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and if Vx' is the perpendicular on Ox ,

$$(16) \quad dz = HG \tan \omega d\omega = 2F \tan \omega \frac{Vx'}{OV} = 2F \frac{x'x}{OV},$$

$$(17) \quad z = 2F \log \frac{OV}{OA} = 2F \log \sec \omega = 2F \log \sec \frac{t}{G},$$

and reckoning the time t backward for the culminating point, it is proportional to the circular sector AOQ , as in the *Principia*.

Projected vertically upward with infinite velocity, the time to the highest point of culmination is $\frac{1}{2}\pi G$, but the height ascended is infinite.

Newton's use of the three letters F , G , H should be followed as the standard notation by all writers on this theory of bomb dropping.

But the Newtonian geometrical method followed above is replaced to-day by the more concise analytical procedure of the Calculus.

The same theory will apply to a number of associated questions, such as the starting and stopping of a train or steamer, the action of the hydraulic buffer, the rise of a bubble through water, the subsidence of sediment or precipitate, the ascent of smoke up a chimney and the settlement down again in the air; as well as for the original application to Newton's experiments with glass globes, let fall in the dome of St. Paul's.

Staple Inn, Dec. 20, 1916.

G. GREENHILL.

507. [Kl. 8. a.] To prove that the area S of a quadrilateral whose sides taken in order are a , b , c , d is given by

$$S^2 = (s-a)(s-b)(s-c)(s-d) - abcd \cos^2 \alpha,$$

where $2s = a + b + c + d$, and 2α is the sum of two opposite angles.

$$S = \Delta + \Delta',$$

$$\Delta \cot B = \frac{1}{4}(a^2 + b^2 - AC^2),$$

$$\Delta' \cot D = \frac{1}{4}(c^2 + d^2 - AC^2);$$

$$\begin{aligned} \therefore \left\{ \frac{a^2 + b^2 - c^2 - d^2}{4} \right\} &= (\Delta \cot B - \Delta' \cot D)^2 \\ &= \Delta^2 \operatorname{cosec}^2 B - \Delta^2 - 2\Delta\Delta' - 2\Delta\Delta'(\cot B \cot D - 1) + \Delta'^2 \operatorname{cosec}^2 D - \Delta^2 \\ &= \frac{1}{4}a^2b^2 + \frac{1}{4}c^2d^2 - \frac{1}{2}abcd \cos 2\alpha - (\Delta + \Delta')^2 \\ &= \dots\dots\dots + \frac{1}{2}abcd - abcd \cos^2 \alpha - (\Delta + \Delta')^2. \\ (\Delta + \Delta')^2 &= \frac{1}{16}\{4(ab + cd)^2 - (a^2 + b^2 - c^2 - d^2)^2\} - abcd \cos^2 \alpha \\ &= \frac{1}{16}\{(a+b)^2 - (c-d)^2\}\{(c+d)^2 - (a-b)^2\} - abcd \cos^2 \alpha \\ &= (s-a)(s-b)(s-c)(s-d) - abcd \cos^2 \alpha. \end{aligned}$$

R. F. DAVIS.