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Investigation of Formulæ for Determining the proper size of Shafts and Piston Rods. By D. M. GREENE, 2d Assistant Engineer, U.S.N.

An article published in a recent number of the *Journal* contains several useful formulæ for determining the proper sizes of various important parts of the steam engine; but as these formulæ are not accompanied by any investigations or explanation of the processes by which they were obtained, they fail to meet the requirements of the intelligent and careful designer of steam machinery.

It is the purpose of the present paper to develop formulæ for calculating the diameters of *Shafts* and *Piston Rods*,—divesting the investigations of such technicalities as would tend to embarrass the general reader.

SHAFTS.—*Wrought Iron*.—*The moment of a pressure tending to produce rotation about an axis, or to resist such motion, is equal to the intensity of the pressure or resistance multiplied by the lever arm with which such pressure or resistance acts.*

In order to determine the relation between the diameters of shafts and their moments of resistance to torsional strain,

Let s = torsional strength per square inch of transverse sectional area of shaft.

$a, a_1, a_2, \&c.,$ = the sectional areas.

d, nd, n_1d = their diameters.

$r, nr, n_1r, \&c.,$ = their lever arms of resistance.

$m, m_1, m_2, \&c.,$ = their moments.

Then,

$$m = a s r \qquad m_1 = a_1 s nr$$

whence

$$m : m_1 :: a r : a_1 nr. \quad . \quad . \quad (1)$$

$$\text{But, } a = .7854d^2 \quad a_1 = .7854n^2d^2$$

Substituting these values in (1) we get

$$m : m_1 :: .7854d^2r : .7854n^2d^2nr :: 1 : n^3.] \quad (2)$$

Similarly,

$$m : m_2 :: 1 : n_1^3 \quad . \quad . \quad (3)$$

From (2) and (3) we get

$$m_1 : m_2 :: n^3 : n_1^3 \quad . \quad . \quad (4)$$

Hence, the moments of resistance of shafts to torsional strain vary directly as the cubes of their diameters.

If d = one inch in the preceding notation, 1, $n, n_1, n_2, \&c.,$ will represent the number of inches in the diameters of the shafts.

If, again, $n = 1$, (4) becomes

$$m_1 : m_2 :: 1 : n_1^3; \\ \text{whence} \qquad m_2 = m_1 n_1^3; \quad . \quad . \quad (5)$$

Or the moment of resistance to torsion of a shaft whose diameter is n_1 inches, is equal to the moment of resistance of a shaft whose diameter is one inch multiplied by n_1^3 .

If now the value of m , for a shaft one inch in diameter be deter-

$$\therefore d = \sqrt[3]{.002 D^2 p s}. \quad (10)$$

for the shaft of a single engine.

For a double engine, with the cranks placed at right angles, we have as before

$$200d^3 = P^1 a^1, \quad (11)$$

in which P^1 represents the pressure upon the two crank pins, and a^1 the lever arms with which they act, when the strain upon the shaft is a maximum; which will be when each crank makes an angle of 45° with the direction of the piston rod.

In this position we shall have,

$$\begin{aligned} a^1 &= a \cos. 45^\circ \\ &= \frac{s}{2} \times .7 \end{aligned}$$

and since there are two cylinders, we shall have, $P^1 = 2 \times .7854 D^2 p$;

$$\text{and } P^1 a^1 = \frac{2 \times .7 \times .7854 D^2 p s}{2}.$$

This value substituted in (11) gives

$$200d^3 = 2 \times .7 \times .7854 D^2 p s.$$

whence,

$$\begin{aligned} d^3 &= \frac{2 \times .7 \times .7854}{400} D^2 p s. \\ &= .0027 D^2 p s. \end{aligned}$$

and

$$d = \sqrt[3]{.0027 D^2 p s}. \quad (12)$$

For an engine with three cylinders, having its cranks placed at angles of 120° , we shall have the maximum torsional strain on the shaft, when one crank is perpendicular to, and the two others each make an angle of 30° with the direction of, the piston rod.

In this case we shall have

$$\begin{aligned} 200d^3 &= P a + P a \sin. 30^\circ + P a \sin. 30^\circ \\ &= \frac{P s (1 + 2 \sin. 30^\circ)}{2} \\ &= P s. \\ &= .7854 D^2 p s. \end{aligned}$$

whence,

$$\begin{aligned} d^3 &= \frac{.7854}{200} D^2 p s. \\ &= .0039 D^2 p s. \end{aligned}$$

and

$$d = \sqrt[3]{.0039 D^2 p s}. \quad (14)$$

Comparing now (10), (12), and (14), supposing D , p , and s constant, we get

$$\begin{aligned} d_1 : d_2 : d_3 &:: \sqrt[3]{20} : \sqrt[3]{27} : \sqrt[3]{39} \\ &:: 1 : 1.1 : 1.25; \end{aligned}$$

whence it appears that diameters of the shafts of single, double, and

treble engines, having cylinders of the same size and using the same pressure of steam, are to each other respectively as 1, 1.1 and 1.25.

Recapitulation.

For Single Engine (10).

$$d = \sqrt[3]{.002D^2 \text{ ps.}}$$

For Double Engine (12).

$$d = \sqrt[3]{.0027D^2 \text{ ps.}}$$

For Treble Engines (14).

$$d = \sqrt[3]{.0039D^2 \text{ ps.}}$$

PISTON RODS.—(*Wrought Iron*).—For all the knowledge we possess in relation to the resistance of columns to crushing strains we are directly indebted to experiment.

It is found that columns of wrought iron will bear without crushing, per square inch:—

When their lengths are less than 12 diameters,	14,225 lbs.
“ “ “ “ between 12 and 24 “	11,877 “
“ “ “ “ greater than 24 “	7,112 “

In practice we may take one half these values.

Let s = the safe resistance of the material per square inch.

d = diameter of the piston rod in inches.

a = area of transverse section in square inches.

Then we shall have for the pressure which any rod will bear with safety

$$P = a s \\ = .7854 d^2 s.$$

Substituting now in this expression, for s , the safe resistances indicated above, we get,

$$P = \frac{14225}{2} \times .7854 d^2 \quad . \quad . \quad (1)$$

$$P_1 = \frac{11877}{2} \times .7854 d^2 \quad . \quad . \quad (2)$$

$$P_2 = \frac{7112}{2} \times .7854 d^2 \quad . \quad . \quad (3)$$

These expressions must be made equal to the pressure upon the pistons. To obtain an expression for the effective pressure upon any piston:—

Let D = diameter of cylinder in inches.

p = effective pressure per square inch upon the piston = pressure per steam gauge vacuum.

Then we shall have for the whole pressure on the piston:—

$$P = .7854 D^2 p.$$

Equating successively the values in (1), (2), and (3) with the preceding, we get

$$1^\circ. \quad \frac{14225}{2} \times .7854 d^2 = .7854 D^2 p.$$

$$\begin{aligned} \text{whence} \quad d^2 &= \frac{2 \times .7854}{14225} D^2 p \\ \text{and} \quad d^2 &= 0.11 D \sqrt{p} \end{aligned} \quad (4)$$

$$2. \quad \frac{11877}{2} \times .7854 d^2 = .7854 D^2 p.$$

$$\begin{aligned} \text{whence} \quad d^2 &= \frac{2 \times .7854}{11877} D^2 p \\ \text{and} \quad d &= .013 D \sqrt{p}. \end{aligned} \quad (5)$$

$$3. \quad \frac{7112}{2} \times .7854 d^2 = .7854 D^2 p$$

$$\begin{aligned} \text{whence} \quad d^2 &= \frac{2 \times .7854}{7112} D^2 p. \\ \text{and} \quad d &= .017 D \sqrt{p} \end{aligned} \quad (6)$$

Steel Rods.—The relative abilities of wrought iron and steel to resist crushing strains are as 1745 to 2518; hence, the transverse sectional areas of piston rods made of these materials, should be inversely proportional to these numbers; or their diameters should be inversely proportional to the square root of of 1745 and 2518.

$$\text{Now,} \quad \sqrt{\frac{1745}{2518}} = .83$$

$$\begin{aligned} \text{and (4) becomes} \quad d &= .83 \times .011 D \sqrt{p} \\ &= .009 D \sqrt{p} \end{aligned} \quad (4)^a$$

$$\begin{aligned} \text{Similarly, (5) becomes} \quad d &= .83 \times .013 D \sqrt{p} \\ &= .011 D \sqrt{p} \end{aligned} \quad (5)^a$$

$$\begin{aligned} \text{and (6) becomes} \quad d &= .83 \times .017 D \sqrt{p} \\ &= .014 D \sqrt{p} \end{aligned} \quad (6)^a$$

Recapitulation.

When	Wrought Iron.	Steel.
$l < 12d$,	$d = .011 D \sqrt{p}$	$d = .009 D \sqrt{p}$
$l > 12d$ and $l < 24d$	$d = .013 D \sqrt{p}$	$d = .011 D \sqrt{p}$
$l > 24d$	$d = .017 D \sqrt{p}$	$d = .014 D \sqrt{p}$

l being the length of the rod.

If different values be assigned to p in the above formulæ, tables may be readily constructed. The three cases taken, correspond to *short*, *medium*, and *long* strokes. Hence the formula for d when $l < 12d$ would apply to the piston rods of propeller engines; but since the speed of piston in the propeller engine is considerable, it would perhaps be better to use the formula (5) or (5)^a, in determining the diameters of their rods; in any other case, a mental comparison between the length and diameter of the cylinder will show which formula should be used.