A Graphic Method of Predicting Occultations.

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[With a plate].

The graphic method of predicting occultations to be described in this article was devised by the writer for his own use several years ago. It is based on the method of Bessel, that is, the path of the place of observation and that of the cylindrical shadow cast by the moon in the light of the star are projected orthographically on a plane perpendicular to the light of the star. It applies to stars for which the usual data are given in the Ephemerides. A remark in the annual report of the Berlin Observatory for 1894 (V. J. S. 1895, pp. 139 and 140) on the preparation necessary for observations of this kind, suggested the idea of making this method known, although it does

not entirely solve the general problem mentioned by Dr. Foerster.

With the method here given the preparatory diagram is made once for all occultations to be determined for a given place of observation. This diagram once made, five minutes are sufficient for the prediction of all the circumstances of an occultation, viz. the times, the position angles and the angles from the vertex.

After the manner of making the diagram has been explained, the graphic construction of the data given by the Ephemeris will be shown, and lastly the circumstances of an occultation will be determined.

I. Construction of the Diagram.

If the diagram were to be constructed for a single occultation, it would contain but a single ellipse, namely the path of the place of observation as seen from the star. But as the diagram is to serve for all the occultations visible at a given place, it must contain the paths of the place of observation as seen from all stars that can be occulted, or at least from stars at certain intervals of declination from $+30^\circ$ to -30° , these intervals being sufficiently small to allow of the easy estimation of all intermediate ellipses. In the accompanying diagram the interval is 5° . These ellipses have the following constants, where φ is the latitude of the place, δ , the declination of the star and τ , its hour angle. The semi-major axes of all are the same, $a = \cos \varphi$, the semi-minor axis of each is $b = \cos \varphi \sin \delta$, and the ordinate of the centre of each is $d = \sin \varphi \cos \delta$. The coordinates of the points of any one are $\xi = a \sin \tau$, and $\eta = d - b \cos \tau$. The hour angle, τ , is the time scale for the place of observation, and is given by the system of parallel lines, the abscissa of each being, of

course, ξ . The earth's radius may be taken as constant, and the geographic may be used for the geocentric latitude, without introducing any sensible error.

It is well to indicate the extremities of the major axes of the ellipses and their points of tangency with the circumference of the earth's disk or the horizon line. This has been done in the diagram for all ellipses except for $\pm 5^\circ$. The ordinates of the former are, of course, d , and those of the latter are $\sin \varphi \sec \delta$.

Only a single copy of the diagram just explained is required for each place of observation, because there is no need of drawing any lines or marking any points on it. The additional lines on the diagram are therefore unnecessary, but they have been drawn in order to obviate the necessity of a second figure. Of course, if one wishes, the diagram may be lithographed and one copy used for each occultation, the necessary lines being actually drawn upon it.

II. The Graphic Construction of the Data given by the Ephemeris.

In order to show the practical use of the diagram, it has been constructed for $\varphi = +52^\circ 3'$, the latitude of Berlin, and the occultation of δ Cancr. on Dec. 22, 1896, has been investigated for that place. The notation and data used are those of the American Ephemeris. As the various Ephemerides give different letters for the same

quantities, it is well to note them here. They are given in the following table. A dash indicates that the corresponding quantity is not given directly, and brackets that it is given but no letter assigned it. At the right are the data for the occultation of δ Cancr. from the American Ephemeris.

N. A.	C. T.	B. J.	A. E.	Notation	Data
$[T]$	T_0	$[T]$	$[T]$	(Washington) M. T. of conjunction	$22^d 10^h 20^m 9$
—	—	—	$[\delta]$	Declination of Star	$+18^\circ 32' 0$
—	H_0	—	H	(Washington) Hour Angle	$-4^h 9^m 2$
q	q_0	q	Y	Intercept of Moon's path on axis of Y	$+0.778$
p'	p'	p'	x'	Moon's hourly motion in RA.	0.544
q'	q'	q'	y'	Moon's hourly motion in δ	-0.196
NS	NS	—	NS	Limiting parallels	$+90^\circ +7^\circ$

To plot the moon's path across the earth's disk as seen from the star, a strip of paper is placed so that its edge cuts off the values of x' and y' on the axes (AC and CB). After marking their extremities on the paper and thus obtaining the length of the moon's hourly motion in its projected orbit (AB), a ruler is placed against the strip and moved parallel to itself until it cuts the value of Y (CG) on the axis of Y . Securing the ruler temporarily by a weight, we take the paper edge, subdivide the moon's hourly motion to sixths or 10 minute spaces and extend the graduation to four hours. This graduation is executed very rapidly by means of a scale of equal parts, e. g., a system of converging lines. It need not extend beyond the limits 0.52 and 0.65 on the scale of the drawing.

The local hour angle of conjunction, $H - \lambda$, is next computed. This is $+1^h 52^m 6$ in our case, the longitude of Berlin from Washington being $-6^h 1^m 8$. This value of $H - \lambda$ is now to be indicated on the graduated paper edge between two full hour marks and the latter properly numbered. A practical rule is to mark the hour angle in one of the two left hand spaces when it is positive, and in one of the two inner spaces when it is numerically less than two hours.

The graduated paper edge is now placed against the ruler so that the value of $H - \lambda$ falls on the axis of Y . Weights are then placed on the paper and the ruler is removed. A little practice will enable one to judge beforehand whether the graduated edge should be up or down, in order that the paper may not cover that part of the diagram which is to be used. But should a reversal be necessary, it may be effected by replacing the ruler against the paper, then applying a second ruler to the first and lastly the paper edge to the second ruler. $H - \lambda$ must then be indicated anew and the numeration of the hours changed accordingly.

III. Circumstances of the Occultation.

In order to investigate the circumstances of the occultation, we need a circle drawn to the radius 0.272 (the ratio of the moon's to the earth's radius) on the scale of the diagram. This should be drawn on some thin transparent material, such as gelatine, mica, or the like. The circumference should be graduated clockwise to 10 degrees. This has not been done on the diagram.

This circle is placed with its zero to the north by means of the parallel lines, and with its centre on the

paper edge (the moon's path). It is then moved along it until the centre reads the same hour angle that the circumference does on the proper ellipse. This will happen in our case when the centre is at D and the circumference is at E on the estimated ellipse corresponding to $\delta = +18^\circ 5$. The common reading of D and E , $+2^h 3^m$, is the hour angle at immersion. The position angle $N'DE$ is 126° . The vertex V' may be found by drawing a line through the moon's centre D parallel to CE . The parallactic angle $N'DV'$ is 30° , and the angle from the vertex is $V'96^\circ E$.

Similarly at the emersion, the moon's centre is at H and its circumference at F , both reading $+3^h 10^m$. The position angle $N''HF$ is 291° , the parallactic angle $N''HV''$ is 36° , and the angle from the vertex is $V''256^\circ E$ or $V''104^\circ W$.

The sidereal time is found, of course, by adding the local hour angles at immersion and emersion, in our case $+2^h 3^m$ and $+3^h 10^m$, to the star's right ascension. The local mean time is simplest found and with sufficient accuracy, by adding them to $T - H$, the (Washington) mean time of conjunction minus the (Washington) hour angle at the same moment. In our case $T - H = 14^h 30^m 1$, and hence the local mean times of the phases are $16^h 33^m$ and $17^h 40^m$. The Berliner Jahrbuch on page 377 gives $16^h 33^m 4$ and $17^h 41^m 2$, and the position angles $126^\circ 9$ and $290^\circ 1$. The error of the time of emersion is larger than is usual with a diagram of the scale given. With a larger scale the error would be less.

Finally, the lines CE and CF are the cosines of the star's altitudes at the times of the phases. The altitudes can be found from the scale given on the positive side of the axis of X .

A thorough acquaintance with the lay of the ellipses on the diagram, and at times a mere mental picture of the same, will tell us at once whether it is worth our while to investigate an occultation at all, and will supplement the tests of visibility given in the Ephemerides (v. g., A. E. p. 520). Thus, for Berlin we see that if Y is nearly $+0.7$ and $H - \lambda$ is very small, the occultation is practically certain for almost all stars. Negative values of Y make all occultations impossible. If Y is nearly $+1.00$, then the occultation of southern stars is very probable if $H - \lambda$ is small and has the opposite sign from y' . The same is true, although in a less degree, for northern stars when Y is about $+0.6$.