THE DISCHARGE OF ELECTRIFIED BODIES BY THE X-RAYS.¹ II.

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THEORY.

THE results thus far given can be explained on the hypothesis given by Thomson and Rutherford.² It has been shown that the condition of the gas which is produced by the passage of X-rays through it, after lasting for an appreciable length of time, disappears. This condition is also rapidly destroyed by the passage of a current. It was, therefore, assumed that the gas through which the X-rays pass becomes dissociated, that the ions normally recombine after a short period, and that they recombine much more rapidly when a current passes through the gas. Since I shall have occasion to consider the theory somewhat in detail, the following sentences are quoted from Thomson and Rutherford:

“If we regard the gas as an electrolyte, then the passage of a quantity of electricity will destroy e/ϵ of the conducting particles, where e is the amount of electricity carried, and ϵ is the amount carried by one particle.

“Let n be the number of conducting particles in a unit volume of the gas, q the rate at which they are produced by the rays, an^2 the rate at which they disappear independently of the passage of the current, i the current through unit area of the gas, l the distance between the electrodes. Then we have

$$\frac{dn}{dt} = q - an^2 - \frac{i}{l\epsilon}$$

so that when the state of the gas is steady

$$0 = q - an^2 - \frac{i}{l\epsilon}.$$

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² Phil. Mag., 42, 392.

"If E is the difference of potential between the plates, U the sum of the velocities of the positively and negatively electrified particles when the potential gradient is unity, we have

$$i = \frac{n\epsilon UE}{l} \text{ or } n = \frac{li}{\epsilon UE}."$$

Substituting this for n in the above equation and solving for i we

$$\text{find } i = \frac{-\frac{1}{l\epsilon} + \sqrt{\frac{4aq l^2}{\epsilon^2 U^2 E^2} + \frac{1}{l^2 \epsilon^2}}}{\frac{a l^2}{2 \epsilon^2 U^2 E^2}}.$$

High Potential.—Now if E is very large $i = q\epsilon l$ and the current is not determined by the potential nor by U , but is proportional to the rate at which conducting particles are formed. If, therefore, q is proportional to the square root of the density of the surrounding gas, we should have the current in the case of a large potential proportional to the square root of the density, and we should also have the limiting currents proportional to the same thing. This is approximately what was found to be the case.

Low Potential.—If, however, E remains very small while q becomes large, $i = \frac{\epsilon UE}{l} \sqrt{\frac{q}{a}}$. It is evident that both U , the velocity of the ions for unit potential gradient, and a , the rate at which the ions recombine when there is one per unit volume, may be functions of the density of the gas. As the density is increased we should expect U to decrease and a to increase. Such a change in either would account for the fact that with small potentials i becomes smaller as the density of the gas is increased.

Density for Maximum Discharge.—If now we make some assumption regarding the dependence of U , a , and q upon the density of the gas, it will be possible to find a value of d which will cause i to have a maximum value. It is apparent from the experimental work that q , the rate at which ions are formed, is roughly proportional to the square root of the density of the gas. Let us, therefore, say that $q = k\sqrt{d}$ where d is the density of the gas, and k is a quantity depending upon the intensity of the X-rays. If we assume that

$\frac{a}{U^2} = k'd^{\frac{3}{2}}$ where k' is a constant, it can be shown that $d_m = \frac{E}{2l} \sqrt{\frac{3}{kk'}}$ where d_m is the density that will cause i to be a maximum.

I have no reason *a priori* for supposing that $\frac{a}{U^2}$ varies as the $3/2$ power of d rather than according to some other power. This is assumed simply because it gives a value for d_m more consistent with the experimental results than any other supposition. It has already been shown experimentally that d_m varies approximately directly as E and inversely as the square root of k . I also plotted curves between i and d assuming that $\frac{a}{U^2} = k'd^{\frac{3}{2}}$ and $q = kd^{\frac{1}{2}}$. These curves agreed fairly well with those found experimentally, the dotted curves in Fig. 6 being the theoretical ones, and the smooth, the experimental.

If the assumption is made that $\frac{a}{U^2} = k'd^2$, then $d_m = C \frac{E^{\frac{2}{3}}}{k^{\frac{1}{3}}}$, where C is a function of l and k . This agrees nearly, but not fully, as well with the experimental results as the other value for d_m . The curves

FIG. 6.

between i and d which we should then have also agree fairly well with the curves found. No other relation between d and $\frac{a}{U^2}$ explains the experimental results so well.

Rate of Recombining.—Thomson and Rutherford state that the rate at which the dissociated ions disappear independently of the passage of the current is proportional to the square of the number dissociated. That this is true rather than that they disappear at a rate proportional to the first power, appears from the experimental

results here given. If the later assumption be made, it can be shown that d_m will not depend at all on k , that is on the intensity of the rays. But experiment shows very clearly that d_m does depend on the intensity of the rays.

This same assumption can also be tested in another way. It has already been shown that if E is large, the current is proportional to q , the number of ions dissociated, but if E is small the current is proportional to the $\sqrt{q}$. It is most natural to suppose that q is proportional to the intensity of the rays, and thus it follows that a change in the intensity of the rays produces a greater effect on the rate of discharge when E is large than when E is small. This was tested and it was found that such was actually the case. With a difference of potential of 60 volts the fall in potential was 4 volts when the intensity was strong, and 2.1 when it was weak. With a difference of potential of 5 volts the fall was 1 volt when the intensity was strong, and .7 when it was weak. The ratio in the former case was 1.9 and in the latter 1.43. The difference in the two cases corresponds very closely with the difference which the theory calls for. This fact is an important one to bear in mind when one is measuring the intensity of X-rays by the rate of discharge which they produce.

Variation in Number of Discharges.—It is very evident that some more direct way of determining the relation between $\frac{a}{U^2}$ and d is desirable. It seemed possible that by varying the number of discharges per second through the tube, some idea might be obtained concerning this relationship. For this purpose a mercury interrupter, having a device for regulating the number of interruptions per second, was substituted for the magnetic interrupter.

It was first found that when the potential of the charged body was large, the rate of discharge was directly proportional to the number of interruptions up to 12 per second, but there was not this relationship when the potential was small. Curves are plotted in Fig. 7, showing the relation between the interruptions per second and the rate at which the electrified body is discharged in gas at different densities. These were taken with an initial potential of 5 volts.

Fig. 7.

It will be noticed that, according to these curves, when a large number of interruptions in the primary circuit takes place, the discharge is more rapid for small densities of the gas than for larger. This is not the case when there are but two or three interruptions per second. It would appear, therefore, that if we should vary the density of the gas, using but few interruptions per second, there would be no maximum current, even though the potential were small. This was found experimentally to be the case.

It is evident that these results are of considerable importance, if a correct explanation for them can be found. In the first place, it is necessary to explain the fact that the rate of discharge is proportional to the number of interruptions per second when the potential is large, and is not proportional when it is small. It has been estimated that the ionized condition of the gas continues for about $\frac{1}{10}$ of a second. It is evident, moreover, that the time will be shorter if a large discharge takes place, and the normal rate of reassociation is increased by the passage of the current through the gas.

If, then, the discharge due to one flash of the rays from the tube is finished before another one occurs, the total discharge will be directly proportional to the number of flashes which are sent out. If,

however, one such discharge is not completed before another occurs, such a direct proportion does not hold. For the rate at which the ions normally recombine is assumed to be proportional to the square of the total number present. When few ions are left from a given flash of the rays, their rate of recombining is small, as compared with the number still present, and the current which they carry before they completely recombine will be appreciable.

If, however, a new supply of ions is suddenly added, the rate of recombining is much larger, the ions left over from the first supply are quickly recombined and the amount of electricity carried by them is inappreciable. In this case the total amount of electricity carried by any one supply of ions is less than it would have been if each supply had acted independently. This would explain the fact that the rate of discharge is no longer proportional to the number of interruptions in the coil, when this number is large.

If this be true, then it appears from the curves in Fig. 6 that at atmospheric pressure the discharges begin to interfere with each other when there are 2 per sec., but at a pressure of 150 mm. they do not interfere with each other until there are about 6 per sec. In other words, they more quickly recombine at lower pressures.

This may be caused either by the rate at which they normally recombine being greater at the lower pressure, or by an increase in the rate at which they recombine due to the current. However, if it were due to the fact that they normally recombine more rapidly, the ions would not have as great a chance to carry electricity, and the amount of electricity discharged would therefore be less at the lower pressure. This is not the case.

We are, therefore, driven to the other explanation, namely, that the current causes the ions to recombine more rapidly when the pressure is low. If the velocity of the ions were greater at the lower pressure, then the ions would carry more current and would also be recombined more rapidly. This explanation would, therefore, account for both of the observed facts.

Modification of Theory—Action Discontinuous.—Let us, therefore, return to the mathematical part of the theory. Thomson and Rutherford assumed that the number of dissociated ions in the gas remained constant, being replenished by the action of the X-rays at a

uniform rate equal to the rate at which they recombined. It would appear from my last set of curves that when the potential is large, the action of the ions is not continuous, but that the gas resumes its normal condition after each discharge of the tube. The assumption in this case is not only not in accordance with the facts, it is not even an approximation. It will, therefore, be of interest to work the theory through on a more correct assumption in this particular.

A graphical representation of what takes place can be given by curves similar to those in Fig. 8.

FIG. 8.

The ordinates correspond to the number of ions present, the abscissæ to the time. Since the rate of discharge is proportional to the number of ions present, this rate at any instant is also proportional to the corresponding ordinate.

The total discharge is proportional to the area of all the curves.

First let us find the quantity of electricity discharged by one supply of dissociated ions. Let b equal the rate at which the ions recombine due to the action of the current. Then $\frac{dn}{dt} = -an^2 - bn$.

Solving this differential equation we find that $n = \frac{n_0 b e^{-bt}}{an_0(1 - e^{-bt}) + b}$ where n_0 is the number of the ions when $t=0$. This would be the number due to one flash of the X-rays. Now $i = \frac{\epsilon UE}{l} n$ and if Q represents the total quantity of the electricity discharged by this one supply of ions,

$$Q = \frac{\epsilon UE}{l} \int_0^\infty n dt = \frac{\epsilon UEn_0 b}{l} \int_0^\infty \frac{e^{-bt}}{an_0(1 - e^{-bt}) + b} dt = \frac{\epsilon UE}{la} \log \frac{an_0 + b}{b}$$

$$\text{But } bn = \frac{i}{\epsilon} \text{ and } i = \frac{\epsilon UE}{l} n \therefore b = \frac{UE}{l^2} \text{ and } Q = \frac{\epsilon UE}{la} \log \frac{an_0 + \frac{UE}{l^2}}{\frac{UE}{l^2}}$$

If this be evaluated for a large value of E , we find that $Q = n_0 \epsilon l$. If there are m discharges per sec. and each one finishes its action be-

fore another one begins, the total discharge per sec. is $mQ = mn_0\epsilon l$. This agrees identically with the result given by Thomson and Rutherford when we remember that q must be considered equal to mn_0 . This also agrees with the fact that in this case the rate of discharge is directly proportional to the number of interruptions per second.

If now we go back to the expression for Q and evaluate it for a small value of E , we find that

$$Q = \frac{\epsilon l n_0 \frac{UE}{l^2}}{an_0 + \frac{UE}{l^2}}$$

and if $\frac{UE}{l^2}$ is very small as compared with an_0

$$Q = \frac{\epsilon UE}{al} \quad \text{and} \quad Qm = \frac{m\epsilon UE}{al}.$$

Thomson and Rutherford found $i = \frac{\epsilon UE}{l} \sqrt{\frac{q}{a}}$ when E was small.

The value for Qm would seem to be incorrect in that it is independent of n_0 . It must be remembered, however, that this does not hold except when n is very large and E very small, and the action is entirely discontinuous. There would be the greatest difficulty in detecting the rate of discharge under these circumstances. It can not be claimed that any experimental data found by myself were taken under circumstances which would satisfy these conditions.

Action Nearly Discontinuous.—Let us next consider the case when the action is nearly but not quite discontinuous. In this case the ions would be nearly recombined before a second discharge occurred. The few remaining uncombined when the second group of ions was supplied would recombine so quickly, due the presence of additional ions, that their presence could be neglected. In that case

$$Q = \frac{\epsilon UE}{l} \int_0^t n dt$$

where t_1 is of such a value that ϵ^{-bt_1} is nearly equal to zero. If this be integrated and evaluated when E is small, we find

$$mQ = m \frac{\epsilon n_0 \frac{UE}{l^2}}{an_0 + \frac{l^2}{l^2}} (1 - \epsilon^{-\frac{UE}{l^2 m}}), \text{ since } t_1 = \frac{1}{m}.$$

It is apparent that this quantity is not proportional to m because of the term $\epsilon^{-\frac{UE}{l^2 m}}$. This term will become larger as U becomes smaller, and since there is greater deviation from the law of strict proportionality in the case of discharge in a gas of greater density, one is led to the conclusion that it becomes smaller as the density of the gas becomes greater.

In the case that $(1 - \epsilon^{-\frac{UE}{l^2 m}})$ differs appreciably from 1, it is evident that this factor will cause Q to increase as U increases. These two conclusions are the same that we have derived by more general consideration of the curves in Fig. 7.

Action Nearly Continuous.—It would be desirable to consider the case when the action of the ions is far from being discontinuous and is not completely continuous. This can be done by making one or two assumptions. The value found for i in this case when E is small is practically the same as that found by Thomson and Rutherford. But it is doubtful whether the theoretical work is at present of any value. It is possible to have a theory with more assumptions than facts, and in this case it certainly would be well to test those assumptions made concerning the rate of normal reassociation and the rate due to the action of the current, by some more direct means than has yet been employed, before a more elaborate theory is constructed.

It is thus evident that I am leaving this subject in a very incomplete condition. Both the fundamental assumptions might be tested by more direct methods and the consequences could be worked out more in detail. However, circumstances necessitate a postponement of further work by myself on this subject for the present. It, therefore, seems better to publish what has already been done and trust that the work may eventually be more fully completed.