II.—THE INFLUENCE OF MATHEMATICAL CONCEPTIONS ON BERKELEY'S PHILOSOPHY.

BY G. A. JOHNSTON.

THAT Berkeley was keenly interested in mathematics is well known. In the *Commonplace Book* a great deal of attention is paid to mathematical questions; it is noticeable, indeed, that Berkeley refers in its pages to mathematicians far more frequently than to philosophers. The extent of his interest in mathematics is indicated also by a group of early writings, *Arithmetica absque Algebra aut Euclide demonstrata*, and *Miscellanea Mathematica*, which includes papers, "de Radicibus Surdis," "de Cono Aequilatero et Cylindro eidem Sphaerae circumscriptis," "de Ludo Algebraico," and "Paraenetica quaedam ad studium matheseos praesertim Algebrae". Both these tracts were written in 1705 and first published in 1707. Belonging to the same period is the essay *Of Infinites*, which deals in part with the infinitesimal calculus. Berkeley deals also with mathematical questions in *The Principles* (1710) and in *De Motu* (1721), and his criticisms of the principles of the calculus in *The Analyst* (1734) and *A Defence of Free-thinking in Mathematics* (1735) are of some importance in the history of mathematics.

In this paper I do not propose to say anything of Berkeley's mathematics as such: what I wish to do is to indicate two respects in which his philosophy was affected by mathematical conceptions.

It is fairly clear that the conceptions of mathematics exercised on Berkeley's mind the same sort of influence as the idea of evolution exerted on the philosophy and literature of the second half of the nineteenth century. When Berkeley began to write, at the beginning of the eighteenth century, mathematics was *the* science. Mathematical work of all kinds had been encouraged by the wonderful results progressively achieved in the previous century and particularly in the immediately preceding two or three decades. Mathematics was revolutionised in 1637 by Descartes' discovery of the so-called cartesian or analytical geometry. The work of

Descartes and Cavalieri, who, following Kepler, developed the use of the principle of indivisibles, was extended and systematised by Wallis, Professor of Geometry at Oxford, in a series of important works extending from 1656 to 1686. The next great advance in mathematics was made when the fluxional or differential calculus was invented almost simultaneously and probably independently by Newton and Leibniz. These and other mathematical discoveries, whose importance was only coming to be fully realised when Berkeley was beginning to write, led to the entire reconstruction of the science, and rendered possible the further extremely rapid progress of pure mathematics and its application to the world of nature in mechanics and physics. At that time the place of mathematical and physical science was almost exactly similar to that occupied 150 years later by biological conceptions. One or two illustrations will help to give point to the analogy. When Richard Bentley was appointed to give the first course of Boyle Lectures on the Being of God, he wrote to Newton for instructions how to read the *Principia*, and in his lectures he applied the conceptions of the *Principia* just as theologians of later date applied the conception of evolution in their apologetics. Again, Locke, in spite of mathematical incapacity, assimilated as best he could the argument of the *Principia*, after having carefully inquired whether the mathematical calculations which he was unable to follow might safely be accepted. Mathematical conceptions form the warp and woof of the thought of the day; and Berkeley, like everybody else, was exposed to their influence.

At two points, one of them of central importance in his philosophy, Berkeley attempted to "apply" mathematical conceptions. By making use of comparatively recently discovered methods of calculation by signs and symbols he sought to give an explanation of nature and its laws by means of the relation of sign and thing signified, and thus establish an Algebra of Nature; and he endeavoured to develop an Algebra of Ethics by applying algebraic methods to the problems of morality.

To take first the Algebra of Nature. According to Berkeley's well-known theory, the relation between cause and effect in nature is a purely arbitrary one. Cause and effect are connected by no necessary tie; they bear to one another merely the relation of sign and thing signified. By experience we learn that such and such ideas are followed or attended by such and such other ideas; certain sequences and concurrences occur regularly and uniformly. The pre-

ceding ideas are not, Berkeley avers, the *causes* of the subsequent ideas; they are merely the *signs* that warn us that they will be followed by certain other ideas. Thus Berkeley's theory of causality becomes a doctrine of signs.¹

This doctrine of signs occupies a highly significant place in Berkeley's philosophy. "I am inclined to think," he says, "the doctrine of signs a point of great importance and general extent, which, if duly considered, would cast no small light upon things, and afford a just and genuine solution of many difficulties."² In particular, he makes use of the conception of signs in his theory of vision and in his account of universality in knowledge. But though the importance of the doctrine of signs to Berkeley's philosophy has been very generally recognised, it has never been made clear, so far as I am aware, that the use of signs in mathematics did much to suggest to Berkeley, or at least to confirm his belief in, the importance of a metaphysical theory of signs.

It must be remembered that Berkeley's great object, as he tells us again and again, is to simplify philosophy and "abridge the labour of study". Now, in mathematics it is the great function of signs to abridge the labour of study and to simplify methods and explanations. This function of signs is apt to be overlooked by us, for we take the use of signs in mathematics simply as a matter of course, and should be unable to conceive a mathematic without signs. But in Berkeley's day the extended employment of signs in mathematics was still almost a novelty, and he takes pains to point out the value of those branches of mathematics which are specially concerned with signs in the simplification of the sciences. "Modern algebra," he says, "is in fact a more short, apposite, and artificial sort of language."³ Now, philosophy has always suffered, Berkeley believes, from the ambiguity and unsuitability of the language with which it has been forced to work. What advances, then, might we not hope for, if we could employ in philosophical investigation a perfectly determinate and suitable terminology? Such a terminology, Berkeley hoped, might be supplied by signs akin to those of algebra. Algebra is *par excellence* the science of signs, and, according to Berkeley, a little attention to algebra and the way in which it uses its signs "may possibly help us to judge of the progress of the mind in other sciences; which, though differing in nature, design, and object, may yet agree in the general methods of proof and inquiry".⁴

¹ In Locke's classification of the Sciences (*Essay*, IV., xxi.) the third division of knowledge is termed "*Σημειωτική* or the doctrine of signs".

² *Alciphron*, ii., 343.

³ *Ibid.*, ii., 344.

⁴ *Ibid.*, ii., 342.

It is possible to realise the impression made upon Berkeley's mind by the conception of signs only if we keep in view the way in which, shortly before his day, signs had come to be used in mathematics and especially in algebra. I shall therefore give very briefly an account of the development of the use of signs in mathematics in the decades immediately preceding the time when these problems began to occupy Berkeley's attention.

The first signs to be used were naturally those of addition and subtraction (+ and -); yet even these were not generally accepted symbols till about 1630. And it was much later before uniformity was reached in the use of the other chief signs. From 1631 onwards English mathematicians used the sign $\times$ to denote multiplication, but many French mathematicians, following the usage of Descartes, indicated the operation by a dot. And it was denoted by Leibniz in 1686 by the sign $\cdot$. A similar lack of agreement existed as to the symbols with which to represent division. It was usually indicated by the method, copied from the Arabs, of writing down the quantities to be operated upon in the form of a fraction by means of a line drawn between them, in any of the forms $a-b$, a/b , or $\frac{a}{b}$. English mathematicians, however, frequently indicated it by a dot. In 1686 Leibniz used the sign $\div$. The symbol $=$ for equality was not commonly used till the time of Newton; previously the word was written out fully, or the signs ∞ or ∞ were used. The sign $::$ to denote the equality of two ratios was brought into common use by Wallis in 1686. The relations *is greater than* and *is less than* were, at the beginning of the eighteenth century, indicated either by our present signs $>$ and $<$, or by $\rhd$ and $\lhd$. The negative symbols $\nmid$ for *is not equal to*, $\nless$ for *is not greater than*, and $\nless$ for *is not less than* had not been introduced in Berkeley's time. The use of indices to denote the power to which a magnitude is to be raised had only comparatively recently become general. As early as 1637 Descartes used indices, but only positive integral ones, e.g. a^2 , a^3 . In 1659 Wallis used and explained fractional and negative indices, e.g. a^{-1} , $a^{\frac{1}{2}}$; and Newton was the first to use an index infinitely large, e.g. a^{∞} .¹

Now practically all these signs, it must be repeated, were comparatively new in Berkeley's student days; and their

¹ The above account of the development of signs in mathematics has been derived from W. W. R. Ball, *A Short History of Mathematics*, p. 212 ff.; F. Cajori, "The Works of William Oughtred," in the *Monist*, July, 1913, p. 441 ff.; and M. Cantor, *Geschichte der Mathematik*, *passim*.

use had hardly yet become common. Still, it was already clear how wonderfully they had contributed to the progress of mathematics. They had enabled it to advance by simplifying its methods, for before they were introduced all mathematical operations had to be written out fully in words, and mathematical demonstrations, unless they could be represented geometrically, were cumbrous and tedious.

Berkeley himself was greatly interested in the use of signs in mathematics. In the *Miscellanea Mathematica* he indulges in a perfect orgy of symbols. The appendix resembles nothing so much as a few pages from Russell and Whitehead's *Principia Mathematica*. And he suggests, in the short paper, "De Radicibus Surdis" (1707), as a simplification of the usual method of representing surd quantities, the introduction of a new symbolic notation of his own. Roots, he points out, might conveniently be represented by Greek letters: β , for instance, would express $\sqrt{b}$, δ would stand for $\sqrt{d}$, and so on. Similarly $\sqrt{bc}$ would be written $\beta\kappa$, and $\sqrt{\frac{\beta\delta\mu}{\epsilon}}$. But he sees that, if this notation were adopted,

it would not enable us to distinguish square roots from cube roots and those of higher powers; and he therefore makes the alternative suggestion that roots should be expressed by the same method of dots as was then used for fluxions, so that $\dot{a}$ would stand for $\sqrt{a}$, $\ddot{a}$ for $\sqrt[3]{a}$, $\ddot{\ddot{a}}$ for $\sqrt[4]{a}$, and so on.

Worthless as this is in itself, it is of value for the light it throws on Berkeley's interest in symbols as such. He was interested in them because they were still so new that changes such as he advocated might yet be suggested with some hope that they would be generally adopted. But another consideration weighed with him. Though fifty or even twenty-five years before his student-days signs had been used by mathematicians with but little uniformity, they had already by the time he began to write become fairly standardised, so that the same symbols always and everywhere meant the same thing. This meaning was, indeed, arbitrary and artificial, but for Berkeley the important thing was that it was a definite and determinate meaning. And by the use of similar signs in philosophy he hoped to be able to introduce exactness and accuracy, and at the same time secure results which could be demonstrated so that all who agreed in the meaning of the signs would be forced to give assent to the conclusions. And finally, he expected that by such an application of signs in philosophy it would be possible to simplify it, and rescue it at once from the "meaningless subtleties" of the

Schoolmen, and from the "occult complexities" of the Cartesians.

When Berkeley speaks of the importance of signs, he is thinking specially of the use of signs in algebra. His interest in algebra is proved not only by his frequent references to it in his works, but also by the juvenile publication, *De Ludo Algebraico*. In this tract, after explaining the algebraic game, which he advocates on the characteristic grounds that it is both as pleasant a recreation as chess and a useful exercise in algebra, he concludes by making the most extraordinary claims for algebra. It may usefully be applied, he says, "to the whole extent of mathematics, and every art and science, military, civil, and philosophical". "Through all of these," he continues, "is diffused the wondrous power of algebra. By all it is regarded as a great and wonderful art, the topmost pinnacle of human knowledge, and the kernel and key of all mathematical science."¹

After thus giving his own testimony to the value of the application of algebra in the sciences, Berkeley proceeds to appeal for confirmation to the evidence of Descartes, Locke, and Malebranche.² Unfortunately the passages in these philosophers to which Berkeley refers shed very little light on the *application* of algebra. Locke, whose *Conduct of the Understanding* Berkeley refers to, speaks very favourably of algebra, but he says nothing of the possibility of applying its methods directly to other sciences.³ And Malebranche, to whom Berkeley also refers, though he expresses himself with more vigour and enthusiasm than Locke, does so with equal vagueness. He merely insists, as Berkeley does, on the ease and simplicity with which by means of algebra we are able to abridge the labour of study; and he declares that algebra, along with arithmetic, forms the foundation of all the sciences, and supplies the means by which they may be acquired.⁴ The point on which both Malebranche and Locke insist is the value of algebra in *simplifying* the sciences in which it is applied.

In Berkeley's day the most notable example of the application of algebra in mathematics was supplied by the analytical geometry of Descartes. Descartes substituted simple algebraic methods, which could be applied universally, for a more or less cumbrous geometry requiring new constructions for each particular problem attacked. Analytical geometry gives us a method of representing

¹ *Works*, iv., 60.

² *Miscellanea Mathematica*, iv., 62.

³ *Conduct of the Understanding*, § 7.

⁴ *Recherche de la Vérité*, VI., i., 5.

curves and curved surfaces by means of simple algebraic equations. Instead of having to draw a special figure for each case of a problem, as we must do in ordinary geometry, it is only necessary to know the general equation to the curve, and any particular property may then immediately be deduced by an application of ordinary algebra. In this way the application of algebraic methods in geometry immensely simplifies what would otherwise be exceedingly complicated operations.

The possibility of applying algebra outside mathematics had occurred to many thinkers in Berkeley's day, and algebraic methods had been applied, often very foolishly and fantastically, to all sorts of problems. Berkeley himself notes its application in medicine and natural philosophy; and he refers, with evident appreciation, to the use that had been made of it in demonstrating the credibility of human testimony. As an example of this he gives a reference to an article in the *Philosophical Transactions of the Royal Society*; and it seems worth while, as an instance of the sort of "application" he considered feasible, to indicate the scope and argument of the article in question.

In this article,¹ which is anonymous, the writer considers the credibility of evidence. He examines, for instance, the reliability of the report that £1200 has been given to him by somebody. He assumes that the credibility of the average report is $\frac{5}{6}$ absolute certainty, and that if it be at second-hand it will be $\frac{5}{6}$, i.e. $\frac{5}{6}$ of $\frac{5}{6}$, and so on. This may be stated algebraically as follows, if we put a for the share of certainty given by a single reporter, and c for what is lacking to make the certainty complete. The degree of certainty at first-hand is $\frac{a}{a+c}$, at second-hand $\frac{a^2}{(a+c)^2}$, at third-hand $\frac{a^3}{(a+c)^3}$, and so on. Other questions considered in the article are the credibility of oral tradition and the accuracy of written tradition involving several copies of the original document. In the same year as this paper appeared (1699) a book was published bearing a title copied from Newton (*Theologia Christianæ Principia Mathematica*), and dealing with the same problems as the article. The author of this book, who may also have been the writer of the article, was one John Craig. He calculates by mathematical methods that the evidences of Christianity, gradually deteriorating, will be

¹ "On the Credibility of Human Testimony," *Philosophical Transactions*, 1699, vol. xxi., No. 257, p. 359.

reduced to nil in 3150 A.D., and that a new revelation will then become necessary.

Berkeley avoided the absurdities and extravagances of such "applications" of algebra, but he was actuated by the same spirit in his attempts to apply algebraic methods to the relations of signs in Nature.

Just as algebraic signs suggest to us, or enable us to infer,¹ the things they signify (e.g. from the collection of signs $x^2 + y^2 = c^2$ we infer, according to the cartesian system, a circle with its centre at the origin), so the signs which we see in Nature suggest to us, or enable us to infer, the things they signify. Thus, to use Berkeley's illustration, a fire which I see suggests to me, or enables me to infer, that if I approach too near to it I shall suffer pain.² Similarly, the noise that I hear suggests to me, or enables me to infer, that some sort of collision or concussion has taken place.

The relation between the sign and the thing signified is not necessary. The sign does not immediately and inevitably suggest the thing it signifies; the relation between them must be learned. To the savage the group of signs $x^2 + y^2 = c^2$ does not immediately suggest a circle; the expression suggests a circle only to the man who has learned the relation between sign and thing signified. So, the fire does not necessarily suggest pain; it suggests pain only to "the burnt child," only to the person who has learned the relation between the sign and what it signifies.

But though the relation is not necessary, it is uniform. It is uniform—to use a modern term which well expresses Berkeley's meaning—within a certain universe of discourse. Within the universe of discourse of cartesian geometry $x^2 + y^2 = c^2$ uniformly enables us to infer a circle, and within the universe of discourse of the Earth fire uniformly enables us to infer pain if we approach too closely.

But Berkeley insists that the relation is an arbitrary one. The choice of the particular group of signs $x^2 + y^2 = c^2$ to represent a circle is perfectly arbitrary. Yet it always enables us to infer a circle, because there is universal agreement among mathematicians as to the meaning of these signs. Similarly, the relation between fire and pain is arbitrary, but the latter always allows us to infer the former, because it has been so decreed by God. It is due to the arbitrary though not capricious will of God that certain natural signs always suggest certain natural things signified. The connexion is purely arbitrary.

¹ Between suggestion and inference there is an important epistemological distinction. See Berkeley's *Theory of Vision Vindicated*, § 42.

² *Principles*, § 56.

And this, Berkeley argues, is all we mean by causality. Causality is not the relation of cause and effect, it is the relation of sign and thing signified. The fire that I see is not the *cause* of the pain I feel on approaching it too closely, it is the *mark* or *sign* that forewarns me of it.¹ Similarly, the noise that I hear is not the *effect* of the collision, but the sign that enables me to infer that a collision has taken place. The sign may thus be either what is commonly called the cause or what is commonly called the effect. If it is the "cause," it suggests, as the thing signified, the "effect"; and if it is the "effect," it suggests, as the thing signified, the "cause".

Berkeley thus implies the strict correlativity of "cause" and "effect," and with such a doctrine as the plurality of causes he would have no sympathy. Every sign in Nature is correlated by God with some one thing which it signifies; there is a pre-established harmony between them, and, as the sign strictly suggests only the one thing signified, so the thing signified suggests only the one sign. A thing signified cannot be signified by a plurality of signs; it is suggested only by its own proper sign.

Nature is systematically organised by God so that signs and things signified preserve this one-one relation. As the Language of Nature—to use Berkeley's own phrase—is a perfect language (for it is the language of God), each word in it stands for some particular thing or idea, and each thing or idea in the universe has its appropriate and particular name. Thus, in the mind of God, Nature is absolutely systematic, and signs and things signified are perfectly adjusted. This divine language constitutes, for Berkeley, the system of the laws of Nature. The language of Nature reveals a "consistent uniform working," and shows that its laws are "connexions established by the Author of Nature in the ordinary course of things".² These laws of Nature, representing a pre-established connexion in the mind of God, are absolutely settled and fixed, and in accordance with them everything in Nature takes place with perfect uniformity.

Now, as Berkeley points out, men often doubt the uniformity of Nature and the universality of its laws. The reason for this is that the laws of Nature are not self-evident. They need to be learned, and the universe may well seem to be a chaos before we have learned its meaning. This meaning is not supernaturally revealed to us at birth. God, it is true, excites in us from time to time certain ideas which are

¹ *Principles*, § 65.

² *The Theory of Vision, or Visual Language Vindicated*, ii., 398.

connected by set rules in his mind, but he does not explain their connexion to us all at once. We must learn by experience, we must acquire God's notation, as we have to learn that of Leibniz and Newton. We understand the laws of Nature, "the set rules or established methods wherein the mind we depend on excites in us the ideas of sense,"¹ only when we are able to interpret God's symbolism, just as we understand the theorems established in the *Principia* only when we are acquainted with the notation which it employs. Hence it is the great task of science to try to understand the divine symbolism. "It is the searching after and endeavouring to understand this Language (if I may so call it) of the Author of Nature that ought to be the employment of the Natural Philosopher."²

In the process of seeking to understand this divine language of the laws of Nature we may often attain some knowledge of the sign without fully or exactly comprehending what it signifies. The sign may suggest *something* to us, and we may be able to make use of it, though we may be quite unable to formulate precisely what it does suggest. Here again the analogy of mathematics makes Berkeley's meaning clear. Such signs or groups of signs as $\sqrt{-1}$ and π mean something, and may be used in mathematical operations, though it is impossible to express numerically exactly what they suggest. So, even though it be impossible to explain precisely what certain signs in Nature suggest, we may make use of these signs, and may indeed maintain that, though we cannot formulate them exactly, there is something that they suggest.

But in general we are able to learn by experience the precise relation between sign and thing signified. God follows certain rules in the organisation of Nature, and, as men succeed in discovering these rules, the relation of sign and thing signified becomes ever clearer. God creates certain organisms in much the same way as men combine letters in words and words in sentences. As the relations of words are clarified when they appear in sentences, so the relations of things are elucidated when they are seen in their proper context as elements in machines or organs in organisms.³

The question may be raised, why, if God is the ultimate and omnipotent cause, he requires organisms of complex structure to produce effects which he could equally well have created by a single fiat of his will. To this question Berkeley's answer is that all the elaborate organisation and mechanism is "for our information". It is not necessary

¹ *Principles*, § 30.

² *Ibid.*, § 66.

³ *Ibid.*, § 65.

for the production of the effects themselves, but it is essential in order that they should occur "according to the laws of Nature".¹ Things must be produced by God by the same methods and in accordance with the same processes, in order that we, perceiving the appropriate signs, may have due warning that the things signified will follow.

Thus the two functions of the laws of Nature or the methods of God's operation are (a) to guarantee the uniformity of experience, and (b) to enable us to use foresight for the benefit of life. Without these two conditions of experience knowledge and action would be alike impossible. But, as it is, we are able to acquire scientific knowledge of Nature, to pass judgments of value on actions, and to predict the future with sufficient accuracy to make practical activity fruitful.²

In all this Berkeley is oscillating uncomfortably between a theocentric and an anthropocentric view of the universe. From the point of view of knowledge the balance dips towards the theocentric theory, but in regard to practice Berkeley is decidedly anthropocentric.

On the one hand, the reality of the universe is due entirely to God. From the human point of view the laws of Nature by which the world is governed seem to have no reality. They are simply convenient names which indicate the regular order with which in our experience sign and thing signified constantly occur. A law of Nature is not even a category which *we* apply.³ It is nothing but an arbitrary relation devised by God for our information. But the apparent unreality of the laws of Nature vanishes when we survey them *sub specie aeternitatis*, for as thus conceived they exist in the mind of God and have perfect reality; they are not only real but the forms in which all reality exists. From the point of view of knowledge the world is necessarily theocentric.

On the other hand, from the practical standpoint, the centre of the universe is man. Though God is the ultimate cause, and acts always in accordance with his own will, all his activity is directed to secure the greatest value for life to finite persons. He goes to the trouble of putting countless cogs in machines, and innumerable organs in organisms (all from his point of view useless), solely for the benefit of life of finite spirits. The whole universe is benevolently ordered by God for man's advantage; and thus, varying a well-known title, we may say *servus servorum Deus*.

We now turn to Berkeley's application of mathematics in ethics. That the possibility of an Algebra of Ethics was

¹ *Principles*, § 62.

² *Ibid.*, § 62.

³ *Ibid.*, § 66.

suggested to him by Locke is made clear in the *Common-place Book*. "N.B.," he says, "To consider well what is meant by that which Locke saith concerning algebra—that it supplies intermediate ideas. Also to think of a method affording the same use in morals, etc., that this doth in mathematics."¹ Now, Locke had himself given a hint of the way in which mathematical methods might be followed in a demonstrative moral science. According to his theory of knowledge, certainty means simply the agreement or disagreement of our ideas, and demonstration consists in making clear that agreement by employing intermediate ideas or mediums. In mathematics algebra had been of use in supplying these intermediate ideas, and Locke is inclined to think that by applying algebra in ethics a demonstrably certain system will be produced. "I doubt not," he says, "but from self-evident propositions, by necessary consequences, as incontestable as those in mathematics, the measures of right and wrong might be made out."² For Locke mathematics and ethics alike are pure *a priori* sciences, independent of the matter of fact of experience. If they had to do with concrete experience, they would consist of (a) simple ideas, or (b) complex ideas of substance, and in neither case would the science be demonstrative, for (a) simple ideas give us knowledge that is "barely particular," from which no universal propositions can be inferred, and (b) the general knowledge we gain from complex ideas of substance is "merely probable".

Berkeley never worked out his algebra of ethics. But he said enough to show that his system would have diverged widely from that of Locke. The difference between their theories of ethics would have been exactly parallel to that which throws into such marked contrast their theories of mathematics. For Locke mathematics is a pure science dealing with relations of universal ideas, and abstracted from all concrete existence. On the other hand Berkeley holds that mathematics is essentially practical. The speculative parts of mathematics, which are concerned with *difficiles nugae*, are cut away by the New Principle, and only those portions of arithmetic and geometry and algebra that are "useful" and "practical" will remain.³

In precisely the same way Berkeley's theory of ethics differs from Locke's. For Locke ethics is a pure science, having as its subject-matter relations of ideas, and omitting all question of the realisation of these ideas in the concrete

¹ *Works*, i., 40.

² *Essay*, IV., iii., 18; cf. III., xi., 16.

³ Cf. *Principles*, §§ 121, 131.

matter of fact of moral experience. But Berkeley's view is very different. Ethics is an applied or practical science: it is concerned throughout with actual conduct; and its subject-matter is moral experience. And its great aim is the improvement of conduct and the advancement of the good cause of the world. Hence, though algebra may be regarded from one standpoint as a pure science, since it deals with signs in abstraction from the things that they signify, the algebra of ethics, being concerned with actual moral experience, must be a department of applied mathematics. That is what Berkeley means by saying, "Morality may be demonstrated as mixt mathematics".¹

Berkeley agrees with Locke that ethics is a demonstrative science. But by that he does not mean, as Locke would have said, that its demonstrability consists in proving relations of ideas by means of intervening ideas.² In Berkeley's view, ethics is not concerned with ideas at all, but with marks or signs; and it is by means of these words or signs that it must be demonstrated. "We have no ideas," he asserts, "of virtues and vices, no ideas of moral actions."³ In other words, we can neither perceive nor imagine virtue or vice in abstraction from concrete virtuous and vicious actions. Thus, if the demonstrability of ethics depends on the consideration of relations between ideas, as Locke maintained, Berkeley fears that it will be impossible to arrive at demonstrative truth in ethics; and he insists that those who agree with Locke that we may have ideas of morals have given themselves, in the demonstration of ethics, an impossibly difficult task.⁴ It is impossibly difficult, because we can have no certainty about ideas, as Locke supposed, but only about words.⁵ We may, indeed, *reason* about ideas, but by doing so we shall never attain demonstrative certainty, for "demonstration can be only verbal".⁶ Perfect demonstration, that is, is possible only when we are dealing with words or signs. And Berkeley states as his conviction that "to demonstrate morality it seems one need only make a dictionary of words, and see which included which".⁷

This utterance in itself is perhaps rather cryptic, but, if we bear in mind Berkeley's general view of the applicability of algebra in the various departments of knowledge, its meaning becomes plain. In his view algebra is "purely verbal" and "entirely nominal";⁸ it deals with relations of arbitrary signs, and when they are employed demonstration is possible

¹ *Works*, i., 46.

² *Ibid.*, i., 40-43.

³ *Ibid.*, i., 36.

⁴ *Ibid.*, i., 38.

⁵ *Ibid.*, i., 43.

⁶ *Ibid.*, i., 50.

⁷ *Ibid.*, i., 39.

⁸ *Ibid.*, i., 47.

because there is uniformity in their use. Though they are arbitrary, their meaning is universally agreed upon, and therefore demonstration by their means is of absolute cogency. Now, words are not so well suited for demonstration as signs, because there is not universal agreement as to the meaning of words. Mathematicians are universally agreed on the meaning of such signs as + and -; but the meaning of the word "truth" or "good" is not a matter of universal agreement. But Berkeley believed that this was not a fatal or ultimate defect in words. It was only in the last half century before he wrote that mathematicians had attained uniformity in the use of signs, and he hoped that it would soon be possible to reach similar agreement as to the use of words. To this end it would be necessary to make a universal dictionary, whose definitions would be sufficiently authoritative to command universal assent. If the meaning of words were settled, propositions in ethics would be demonstrated as readily as propositions in mathematics. It is universally agreed among mathematicians that such propositions as

$$2 + 2 = 4$$

$$\text{or } \log(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

are true.¹ In these cases the meaning of all the terms used is a matter of universal agreement. And if similar agreement existed as to the meaning of words, then such ethical propositions as "Man is free" and "God ought to be worshipped" would be universally admitted to be true, for they would be absolutely demonstrable. The latter proposition, for instance, would be readily demonstrated, as Berkeley says, "when once we ascertain the signification of the words God, worship, ought".²

Berkeley's former example of a demonstrable proposition in ethics supplies a good illustration of what he means by saying, as he frequently does, that ethics deals with the relation of *inclusion*. He mentions that it is part of the task of demonstration in ethics, after we have compiled our universal dictionary, to see which words *include* which. And elsewhere in the *Commonplace Book* he points out that ethics is concerned with "definition, or inclusion, of words,"³ and that it deals with "signification, by including".⁴ What he means by this is that if we take such a

¹ This series was discovered independently in the seventeenth century by Mercator and Saint-Vincent, and though it was not used as an example by Berkeley it serves well to indicate his meaning.

² *Works*, i., 41; cf. i., 32.

³ *Ibid.*, i., 55.

⁴ *Ibid.*, i., 37.

proposition as "Man is free," it is possible to demonstrate it when we know that "free" is included in "man". Given definitions in our universal dictionary such that the definition of "free" is comprehended within that of "man," and the proposition "Man is free" is universally demonstrable. In this theory of the nature of inclusion Berkeley has been influenced by mathematical analogies. The expression $\log(1+x)$ includes the series $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 \dots$. The series is analysed out of it. So, Berkeley believes, by an application of mathematical methods in ethics we shall be able to demonstrate relations of inclusion and exclusion between words.

Berkeley's mathematical theory of ethics is in harmony with his general philosophical position. For, according to his theory of knowledge, we reason on a particular which stands for all other particulars of the same kind. As representing other particulars it becomes a sign, and performs the functions of universality. But Berkeley insists that this function is not an idea, and he objects to Locke's theory of ethics on the ground that the abstract ideas which he had posited do not exist either in mathematics or in ethics. It is impossible, Berkeley has shown, to frame an abstract idea (which means, in his terminology, an abstract concrete percept or image) of triangle. Equally impossible is an abstract idea of justice. In ethics we are never concerned with the abstract, but always with particular instances of just or unjust actions. What we do is to take this or that just act, ignore all irrelevant features, and make it stand for all other just acts. On these particular cases we may reason in precisely the same way as we do in mathematics. In mathematics we give names to these particulars, and these names or signs are universal, or rather, perform the functions of universality. Similarly in ethics signs are used, these signs being words and not ideas.

The only obstacle mentioned by Berkeley in the way of such a science of ethics is the very great difficulty of reaching agreement with regard to its definitions. The definitions which mathematics employs are not questioned, because the student comes to them with no preconceived ideas or prejudices. He is willing to take them on trust. But in ethics it is otherwise. Men approach the subject with pre-suppositions of their own, and, clinging to these primitive convictions, they refuse to come to any agreement in the definition of terms.

One very real difficulty which Locke raised is denied by Berkeley. Locke pointed out that the complexity of moral

ideas increases the difficulty of dealing with them on the mathematical method. But in this difficulty Berkeley sees nothing. Yet if we extend the term "complexity" to include the relations and context of moral experience,¹ the difficulty becomes a very pertinent one. On Berkeley's theory, if we take a particular triangle it is possible to abstract what is irrelevant to its triangularity, and the particular may be taken to stand for or represent all other particulars of the same kind. And, as we have seen, Berkeley thinks the same thing may be done in ethics. But, of course, it is not thus possible to isolate a particular just act. If it be cut loose from its context, it may no longer be a just act, for its justice may consist precisely in the complex relations in which it stands to its environment. But though Berkeley was not aware of this difficulty in the days of the *Common-place Book* it is clear from *Alciphron* that he came to appreciate it later. And this may well have been one of the reasons why he abandoned the project of writing a mathematical treatise on ethics.

¹ This involves a departure from Locke's meaning of the term.