VIII. The Calculation of the Probable Errors of Certain Constants of the Normal Curve.

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Anything which serves to lighten the labour of computation incident to biometric investigations is very welcome. On this account I venture to call attention to a point in which the "Tables for Facilitating the Computation of Probable Errors," recently published by Miss Winifred Gibson* will be found very useful.

It has been shown by Pearson† that in order to determine whether a given frequency distribution deviates sensibly from the normal or Gaussian law, it is necessary to know the probable errors for the normal curve of the following constants: the skewness, $\sqrt{\beta_1}$, $\beta_2 - 3$ the kurtosis and the "modal divergence." The formula for the probable error of the skewness, when n is the total number of individuals, is

$$\text{P.E.}_{\text{SK}} = .6744898 \sqrt{\frac{3}{2n}}.$$

In a recently published note‡ I pointed out the fact that, after having calculated the value of this expression for any given n , in order to obtain the probable errors of $\sqrt{\beta_1}$, $\beta_2 - 3$ and the "modal divergence" for the same distribution it was merely necessary to multiply the calculated value successively by 2, by 4 and by σ (i.e. the standard deviation of the distribution). We have then only to find an easy way of getting the value of the expression

$$.6744898 \sqrt{\frac{3}{2n}},$$

in order to make the whole process of testing any distribution for normality very simple indeed.

It is clear that we may write

$$.6744898 \sqrt{\frac{3}{2n}} = .6744898 K \sqrt{\frac{1}{n}} = \frac{.6744898}{\sqrt{n}} K = \chi_1 K,$$

where

$$K = \sqrt{\frac{3}{2}} = 1.2247449\ddot{5},$$

and χ_1 is the χ_1 of Miss Gibson's Table I.

Therefore to test the approach to normality of any distribution we have merely to perform the following operations:

(i) Look out χ_1 for the given n from Miss Gibson's tables and multiply it by the factor 1.2247449, using as many places of decimals as necessary. This gives the probable error of the skewness for the normal curve.

(ii) Multiply the result by 2, which gives the probable error of $\sqrt{\beta_1}$ for the normal curve; multiply this result again by 2, and so obtain the probable error of $\beta_2 - 3$; finally multiply the probable error of the skewness by the σ of the distribution and obtain in this way the probable error of the modal divergence.

The relative divergence from zero of the skewness, $\sqrt{\beta_1}$, $\beta_2 - 3$ and the distance from mean to mode in comparison with their probable errors, measures the probability that the given distribution does not follow the normal or Gaussian law.

* *Biometrika*, Vol. iv. pp. 385—393.

† *Biometrika*, Vol. iv. pp. 169—212, and elsewhere.

‡ *Science*, N. S. Vol. xxii. p. 802.

§ From Barlow's Tables.