XLVI. A reciprocal relation in diffraction

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XLVI. *A Reciprocal Relation in Diffraction.*

By A. A. MICHELSON*.

SUPPOSE the vibration at the surface A of a sphere whose centre is at O to be a known function of x and y , the origin being a point P on the sphere,

$$V = \phi \cos nt + \psi \sin nt. \quad . \quad . \quad . \quad (1)$$

Then the vibration† on a sphere B passing through O whose centre is at P will be

$$W = -\frac{1}{\lambda f} \iint \phi dx dy \sin n(t - \tau) - \frac{1}{\lambda f} \iint \psi dx dy \cos n(t - \tau) \quad (2)$$

Putting

$$\tau = \frac{f}{a} \left(1 - \frac{x\xi + y\eta}{f^2} \right), \quad t - \frac{f}{a} = t_1, \quad \frac{n\xi}{fa} = u, \quad \frac{n\eta}{fa} = v,$$

$$W = -\frac{1}{\lambda f} \left[\iint \phi dx dy \cos(ux + vy) - \iint \psi dx dy \sin(ux + vy) \right] \sin nt - \frac{1}{\lambda f} \left[\iint \phi dx dy \sin(ux + vy) + \iint \psi dx dy \cos(ux + vy) \right] \cos nt \quad (3)$$

or

$$W = P \sin nt_1 + Q \cos nt_1. \quad . \quad . \quad . \quad (4)$$

If now a spherical mirror be made to coincide with the sphere B an image of the source will be formed at A .

This image may also be considered as the resultant of the vibrations at B . Hence, if we designate by DW the

* Communicated by the Author.

† Scientific Papers of Lord Rayleigh, vol. iii. p. 80. The results given by Lord Rayleigh for the *intensity* of the diffraction figure in the focal plane do not apply to the phase of the vibration. This restriction is removed if the surface considered be the sphere B ; for the distance between two points on the spheres is

$$\rho^2 = (x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2,$$

$$\text{or if } f^2 = x^2 + y^2 + z^2 = \xi^2 + \eta^2 + (\zeta - f)^2$$

$$\rho^2 = f^2 - 2x\xi - 2y\eta - 2z\zeta + 2f\zeta.$$

If ξ and η are small, ζ will be of the second order, and so is $f - z$, so that $(z - f)\zeta$ is of the fourth order and may be neglected. Hence

$$\rho^2 = f^2 - 2x\xi - 2y\eta$$

$$\text{or approximately} \quad \rho = f - \frac{x\xi + y\eta}{f}.$$

operation by which W is obtained from V ,

$$\left. \begin{array}{l} W = DV \\ V = DW \end{array} \right\} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (5)$$

or $V = DDV. \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (6)$

These equations hold for light of any colour, and—with corresponding extension of the definition of D —to any combination of colours.

The analogy with Fourier's formula is apparent. In fact, the second of equations (5) is, putting $n/fa = p$,

$$\begin{aligned} V = & -\frac{1}{\lambda f} \left[\iint Q d\xi d\eta \cos p(x_1\xi + y_1\eta) - \iint P d\xi d\eta \sin p(x_1\xi + y_1\eta) \right] \cos nt_2 \\ & -\frac{1}{\lambda f} \left[\iint Q d\xi d\eta \sin p(x_1\xi + y_1\eta) + \iint P d\xi d\eta \cos p(x_1\xi + y_1\eta) \right] \sin nt_2 \end{aligned}$$

Substituting the values of P and Q from (4),

$$\begin{aligned} V = & \frac{1}{\lambda^2 f^2} \iiint dx dy d\xi d\eta \psi(x, y) \sin [(x_1 - x)p\xi + (y_1 - y)p\eta] \left. \begin{array}{l} \\ \\ \end{array} \right\} \cos nt_2 \\ & + \frac{1}{\lambda^2 f^2} \iiint dx dy d\xi d\eta \phi(x, y) \cos [(x_1 - x)p\xi + (y_1 - y)p\eta] \left. \begin{array}{l} \\ \\ \end{array} \right\} \cos nt_2 \\ & + \frac{1}{\lambda^2 f^2} \iiint dx dy d\xi d\eta \phi(x, y) \sin [(x_1 - x)p\xi + (y_1 - y)p\eta] \left. \begin{array}{l} \\ \\ \end{array} \right\} \sin nt_2 \\ & + \frac{1}{\lambda^2 f^2} \iiint dx dy d\xi d\eta \psi(x, y) \cos [(x_1 - x)p\xi + (y_1 - y)p\eta] \left. \begin{array}{l} \\ \\ \end{array} \right\} \sin nt_2 \end{aligned}$$

The first and third integrals are identically zero, so that putting

$$u = \frac{n\xi}{fa} \quad \text{and} \quad v = \frac{n\eta}{fa}$$

and disregarding the phase difference between t and t_2 , we have by (1)

$$\frac{4\pi^2}{f^2} \phi(x_1, y_1) = \iiint du dv dx dy \phi(x, y) \cos [(x_1 - x)u + (y_1 - y)v]$$

This, disregarding the intensity factor f^2 , is the Fourier formula extended to two dimensions.

Formulæ (5) express the fact that if

W is the diffraction image of V , then

V is the diffraction image of W .

In applying the formulæ it must be remembered that V and W represent the vibration—not merely the intensity.