shortened by tables of radices to two or three digits, if multiplication tables are available.

The general neglect of such a simple and convenient method may have been due to the idea that Briggs' radix method required the use of large and expensive tables, which is far from being the case.

Flower's pamphlet is very scarce and very confusing. He possibly calculated the logarithms of the radices by the aid of a table of the successive square roots of ten, and speaks almost as though he considered the nine digits to be roots of ten. He also mixed up the difficulty he found in calculating the logarithms of the radices with the much more simple matter of using them when found.

Atwood's pamphlet is still more scarce; there are copies in the libraries of the Royal and Royal Astronomical Societies, but not in the British Museum; since it deals with natural logarithms only, it would be of little use to practical computers. It seems worthy of reprinting.

A copy of Flower's tract came into the hands of Leonelli, who published his *Supplément Logarithmique* at Bordeaux in 1802, and an edition was published at Dresden by Leonhardi in 1806. He gave tables of natural and common positive radices to twenty places, and also a table of common two-figure radices to fifteen places. Gray claimed as new a two-figure negative table, 1846, and a two-figure positive table, 1848. The *Supplément* was also so scarce as to be almost unknown until Hoüel reprinted the tables of radices in 1858 and 1866, and the whole work in 1876. Schrön also reprinted the positive radices to sixteen places in 1859.

Perhaps the most convenient and powerful tables of three-figure radices are those of Gray, 1876, to twenty-four places. The one-figure positive and negative tables of Thoman to twenty-seven places, Paris, 1867, seem to be out of print and difficult to procure second-hand.

SYDNEY LUPTON.

THE SIMPLE PENDULUM.

THE current elementary discussion of the Simple Pendulum is unsatisfactory, in that the problem is not reduced with sufficient directness to a case of S.H.M. It has to be artificially prefaced by consideration of a curvilinear motion in which the tangential resolute of the acceleration is proportional in magnitude to the arcual distance from a point of the path; and the discussion of this motion raises new points of difficulty altogether out of proportion to its significance in this connection. The net result is that the student's appreciation of this first of the important applications of S.H.M. to "small oscillations" is lost, in dismay at finding that, even after he has mastered the essential difficulties of the S.H.M. itself, the first good application of it brings yet another awkward hurdle. And the physicist or engineer may well make this another case for railing impatiently at the devices for "dodging the Calculus" by which elementary theory is so apt to be obscured. Nevertheless, the teacher of Mathematics knows how important it is to postpone such Calculus difficulties as, *e.g.*, to pave the way to the complete primitive of the S.H.M. differential equation and its uses (one of which gives the only clear-cut way of handling the ordinary discussion of the Simple Pendulum). But the postponement must not be obtained at the disproportionate cost of artificial complications which, for the sake of "elementary" treatment, cast a fog round the important features of the argument, without contributing anything of independent value to the student's store of knowledge.

The elementary treatment of S.H.M. is well worth preserving. The relation of S.H.M. to the uniform circular motion is of fundamental importance to a sound knowledge of S.H.M.; and the general gain from the elementary investigation of the S.H.M. equations is a better understanding of the fundamental dynamical method of resolution—from the opposite point of view to that which obtains in the other elementary case, viz. that of the (parabolic) motion of constant acceleration.

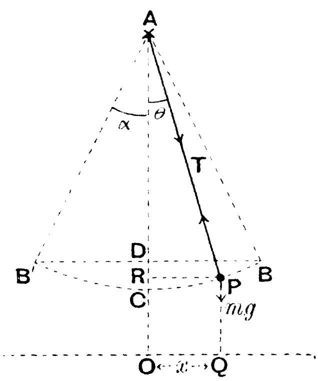
It is this principle of resolution that wants emphasising in the theory of the Simple Pendulum. The horizontal resolute-motion is what really interests us, in the motion of a pendulum; and consideration of it keeps a close analogy with the parent-theory of the S.H.M.: another case of a straight motion related as resolute to a circular motion.

(i) With an obvious notation (see diagram), if f_x denote the measure of the horizontal resolute of the acceleration, the equation of horizontal resolution is

$$m\,.\,f_x = -T\,.\sin\theta$$
$$= -T\,.\,\frac{x}{l}.$$

This is not an equation of S.H.M., since T will obviously vary during the motion; but for a *small* oscillation a first approximation may be obtained by neglecting squares and products of small quantities, i.e. *quantities which would have the zero value if the pendulum were at rest in its equilibrium-position AC.* Hence, since x and f_x are small quantities, as thus defined, and since T differs from $m\,.\,g$ (its equilibrium value) by a small quantity, it follows that the first approximation to the equation of horizontal resolution is the S.H.M. equation

$$f_x = -(g/l)\,.\,x.$$



Thus *the horizontal motion is approximately a* S.H.M., and the measure of its period $= 2\,.\,\pi\,.\sqrt{l/g}$.

(ii) The discussion may be made complete by actually expressing T in terms of a geometrical variable, as follows. The figure explains itself.

Using the equation of normal resolution

$$m\,.\,(v^2/l) = T - m\,.\,g\cos\theta,$$

and the energy equation

$$m\,.\,v^2/2 = m\,.\,g\,.\,l\,.\,(\cos\theta - \cos\alpha),$$

in which α specifies the angular amplitude of the oscillations, we deduce the equation

$$T = m\,.\,g\,.\cos\theta + 2\,.\,m\,.\,g\,.\,(\cos\theta - \cos\alpha)$$
$$= m\,.\,g\,.\,(3\,.\cos\theta - 2\,.\cos\alpha).$$

Hence $$T - m\,.\,g = m\,.\,g\,.\{2(1-\cos\alpha) - 3(1-\cos\theta)\}$$
$$= m\,.\,(g/l)\,.\,(2\,.\,h - 3\,.\,z),$$

if h, z specify the heights of B, P (or D, R) above C;

and, therefore, $$-m\,.\,g\,.\,h/l \leqslant (T - m\,.\,g) \leqslant 2\,.\,m\,.\,g\,.\,h/l.$$

The error in taking $m\,.\,g$ for T is therefore a small quantity of the order of h; and therefore a small quantity of the second order relative to the measure of the horizontal amplitude of the oscillations. And the S.H.M.

approximation to the equation of horizontal resolution involves the neglect of a term which is of the third order relative to the terms retained.

Note that $T=m.g$ when $z=2.h/3$, giving a simple specification of the position in which the tension of the moving pendulum is equal to the equilibrium tension. D. K. Picken.

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ANSWER TO QUERY.

[**71. vol. v. p. 330**] Mr. Shovelton's proof (vol. vii. p. 153) may be shortened by using central-difference operators. The theorem itself may be made clearer by splitting it up into two:

(I) If $\phi(x)$ is an odd polynomial in x, and

$$\phi(x+\tfrac{1}{2}) \equiv A_0 + A_1(x, 1) + A_2(x, 2) + A_3(x, 3) + \ldots,$$

then
$$A_0 - \tfrac{1}{2}A_1 + (\tfrac{1}{2})^2 A_2 - (\tfrac{1}{2})^3 A_3 + \ldots = 0.$$

(II) If $\psi(x)$ is an even polynomial in x, and

$$\psi(x) \equiv B_0 + B_1(x, 1) + B_2(x, 2) + B_3(x, 3) + \ldots,$$

then
$$B_0 - \tfrac{1}{2}B_1 + (\tfrac{1}{2})^2 B_2 - (\tfrac{1}{2})^3 B_3 + \ldots = \psi(0).$$

To these we may add:

(III) If $\psi(x)$ is an even polynomial in x, and

$$\psi(x+1) \equiv C_0 + C_1(x, 1) + C_2(x, 2) + C_3(x, 3) + \ldots,$$

then
$$C_0 - \tfrac{1}{2}C_1 + (\tfrac{1}{2})^2 C_2 - (\tfrac{1}{2})^3 C_3 + \ldots = \psi(0).$$

The following properties are involved, the operators being for increment 1 in x:

(i) If $F(x) \equiv P_0 + P_1(x, 1) + P_2(x, 2) + P_3(x, 3) + \ldots$, then, as Mr. Shovelton points out, $P_0 = F(0)$, $P_1 = \Delta F(0)$, $P_2 = \Delta^2 F(0)$, ...

(ii) $1 + \tfrac{1}{2}\Delta = \tfrac{1}{2}(E+1) = \tfrac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})E^{\frac{1}{2}} = \mu E^{\frac{1}{2}}$.

(iii) If $\phi(x)$ is an odd polynomial in x, then $f(\delta^2)\phi(0) = 0$; and, in particular, $\mu^{-1}\phi(0) = 0$.

(iv) If $\psi(x)$ is an even polynomial in x, then

$$f(\delta^2)\psi(x) = f(\delta^2)\psi(-x) = f(\delta^2)\tfrac{1}{2}\{\psi(x) + \psi(-x)\};$$

and, in particular,

$$\mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\tfrac{1}{2}\{\psi(\tfrac{1}{2}) + \psi(-\tfrac{1}{2})\} = \mu^{-1}\mu\psi(0) = \psi(0).$$

Hence we have the following:

(I) $A_0 - \tfrac{1}{2}A_1 + (\tfrac{1}{2})^2 A_2 - (\tfrac{1}{2})^3 A_3 + \ldots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\phi(\tfrac{1}{2}) = \mu^{-1}E^{-\frac{1}{2}}\phi(\tfrac{1}{2}) = \mu^{-1}\phi(0) = 0.$$

(II) $B_0 - \tfrac{1}{2}B_1 + (\tfrac{1}{2})^2 B_2 - (\tfrac{1}{2})^3 B_3 + \ldots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\psi(0) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\mu\psi(0) = \psi(0).$$

(III) $C_0 - \tfrac{1}{2}C_1 + (\tfrac{1}{2})^2 C_2 - (\tfrac{1}{2})^3 C_3 + \ldots$

$$= (1 + \tfrac{1}{2}\Delta)^{-1}\psi(1) = \mu^{-1}\psi(\tfrac{1}{2}) = \mu^{-1}\psi(-\tfrac{1}{2}) = \mu^{-1}\mu\psi(0) = \psi(0).$$