A Case of Three Rotating Lines and the Point "O"

Author(s): Glanville Taylor

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7. Any irrational number can be approximated to by a decimal approximation.

Let l be the irrational number.

Take matches of unit length. We can then find by trial the two integers between which l lies. Next take matches 10^{-1} of a unit in length. We can then find by trial the two consecutive decimals of the first order between which l lies. Next take matches 10^{-2} of a unit in length. We can then find by trial the two consecutive decimals of the second order between which l lies.

We can proceed in this way to any required degree of approximation.

(To be continued.)

A CASE OF THREE ROTATING LINES AND THE POINT "O."

I.

1. Let AP, BQ, CR be three lines through the vertices of the $\triangle ABC$ which rotate with equal angular velocity ω , and which make equal angles with the opposite sides, the initial position being that of the three altitudes of ABC . Then the three circles AQR, BRP, CPQ , which meet in the well-

FIG. 1.

known point O , will be shewn to form three coaxial systems; and among other things we shall shew that the locus of O is the circle on GH as diameter, G being the centroid and H the orthocentre.

2. Let $(BQ, CR), (CR, AP), (AP, BQ)$ be the points L, M, N . These points lie on the three circles respectively, and if DEF be the $\triangle$ formed by parallels

through A, B, C to the opposite sides, the points L, M, N lie on the three equal circles BCD, CAE, ABF , these latter being concurrent at H .

It therefore follows that since (by hyp.) the angle $HAM = HBN = HCL = \theta$ say, we have $HL = HM = HN$, or H is the circumcentre of LMN . Also the $\triangle LMN$ is evidently similar to ABC and DEF . Moreover H is the circumcentre of DEF , which is itself one of the members of the family LMN .

The circles LMN therefore form a concentric system which expands and contracts, the radius varying from nothing to HD while the three lines AP , etc., have turned through a right angle.

We may note, too, that the points L, M, N move with angular velocity 2ω about the centres A', B', C' respectively, or with angular velocity ω about H collectively.

3. Let AD meet the circle A' in S , and let T, V be corresponding points for BE and CF .

Then $\angle SLQ = \angle SDB = \angle SAQ$. Hence the variable circle $ARLQ$ passes through the fixed points A and S , thus forming a coaxal system. Similarly for the other two circles.

The three radical axes are the medians of ABC , but obviously G is not a radical centre.

Since HD, HE, HF are diameters, $\therefore HS \perp SG$, etc.; and hence S, T, V lie on the circle, diameter GH .

It is seen that $\triangle STV$ is similar to the $\triangle$ formed by the medians.

4. To shew that O lies on this circle GH , we have (see fig. 2)

$$\angle SOR = \pi - GAB, \quad \angle TOR = \pi - GBA;$$

$$\therefore \angle SOT = GAB + GBA = \angle SGT, \text{ which proves the statement.}$$

II.

5. Let d_1, d_2, d_3 be the diameters of the circles AQR , etc., say (O_1, O_2, O_3). Then, since $\angle ARO = \pi - ASO = \angle OSG = \angle OHG$,

$$\therefore AO = d_1 \sin OHG = \frac{d_1 \cdot OG}{GH};$$

$$\therefore d_1 = \frac{AO}{OG} \cdot GH; \text{ whence also } d_1 : d_2 : d_3 = AO : BO : CO.$$

6. The sides of the $\triangle O_1O_2O_3$ are $\perp$ to OP, OQ, OR , the common chords. Hence $\angle O_1 = \pi - ROQ = A$; or the $\triangle O_1O_2O_3$ is similar to ABC . The $\triangle O_1O_2O_3$ is therefore one of a system of $\triangle^{\text{les}}$ of given species (ABC) inscribed in the $\triangle E_1E_2E_3$, formed by the lines of centres, which latter is similar to the $\triangle$ formed by the medians of ABC .

7. Since $O_1O = O_1A$ we have

$$AO_1 : BO_2 : CO_3 = OO_1 : OO_2 : OO_3 = d_1 : d_2 : d_3 = AO : BO : CO;$$

$$\therefore \triangle^{\text{les}} AOO_1, BOO_2, COO_3 \text{ are similar.}$$

Hence the point O is the centre of similitude (c.s.) of $\triangle^{\text{les}} O_1O_2O_3$ and ABC .

8. Again, by Art. 6 and the "point O theorem," the $\odot^{\text{les}} E_1O_2O_3, E_2O_3O_1, E_3O_1O_2$ meet in a fixed point E_4 , which is the c.s. of all the $\triangle^{\text{les}}$ of the system, $O_1O_2O_3$.

9. To find E_4 , we have $\angle O_2E_4O_3 = \pi - E_1 = \angle BGC$, etc. But $\triangle O_1O_2O_3$ is similar to ABC ; hence E_4 corresponds to G , and is therefore the common centroid of all the $\triangle^{\text{les}} O_1O_2O_3$.

Again, the $\triangle K_1L_1M_1$ formed by the mid-points of HA, HB, HC is similar and similarly situated to ABC ; it is also the initial position of $O_1O_2O_3$. E_4 is therefore the centroid of $K_1L_1M_1$, and hence corresponds to G of $\triangle ABC$. But H is the c.s. of $K_1L_1M_1$ and ABC . $\therefore E_4$ is mid-point of HG ; that is the $\odot^{\text{re}}$ of the circle GH .

10. We may note that since $E_4K_1 \parallel GA \perp E_2E_3$, and similarly for E_4L_1, E_4M_1 ; $\therefore K_1L_1M_1$ is the minimum $\triangle^{\text{le}}$ of the system $O_1O_2O_3$.

III.

11. Since E_4O_1 is the $\perp$ bisector of SO , it follows that E_4O_1 rotates with the same angular velocity as SO , i.e. with the angular velocity of O about any fixed point (say G) on the circle GH .

In other words, the $\triangle O_1O_2O_3$ rotates about E_4 with the angular velocity of GO about G .

12. Now (figs. 1, 2) $\angle NOC = NPC = \pi - BPM = BOM$;

$$\therefore \angle MON = BOC.$$

Similarly

$$\angle NOL = COA, \angle LOM = AOB;$$

$$\therefore O \text{ is the c.s. of } \triangle^{les} LMN, ABC.$$

But O is the c.s. of $\triangle^{les} O_1O_2O_3$ and ABC ;

$$\therefore O \text{ is the c.s. of } \triangle^{les} O_1O_2O_3, LMN, \text{ and } ABC.$$

13. Now in $\triangle^{les} ABC, LMN$, since the circumcentres are corresponding points,

$$\therefore O' \text{ corresponds to } H; \text{ and, } O \text{ being the c.s.,}$$

$$\therefore \frac{OO'}{OH} = \frac{BC}{MN},$$

and

$$\angle O'OH = \angle^e \text{ between } BC \text{ and } MN = APC;$$

$$\therefore \angle O'OG = O'OH - \frac{\pi}{2} = APC - \frac{\pi}{2} = HAP \equiv \theta.$$

14. Let $\angle OGH = \phi$.

Then

$$\frac{OH}{OH'} = \frac{\sin OO'H}{\sin O'OH} = \frac{\sin \phi - \theta}{\cos \theta},$$

or, if

$$K = \text{radius of circle } GH,$$

$$2K \sin \phi \cos \theta = 3K \sin \phi - \theta,$$

whence $\tan \phi = 3 \tan \theta$, giving a relation between the angular velocity of GO and that of AP , viz. ω or θ . Hence the motion of O round its circle is determined.

(To be continued.)

REFORM OF MATHEMATICAL TEACHING IN GERMANY.*

I PROPOSE in this paper to deal with the development during the last twenty or thirty years of the dominating ideas of the reform of mathematical teaching in Germany, and must beg for your indulgence if the theme is one which is already familiar.

In the *Mathematical Gazette* of December last there appeared a translation of the so-called Meraner Lehrplan of 1905, which, with the appended notes, formed a kind of Declaration of Rights of the Reform Party. It is my intention to give some account of the movement which led to its publication, to indicate briefly the substance of these appended notes, and to give some details of the way in which the general ideas have been introduced into the curricula.

The most remarkable feature in this development is the manner in which the contentions of the first reformers have been so largely accepted and pressed forward not only by mathematicians but by men of general culture whose bent has been towards humanistic or scientific rather than purely

* Paper read before the London Branch of the Mathematical Association by E. Allan Price, B.A.