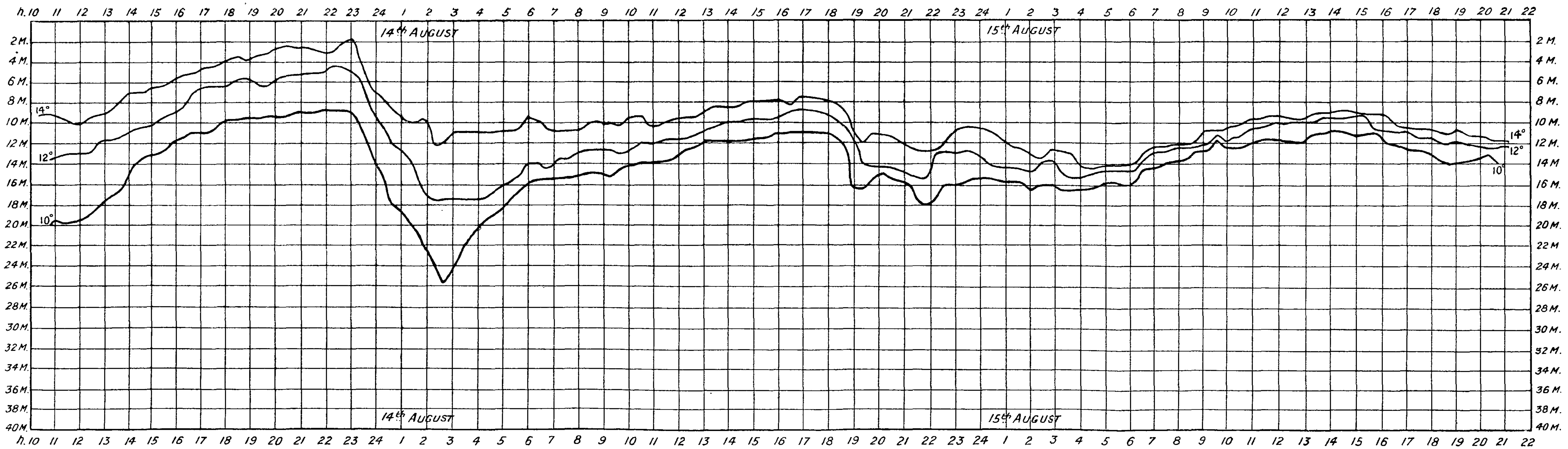

h. 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
14th August
15th August
2 M.
4 M.
6 M.
8 M.
10 M.
12 M.
14 M.
16 M.
18 M.
20 M.
22 M.
24 M.
14°
12°
10°
1
2
Diff 0
-2
-4

Fig. 8.


h. 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
14th August
15th August
2 M.
4 M.
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24 M.
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32 M.
34 M.
36 M.
38 M.
40 M.
14°
12°
10°

Fig. 9.