PRACTICAL ASPECTS OF LOW FREQUENCY ELECTRICAL RESONANCE.

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Mr. President and Gentlemen of the Institute:—A large part of the subject of the following discourse was discussed by me, but in a different way, in three papers. Two of these appeared in the April and May numbers of the *American Journal of Science*. The third will appear in the June number of the same journal. The method which I have adopted in the following discussion seemed preferable to the mathematical method which I followed in those papers. It is probably just as exact, and certainly a much clearer way of viewing the variable flow of electricity, especially those features of it, which have a more or less direct practical bearing.

1. ON THE NATURAL PERIOD OF AN ELECTRICAL CIRCUIT.

An electrical circuit possessing self-induction and capacity behaves in a great many respects as a body does in consequence of its inertia and elasticity. The fundamental reason for this analogy is simply this:—The electromagnetic energy of a coil through which a current flows, has all the characteristic properties of the kinetic energy of a moving body, whereas the energy of the static charge of a condenser has all the characteristic properties of the potential energy of a strained elastic body. If the neutral state of such an electrical circuit is disturbed, it will return to it again after performing a certain number of oscillations about the position of its neutral state. But a return to the neutral state is impossible until the energy which is spent upon the circuit to disturb its neutral state has left the circuit, or to use a more technical expression, until the energy has been dissi-

pated or given off to some other circuit. The two principal causes which produce dissipation and compel the circuit to return to its neutral state again, are frictional resistances and radiation. Just as in the case of vibrating bodies, so also in the case of electrical oscillations, losses due to radiation, especially when no other electrical circuits are near, are exceedingly small when the oscillations are slow. In Herzian oscillations they are quite considerable. In oscillations of the Tesla frequency they are probably not negligible. My remarks refer to electrical oscillations of long period, therefore losses due to frictional resistances are the only losses which I shall consider. Consider now an electrical circuit consisting of a coil *A* and a condenser *B* (Fig. 1) in series with it. It is a circuit with localized self-induction and capacity. I trust that my discussion will lose as little in its generality as it will in its practical bearing if I confine it to such circuits only.

FIG. 1.

Let a sudden electrical impulse disturb the neutral state of this circuit; electrical oscillations will result. These oscillations follow laws practically identical with the laws of the motion of a slowly vibrating body. Their period is constant, as we all know, and it is in general completely determined by the *electromagnetic moment of inertia* and the *dielectric elasticity* of the circuit—that is, by its coefficient of self-induction and its capacity. When, however, frictional losses due to ohmic resistance, magnetic and dielectric hysteresis are large, then the period of this circuit is no longer defined by the self-induction and capacity alone, but it is also influenced by these frictional losses.

When ohmic resistance and hysteresis losses are small enough, then the natural period of the circuit is given by the well-known formula

$$T = \frac{2\pi}{10^8} \sqrt{LC}$$

Where T is the natural period of the circuit in seconds, L its coefficient of self-induction in henrys, and C its capacity in microfarads. For instance, a large Bell telephone whose coefficient of self-induction is 0.5 henrys when connected in series to a condenser of 1 microfarad capacity will have a natural period of very nearly $\frac{1}{225}$ seconds, that is to say, an electrical disturbance would set up oscillations in it, 225 of which would take place in one second. If a permanent magnet were brought into the vicinity of the telephone coil and then suddenly removed, the telephone would sing a note whose pitch would be a little below the well-known note C. But it would not sing it very long. For since the ohmic resistance is 100 ohms these oscillations would disappear almost entirely after

FIG. 2.

10 complete oscillations, somewhat in the manner represented in diagram Fig. 2, that is to say the telephone would sing only during about $\frac{1}{22}$ part of a second. By diminishing the resistance we could prolong its song. But diminish the resistance as much as you please, the pitch of the note of the telephone will remain the same, because, as I said, the natural period of the telephone circuit just described is within wide limits independent of the ohmic resistance.

2. ON THE TUNING OF AN ELECTRICAL CIRCUIT.

To change the note, say to make it higher, it would be necessary to diminish the capacity of the condenser. When a piano tuner wishes to raise the pitch of a piano string he gives it more

tension; so in *tuning an electrical circuit*, in order to change its pitch, it is necessary to change its *electrical elasticity* that is, its capacity. But there are other ways of tuning an electrical circuit, just as there are different ways of tuning musical instruments. Consider a reed pipe, say a clarinet. The musician places a little bit of wax on the reed. When the instrument is too low in pitch he takes off some of the wax, so as to diminish the moment of inertia of the reed, and when the pitch of the instrument is too high, he sticks on more wax so as to increase the moment of inertia. At any rate this used to be the method of old-fashioned country musicians. And so it is in tuning an electrical circuit. Instead of varying its *electrical elasticity*, that is, its capacity, we can vary its *electro-magnetic moment of inertia*, that is to say, its coefficient of self-induction. To show how this may be done in the telephone circuit just mentioned, insert into this circuit a small coil, *a*, [Fig. 3] an auxiliary coil, with a removable iron core *c* made up of very fine iron wire. In doing this

FIG. 3.

we do exactly what the country musician does when he puts wax on the reed of his clarinet. If the electrical pitch of the circuit does not suit, say it is too high, then simply put on more electro-magnetic wax, that is to say, insert the iron core and move it back and forth until the correct position is found which will give the correct electrical moment of inertia, that is to say, the correct coefficient of self-induction. This is, briefly stated, what I mean by the expression *tuning an electrical circuit*. From the simple expression given above for the natural period of an electrical circuit, it is evident that the tuning of an electrical circuit, if not simpler, is certainly quite as simple a process as the tuning of a musical instrument.

3. ON THE DETERMINATION OF THE PITCH OF AN ELECTRICAL CIRCUIT.

If we wish to know the pitch of a musical instrument, say of a tuning fork, to choose a simple illustration, we simply give it

an impulse, say a tap with the finger, and then listen to the vibrations, which in general will last for several minutes, and give us sufficient time to make up our minds as to what the vibrations sound like. In examining the pitch of an electrical circuit it is more convenient to adopt a different method. The reason is that as a rule electrical oscillations are, as pointed out in the example above, much more damped, so that the oscillations resulting from a single impulse would not last long enough to give us sufficient time to see what they look like, or to listen and hear what they sound like. The method suggested by the stroke of a violin bow over the string is preferable. The stroke of the bow produces a series of impulses which quickly succeed each other and maintain the string in uniform vibration. It is interesting now to observe that the same thing can be done with an electrical circuit. Consider the circuit *A B C* Fig. 4. The condenser *B* represents the tension on the violin string, coil *A* represents its inertia. The air-gap *c* stands for the point where the bow by its stroke excites

FIG. 4.

the string. If you now wish a musician who plays with a one-sided stroke, use a high potential direct current generator *D*, and for an alternating stroke substitute an alternator. The discharges at the air-gap *c* succeeding each other quickly enough, and at proper intervals will maintain in the circuit *A B C* practically uniform electrical oscillations. This is one of Tesla's favorite circuits, and I have no doubt but that he will accomplish great things with it yet.¹ Bring a few turns of wire of a telephone circuit into inductive influence of this circuit, and you will have in the telephone a musical note of exactly the same pitch as the pitch of the electrical oscillations in the circuit *A B C*. The note is not perfectly pure. It is marred by the noise of the spark discharge of the air-gap *c*. Neither is the note of the violin string pure; there is always more or less of the scraping noise of the bow. Just as it is necessary to keep the bow well rosined

1. It must be observed, however, that Hertz, in 1887, produced his oscillations by a circuit of this identical form.

so as to give it a good grip upon the string, so it is necessary to apply a strong current of air or the action of some other of Tesla's devices upon the air-gap *c*, otherwise an arc is formed and the generator *D* loses its grip upon the circuit *A B C*.² If we had a number of different coils with condensers like *A* and *B* alongside of each other, and arranged in such a way as to be able to place them at will, now the one and now the other, and now again a number of them in multiple under the action of the electrical bow which the generator *D* keeps moving over the air-gap *c* and at the same time vary the capacities, we should be able to change the electrical vibrations of our system with that ease, precision and grace which the violin player displays when with the one hand he guides his obedient bow, while the busy fingers of the other hand glide over the trembling strings, eliciting from them delightful notes which blend into pleasing harmonies. So with our system of properly tuned electrical circuits, we could produce harmonies, but they would be harmonies of silence, harmonious oscillations in the ether that affect neither eye, nor ear, nor taste, nor smell. But bring a part of a telephone circuit into inductive action of our harmonic system, and let a skilled experimentalist manipulate a properly constructed keyboard which controls the coils and condensers of the various circuits, and harmonies which before were as silent as the grave, will now agitate the responsive diaphragm of the telephone and produce music that could be made to re-echo in every telephone in the United States.

But after all, such an arrangement when used for such a purpose, would be a mere toy in comparison to the purpose for which our distinguished colleague, Mr. Tesla, employs it. To convert high-potential but small current electrical energy into low-potential big current energy, or *vice versa*, accompanied by all possible variations in the frequency of oscillations was the purpose for which Mr. Tesla constructed the device. To a physicist who delights, not less than the engineer, in neat, simple devices for the accomplishment of big and brilliant effects this device of Tesla naturally appeals more than all his other ingenious inventions. Many a delightful hour have I spent in watching experiments on a circuit like the one in Figure 4. The

2. The importance of blowing out the arc for the production of powerful oscillations seems to have been first recognized by H. Classen, Wiedemann Annal. d. Physik und chemie, Band xixxx, p. 647, 1890.

coil A consisted of a short, stout copper wire; the condenser B consisted of a battery of Leyden jars, which my distinguished teacher, Professor Rood, of Columbia College, kindly lent me. The wires *aa* were thin copper wires connecting the air-gap c to the poles of an induction coil which is now in the hospital. It is a delightful sight to see the stout wire aglow under the powerful agitation of the rapid oscillations, whereas the thin wires *aa* adjoining them remained perfectly cool. Twist now the thick wire A into a few convolutions, say ten or twelve, and surround them by a few hundred turns of fine wire and you will have the now well-known oscillatory transformer with which Mr. Tesla and Professor Elihu Thomson produced some brilliant effects, a transformer that will give you any number of volts especially if,—and now I am going to touch a point which forms the central point of my discourse,—especially if the thin wire coil contains *capacity in series with it, so that the natural period of this circuit is the same as the period of the thick wire coil, that is if the two circuits are in resonance.*

4. ON RESONANCE.

Here again I have borrowed a term employed in music. But a few simple considerations will show you that it is very natural that I should, for the phenomena of sound and those of oscillatory flow of electricity are governed by nearly the same laws. Very high frequency electrical oscillations would in all probability be identical with light, as first announced by immortal Maxwell. It is, therefore, not surprising to find that low frequency electrical oscillations should resemble so much the other group of oscillatory phenomena which next to light pleases our senses best, namely, the phenomena of sound, especially agreeable sound, that is music.

To gain a clear conception of what is meant by electrical resonance consider the following simple mechanical analogon—I call it the *torsonal pendulum*:—I used it often with my students in discussing alternating current phenomena and they liked it very much. A heavy bar A (Fig. 5) is suspended on a stiff elastic wire B, which is attached to a plate c whose weight may be neglected. This plate c slides in a groove *aa* which in consequence of friction acts like a brake. Suppose now that the friction between c and *aa* is such that when the angular velocity of c is ω the rate at which heat is generated by the friction between c and

aa is equal to $\omega^2 R$ where R is independent of the angular velocity. This torsional pendulum resembles then very much an electrical circuit having localized self-induction, capacity and ohmic resistance. The moment of inertia of A , the elasticity of B , and the friction in C act exactly the same as the coefficient of self-induction of the coil, the capacity of the condenser, and the ohmic resistance of the circuit. Let $\frac{1}{B}$ stand for the elastic capacity of the wire, that is if the wire be twisted through an angle θ , then the moment of the elastic force which opposes this twisting is $\frac{\theta}{B}$. Let I stand for the moment of inertia of the weight A , then as long as the frictional resistance is within cer-

FIG. 5.

tain limits we shall have for the natural period of the pendulum

$$T = 2 \pi \sqrt{I \times B}.$$

You see that this expression is exactly the same as the one which expresses the natural period of the circuit in terms of its coefficient of self-induction and capacity.

If this pendulum is set in motion by a single impulse it will oscillate with a constant period. The first elongation will be largest, and the successive elongations will be smaller and smaller, just as represented in Fig. 2, until the pendulum is reduced to rest again. This will happen when the energy of the impulse has been entirely dissipated into heat, which is the work done against frictional resistances in the break at C . The smaller this frictional resistance, the longer will the pendulum swing in consequence of the im-

pulse, before it is reduced to rest. On the other hand we can increase the resistance in C to such an extent as to make the swing a-periodic or dead beat, as is for instance the case in the Weston voltmeters and ammeters.

Repeat the impulse at regular intervals, alternating them, the pendulum will keep on swinging; but you can easily see that if the intervals of the impulses are measured off in such a way that every time the pendulum passes through its position of equilibrium, the impulse strikes it, and strikes it in the direction in which the pendulum is moving, then the amplitude of the swing will be much larger than if the intervals of the impulse are not measured off in such a way. In other words when the period of the impulses is the same as the natural period of the pendulum then the swing is largest. The impulses are said then to *be in resonance* with the pendulum.

The same effect will be produced if for the impulses we substitute a periodically but gradually alternating force, say a simple harmonic force, having the same period as the natural period of the pendulum.

Such a force when acting upon the torsional pendulum just described will continually increase the amplitude of the swing, until the swing is so large that the work done during a swing against the frictional resistances, is just equal to the work of the moving force during that time. From that point on, the pendulum will swing with constant amplitude. It is evident, therefore, that the larger *the amplitude of the moving force and the smaller the frictional resistances, the larger will be the amplitude of the swing*. But a large amplitude implies two things:—1st, *A large torsional reaction* in the suspension wire, and secondly, *a large velocity* whenever the pendulum passes through its position of equilibrium.

The bearing of this mechanical analogon upon the electric circuit having self-induction and capacity is very direct, as I shall presently point out. Let a simple harmonic E. M. F. $E \sin pt$. act upon such a circuit. I shall presently consider a complex harmonic E. M. F., and also circuits possessing Foucault current losses, and losses due to magnetic and dielectric hysteresis. They will form the last, and in my opinion the most important part of my discourse. A simple harmonic E. M. F. acting upon an *elastic electrical circuit* in which the only frictional losses are those due to ohmic resistance, will when its period is the same as the natural

period of the circuit, that is when *it is in resonance with the circuit*, continually increase the amplitude of the electric displacement, that is the amplitude of the condenser charge and therefore also of the current, until the work done against the ohmic resistance is exactly equal to the work of the impressed E. M. F. From this point on the amplitude of the current will remain constant and equal to the amplitude of the impressed E. M. F. divided by the ohmic resistance. Or, to state it in terms which are more generally employed: *Capacity and self-induction neutralize each other when the circuit is in resonance with the impressed E. M. F.*

It is evident also that in a resonant flow, there can be no difference in phase between the current and the impressed E. M. F. Since in this case the current at any moment depends on the E. M. F. and the resistance, and on nothing else.

It is clear, however, that if losses due to Foucault currents, magnetic and dielectric hysteresis are present, then the current cannot be made equal to the ratio between the E. M. F. and the ohmic resistance by any combination of capacity and self-induction, although as a simple reflection will show, the difference in phase between the current and the E. M. F. may be reduced to zero in this case also,¹ in which case we should have no false current, as it is generally called. I shall presently discuss this point a little more fully.

5. RESONANCE IN CIRCUITS WITH AN IMPRESSED COMPLEX HARMONIC ELECTROMOTIVE FORCE.

If the impressed E. M. F. is a complex harmonic, then the circuit may be brought into resonance with any of the component harmonics. I have an apparatus here which is capable of producing E. M. F.'s of almost any complexity. I call it an *electrodynamic current interruptor*, for want of a better name. It consists of a phosphor-bronze wire, the vibrator, *a*, *b*, *e*, (Fig. 6), stretched between the poles of a Weston horseshoe magnet *d*. A short, thin, amalgamated copper wire *b*, the dipper, is soldered on the vibrator. The vibrator is stretched like the wire of a monochord, over two hard rubber bridges *a* *e*, and its tension can be varied by a screw and lever *f*. The dipper *c* whenever it

1. Professor Duncan, of Johns Hopkins University informed me yesterday that he deduced the same result in the course of an investigation, a short account of which he read before this Institute at its General Meeting in Chicago in 1892, "Note on some Experiments with Alternating Currents." TRANSACTIONS, vol. ix, p. 179.

dips into the mercury cup closes an electrical circuit $F f e b c g A$, and the vibrator is then repelled upward by the force of repulsion between the magnet and the current flowing through the vibrator. One gravity cell f is sufficient to work this apparatus. It is evident that the fundamental period of the interrupted current will be equal to the period of the vibrator. To change this period, say to make it equal to the period of this tuning fork, which is 256 complete vibrations per second, I simply strike the tuning fork, hold it then near my ear and by turning the screw f of the stretching lever I vary the tension of the vibrator until the rate of the vibrator and that of the tuning fork are in perfect unison, which can be easily recognized by watching the beats. This whole process of tuning takes up, as you see, only

FIG. 6.

a few seconds. In series with the cell f and the vibrator, we have a small coil A , containing a well packed bundle of very thin iron wire, and in shunt with A is a condenser D . This condenser performs the function of bringing this primary circuit into partial resonance with the vibrator which is recognized by the circumstance that at the moment of resonance the spark at the dipper is a minimum.

A simple reflection will show you that the current generated by this interrupter is a complex harmonic. To analyze it into its component harmonics I shall adopt the method employed in acoustics.

A complex sound is analyzed by the well-known resonators of Helmholtz. These resonators are constructed in such a way that

they will respond to one vibration only, and also in a measure to its upper harmonics. To any other vibration they are practically mute. By a number of such resonators we can find out, in the manner first pointed out by Helmholtz, all the vibrations which are contained in a complex sound.

The secondary circuit B C T E, given in the diagram of Fig. 6, performs the function of such a resonator, it is in fact *an adjustable electrical resonator*; c is a small auxiliary coil which you see here; e is a Marshall condenser, and t is a telephone whose note tells me to which of the component harmonics the electrical resonator responds. The telephone is silent now, at any rate as far as you can tell, because the condenser plugs are all out. Now, with this plug, I insert a small capacity. The note which you hear is evidently much higher than even the octave of the vibrator. The electrical flow in the resonating circuit is approximately the same as if we had in it a simple harmonic E. M. F. of the same frequency as this note. I insert another plug, so as to increase the capacity. The note which you now hear is deeper. The difference between the two notes is rendered very striking by making and breaking the contact of the second plug in rapid successions, as I do now. The sound of the two notes, succeeding each other at regular intervals, resembles very much the sound of bagpipes. I insert another plug, so as to increase the capacity still more. The note which you now hear is still deeper. It is the note of the fundamental electrical vibration, which is the same as the vibration of the vibrator. I know that, because, as I continue inserting more and more plugs, you do not perceive any further change in the sound. The capacities which I introduced by the three plugs are to each other very nearly in the ratio of 1 : 9 : 25, hence the frequencies of the three harmonics contained in the impressed E. M. F. and which I have detected by the resonating secondary circuit are to each other in the ratio of 1 : 3 : 5.

The impressed E. M. F. can therefore be written

$$E = E_1 \sin pt + E_3 \sin 3 pt + E_5 \sin 5 pt.$$

Still higher harmonics are present, but they are too weak to be heard all over this room.

I shall presently discuss a method by means of which we not only detect the presence of these harmonics, as I have just done by means of this telephone, but also determine the relative

strength of each. The method imitates in a certain measure Prof. Langley's bolometric method of determining the distribution of energy in the spectrum of a given complex luminous vibration.

Before making the next step in my discourse I wish to call your attention to an application of resonant circuits (in connection with the electro-dynamic interrupter) to a method of producing simple harmonic currents of constant and easily determinable frequency.¹ I simply tune the vibrator by means of a standard tuning fork, and then weed out the upper harmonics by successive transformations. In this connection I ought to mention that Professor Duncan of Johns Hopkins University informed me several weeks ago that he was also weeding out harmonics. I now remember that I got the expression "weeding out" from him, but I have used it in my papers on "Low Frequency Resonance" without giving him credit for this beautifully selected expression.

6. ON THE RESONANT RISE OF POTENTIAL.

The question arises now, how are we to tell experimentally whether a circuit is in resonance to an impressed E. M. F. or not? Several methods suggested themselves to my mind in the course of my work. I selected the one which appeared to me to be the most sensitive and most interesting. *It also suggests a new way of transforming electrical energy which some day may perhaps be of some practical importance.* I wish to discuss this method now, but only very briefly.

As I observed before in the discussion of the motion of the torsional pendulum, represented in Fig. 5, when the moving force and the pendulum are in resonance then both the amplitude of oscillation and the amplitude of the velocity reach their maximum value. And so it is also with a circuit possessing self-induction and capacity. By varying gradually the one or the other, the current (which corresponds to the velocity of the pendulum) will continually increase. But the difference of potential in the condenser (which corresponds to the elastic reaction of the suspension wire) will also increase continually. When the point of resonance has been reached then evidently both the current and the difference of potential in the condenser have reached

1. Pupin, "Electrical Oscillations of Low Frequency and their Resonance," *American Journal of Science*, April, 1893.

their maximum values. This maximum difference of potential in the condenser can be many times greater than the impressed E. M. F., because just as in the case of the torsional pendulum the elastic reaction of the suspension wire is an accumulative effect of the moving force, so in the electrical circuit, the potential difference in the condenser is an accumulative effect of the impressed E. M. F. Now since a resonant electrical flow, which we have just considered, consists simply of a transformation of electrokinetic into electrostatic energy, and *vice versa*, during each half period accompanied by a frictional loss due to ohmic resistance, it is evident that the amplitudes of these two kinds of energies must be equal to each other, that is, we shall have in the usual notation

$$\frac{1}{2} P^2 C = \frac{1}{2} L \frac{E^2}{R^2}.$$

But since owing to resonance we have $p^2 L C = 1$, the preceding relation reduces to

$$P^2 = p^2 L^2 \frac{E^2}{R^2} \quad \text{or} \quad P = p L \frac{E}{R}.^{(1)}$$

It is evident, therefore, that with a little more than ordinary frequency, say 350 periods per second, and an inertia coil of considerable self-induction and small resistance high potentials can be obtained in the condenser by bringing the circuit in resonance with the impressed E. M. F.. *Connecting, therefore, the condenser to a voltmeter we can tell very accurately when in the course of our tuning of the circuit we have reached the point of resonance, because, as I shall presently show you, the voltage goes up very rapidly when the condenser capacity or the self-induction are near the point of resonance. It is this electrometric method which I have employed in my investigations on "Low Frequency Resonance."*²

The same method can be, and has been employed by several investigators in the study of Herzian oscillations.

To show you how strong this resonant rise of potential can be I have within the last few days wound these two inertia coils (see coil *a' b'* in Fig. 8) and borrowed this most excellent mica condenser from our well-known electrician, Mr. Wm. Marshall, for which kindness I feel very grateful to him. The impressed E. M. F. is ob-

1. This relation was first obtained, I believe, by Prof. Oliver Lodge. (See a letter by Prof. O. Lodge, London *Electrician*, April 24th, 1891.)

2. *American Journal of Science*, April, May and June, 1893.

tained in the following way: A small 16-pole alternator running at about 2800 revolutions per minute and generating an E. M. F. of about 600 volts feeds the primary of this small transformer (*c d*, Fig. 8.) Diagram Fig. 7 gives the form of armature *A* and field *B* of

FIG. 7.

the machine. Coils *a b c d e* show how the machine is wound. The transformer consists of a hard rubber spool wound with about 3,000 turns (Fig. 8) of No. 20 A. W. G. wire for the primary. The secondary *h k* consists of about 500 turns of No. 16 wire. A well packed bundle *e* of very fine iron wire is the iron core. A well packed bundle *e* of very fine iron wire is the iron core.

FIG. 8.

This transformer is indicated by *c* in Fig. 7. In series with the secondary we have inertia coils *D* and the above mentioned mica condenser *E* of 0.2 M. F. capacity. The inertia coils consist of hard rubber spools like *a' b'* Fig. 8 wound with No. 14 wire. The

total self-induction of the secondary circuit is about 1 henry, *without iron in the inertia coils*. The poles of the condenser E are connected to a Sir William Thomson electrostatic voltmeter F and also to a vacuum tube G with condenser electrodes, that is to say tinfoil coatings on the outside of the tube. (Indicated by the shaded part in the diagram.) I now start the machine and by weakening the field of the driving motor, I gradually increase the speed of the machine. As you see the voltmeter needle is steadily advancing with the speed. To get the greatest deflection of the needle I ought to have about 350 periods per second. Now I shall advance to very near this speed. The voltmeter indicates now 2,000 volts. The impressed E. M. F. in this secondary resonating circuit is only about 100 volts. Instead of increasing the speed so as to reach the point of perfect resonance I prefer to leave the speed constant, and by gradually inserting these five iron wires to increase the co-efficient of self-induction until that value is reached which with the given speed and capacity brings the circuit in resonance with the impressed E. M. F. I am doing this now and watching the voltmeter needle at the same time. Now the voltmeter indicates 3,000 volts, which is 30 times the value of the original E. M. F. This is the point of resonance, because if I push the iron wire lower, the needle goes back. The point of resonance is very sharply defined, because the slightest motion of the iron wire one way or the other makes quite considerable difference in the reading of the voltmeter. The point of resonance can be shown to a large audience like this much more easily by connecting the poles of the condenser to the tinfoil electrodes of the vacuum tube G. A discharge starts in this tube at about 2,000 volts. Now as I insert these iron wires into the inertia coil you see the intensity of the discharge increases. [The room was darkened.] The point of maximum resonance is clearly marked by the intensity of the discharge, as you see. Now I insert the iron wire too far; as you see the discharge is stopped. I can start it again by raising the iron and stop it again by taking the iron wire entirely out, as you see.

By working with high frequencies and small ohmic resistance any voltage within practical limits can be obtained. Ohmic resistance, however, is not the only thing which limits the resonant rise of potential. *Dielectric hysteresis* and the *peculiar behavior of iron when under the inductive action of a rapidly alternating current*, modify this rise very much. The other day I employed

this home-made condenser in parallel with that Marshall mica condenser. I obtained only 2,000 volts rise, whereas preliminary calculation led me to expect at least 4,000 volts. *But the smell of melting wax reminded me forcibly of the cause of this discrepancy. The home-made condenser got so hot in about two minutes that all the wax (which when boiling hot was compressed between its mica plates to drive out the air and moisture) between its plates was melted.* The energy which was thus used up in overcoming the dielectric hysteresis of the wax had prevented the resonant rise from reaching that value which it ought to have reached according to calculation. Taking this condenser out and using the mica condenser alone the rise went right up to a point considerably above 3,000 v. Paraffin and glass behave in a similar way *So that owing to dielectric hysteresis alone the resonant current (which takes place when capacity and self-induction neutralize each other) is never equal to the electromotive force divided by the ohmic resistance, and it can be very much less. The presence of iron is incomparably more powerful in destroying the resonant rise of current and potential, as will be seen presently. This is very unfortunate in view of the brilliant expectations which not a few electricians hoped to realize from the employment of condensers in the running of alternating current machinery.*

It is well to observe here that if we substitute a Weston voltmeter for the Thomson Electrostatic Voltmeter F Fig. 7, then we shall have practically the condenser in parallel with the inertia coils. It is easy to see from purely theoretical considerations that the resonant rise will be less in this case. Experiments show that the rise can be then very much less if the resistance of the voltmeter circuit is reduced. *The bearing of this observation on the so-called Ferranti effect with and without a load in the mains needs, I venture to suggest, no further discussion.*

7. EFFECT OF IRON ON THE RESONANT RISE OF POTENTIAL.

I have already remarked that iron behaves in a peculiar manner when under the inductive action of a resonant flow of electricity. Allow me to call your attention to the diagram of Fig. 9. This curve was obtained with the machine and the circuits which are given in Fig. 7. Except that the inertia coil D had smaller self-induction. Keeping the speed of the machine and the self-induction of the circuits constant, the difference of potential in

the condenser was varied by varying gradually the capacity of the condenser. Taking then the capacities for abscissæ and the corresponding voltmeter readings for ordinates, the curve in Fig. 9 was obtained. As you see the curve has two maxima. The corresponding capacities are to each other as 1 : 9. Hence the

FIG. 9.

impressed E. M. F. can be represented by

$$E = E_1 \sin pt + E_3 \sin 3 pt.$$

A comparison of the smaller maximum to the larger shows that E_3 , the amplitude of the upper harmonic, is very small in comparison to E_1 , the amplitude of the fundamental. This is sur-

prising, because from the construction of the alternator one would expect a much stronger deviation of the impressed E. M. F. curve from the simple harmonic.

A similar *harmonic analysis* of the impressed E. M. F. obtained by means of the electrodynamic interruptor gave curves I, II and III Fig. 10. Curve I was obtained by inserting a certain capacity in shunt with the primary coil, and then varying the capacity in the secondary and recording the corresponding voltmeter reading. Capacity was measured off as abscissa and the corresponding voltmeter reading as ordinate. Curves II and III were obtained in the same way but with different capacities in the primary circuit. The impressed E. M. F. in each case can be represented by the following complex harmonic:

$$E = E_1 pt + E_3 \sin 3 pt + E_5 \sin 5 pt.$$

But the amplitudes, E_1 , E_3 and E_5 are different in each case.

FIG. 10.

You see, therefore, that this method of harmonic analysis by the resonant rise of potential is very sensitive in detecting upper harmonics. Hence the natural conclusion that curve Fig. 9 indicates no strongly developed harmonics, simply because they were not present. Yet it is very difficult to suppose that my alternator generates practically a simple harmonic E. M. F. when one considers the shape of its armature core.¹ This point evidently deserved additional investigation.

I inserted a few iron wires into the inertia coil D, Fig. 7, and found that the upper harmonic, weak as it was, disappeared entirely. The iron wires became hot in a very short time. I suspected that iron when subjected to strong magnetization wipes out the upper harmonics in a resonant circuit. With the electrodynamic interruptor the magnetization of the iron cores is very weak, hence the persistence of upper harmonics. This suspicion was verified in subsequent experiments.

1. The armature core was a Crocker-Wheeler tooth armature.

When the inertia coil *D* was packed with iron wire then the resonant rise was very much diminished as is evident from the curve in diagram Fig. 11.

Various considerations led me to the conclusion that Foucault current and hysteresis losses were not the only causes of this remarkable diminution of the resonant rise due to the presence of iron. I will mention one only. In one of my experiments I used the primary coil of a small *closed magnet circuit transformer* for the inertia coil and found that as far as I could detect with my apparatus *no resonant rise* could be obtained with 100 volts of impressed *E. M. F.*, no matter how low the frequency was which my machine could produce. About 75 periods per second was the lowest frequency which I employed. If, however, the

FIG. 11.

secondary coil of this transformer was short-circuited then the resonant rise appeared, but not nearly as strong as it ought to have been, even Foucault currents and hysteresis losses taken into account. The self-induction of the primary (with secondary short-circuited) calculated from the capacity which produced resonance was far too high. *There were unmistakable evidences in this, and in all my other experiments in which iron was present in the inertia coils, that the lower the frequency the stronger were the resonance effects.*

When an alternator giving 125 periods per second was substituted for the alternator in Figure 7, (which gave about 350 to 375 periods per second,) then the resonant rise with iron wire core in the inertia coil *D* was one-half of that when no core was

used, the impressed E. M. F. being 45 volts. If, however, the impressed E. M. F. was raised, then this difference in the two cases continually diminished. When the impressed E. M. F. was 200 volts, then the resonant rise with iron was more than without it.

There seems to be in the iron a certain reluctance against getting into full swing when under the inductive action of a resonant current. The higher the impressed E. M. F. and the lower the frequency the easier it is to overcome this reluctance. *When the iron has once been set into full swing, then it is possible by careful manipulation to lower the impressed E. M. F. and increase the frequency without getting the iron out of this swing. In a closed magnetic circuit this reluctance seems to be exceedingly great.*

With a frequency of 50 periods per second the resonant rise

FIG. 12.

with iron was invariably more than without it. Two features in the peculiar behavior of iron were brought out by the resonance experiments with this low frequency machine. These two features, I venture to suggest, deserve more than ordinary attention.

The first peculiar feature is exhibited in the two curves Figure 12. Curve II was obtained when the whole iron wire core (about 500 wires each 40 cm. long and 1 mm. in diameter) was inserted into the inertia coil. The other curve was obtained when only one-half of the core was employed. *The frequency was maintained constant.* From the critical capacity in the two cases it is evident that there was much less difference in the coefficient of self-induction in the two cases than one might expect from the

difference in the quantities of iron, that is, if we suppose that in this case also the natural period of the circuit is independent of the frictional resistances due to magnetic hysteresis. But even if we drop this hypothesis, still the small difference in the capacity in the two cases appears as puzzling as ever. We have resonance in this case also as in any other case, but it is evident that the conditions of resonance are not determined by self-induction and capacity alone. The question arises then, what does resonance mean in this case? Well, it means the largest current and the highest rise of potential, but this maximum potential and current are not necessarily accompanied by zero difference in phase nor by neutralization of self-induction with capacity.¹ The practical bearing of this result needs no further comment.²

There is another point to which I wish to call your attention in connection with curves I and II of Fig. 12. It is this: Although curve II was obtained with twice the quantity of iron and therefore, with at least twice the losses due to Foucault currents and hysteresis, [since the magnetizing current which circulated in the inertia coil with resonance corresponding to curve II was considerably larger than that corresponding to resonance of curve I,] yet the rise of potential in curve II was larger than that in curve I. I do not wish to take up too much of your time in showing you all of the difficulties which one encounters in endeavoring to explain these matters satisfactorily. Suffice it to point out merely that these difficulties do exist and that they deserve our most careful attention.

Before making the next step in my discourse I wish to remark that a circuit consisting of the armature of an alternator, a large *inertia coil without iron* and a condenser, all in series, *does not exhibit any of the peculiarities just mentioned, at any rate not to any considerable extent. Such a circuit can be tuned and a high resonant rise of potential can be thus produced, provided, of course, that this rise of potential is not accompanied by so heavy a current as to produce a serious armature reaction. Foucault current and hysteresis losses in the armature iron cannot, of course, influence this resonance, since they are taken care of by the motor which rotates the armature.*

1. Professor Duncan informed me yesterday that he arrived at this conclusion in a somewhat different way. See paper already referred to.

2. Especially when Dr. Duncan's more definite conclusions on this point are taken into account.

The second peculiar feature in the behavior of iron when it is under the inductive action of the resonant current of an inertia coil will be exhibited to you in the following experiment: The arrangement is just the same as that given in the diagram of Figure 7, with the following exceptions: This alternating current ammeter is in the resonant circuit. The motion of its index, which I hope can be seen all over the room, will indicate to you the progress of tuning. I watch it on this side by means of a multi-cellular voltmeter which is denoted by V in the diagram of Figure 7. This four-pole 1 H. P. alternator which Dr. Laudy, of Columbia College, kindly lent me, takes the place of the 16-pole alternator. Inertia coil, alternator and condenser are all in series. The driving motor can easily take care of 2 H. P., so that the speed will remain constant during all variations of the load since as you will presently see the load will never be over 150 watts. The condenser has now 16 M. F. capacity plugged in. The frequency is about 50 periods per second. The impressed E. M. F. is 45 volts. To bring the circuit in resonance with the impressed E. M. F. it is necessary to insert iron wires into the inertia coil. I insert now about 400 wires. It is, as I can see from the voltmeter reading, considerably too much and, therefore, there is no resonance as you can see from the ammeter. I take the iron wire bundle and raise it slowly, and now I can feel by the pull on the bundle and also by the voltmeter reading that I have reached the point of resonance. You can see it by the ammeter deflection. The ammeter indicates 1 ampere; in reality it is about 2 amperes. [This instrument is a lecture room model and its reading must be multiplied by a large correction factor.] Now I lower slowly the iron wire bundle again; the resonance persists. I have reached now the same position from which I started and when there was hardly any resonance effect. But as you see this time the resonance effect is very strong. *But it is a resonance which resembles very much a supersaturated zinc sulphate solution. A very small crystal thrown into such a solution will cause a sudden precipitation of innumerable small crystals. So it is with this resonance. If I now bring a single iron wire near the bundle, its mere approach upsets the resonance almost completely, as you see from the sudden drop of the ammeter needle. But if I add this wire whose mere approach upsets the resonance almost completely and even five other wires to the bundle and then by gently raising it start the resonant flow I can then lower it again*

without disturbing resonance, as you see it from the deflection of the needle.

I shall vary now the capacity by taking out carefully one plug after the other. But I must take out these plugs without causing a spark. Here is one plug out, and here another and four more. I have reduced the capacity by 1 M. F. or nearly 7 per cent. of the total capacity. There is hardly any change in the resonance effect, as you see from the ammeter needle. Now I shall carefully put back the plugs and even more than I have taken out, but carefully avoid sparking. As you see, resonance still persists without any apparent change. *Now I take out one plug and vary the capacity by only 0.1 M. F., but I take it out in such a way as to produce a strong spark by the removal of the plug. The experiment does not succeed. But here it succeeds at the removal of a 0.3. M. F. plug. The collapse of the resonance effect is as you see almost complete. Generally I succeed with even less than a 0.1 M. F. plug.*

The same peculiarities appear also at higher frequencies, but they are not nearly as clearly defined at frequencies over 100 as they are at those of less than 50 periods per second. They are certainly not due to any change in the speed of the machine, produced by the variation of the load due to the rise of the resonant current. Nor is it probable that they are due to armature reactions. This last point, however, deserves a closer examination than I have been able to give it as yet.

One more point I feel that I ought to mention before I end my discourse. It is a point which I think may have some practical importance sooner or later. *When a large inertia coil without iron is joined in series with the armature of an alternator and also with a condenser and then the circuit is tuned, so as to analyze harmonically the shape of the impressed E. M. F. curve, I find in all the machines that I have tried so far that this curve is almost perfectly free from upper harmonics, even with frequencies of 50 periods per second. If, however, the armature is closed by the primary of a transformer then upper harmonics will appear, as for instance, in the arrangement in Figure 7. I do not think that these upper harmonics are entirely due to the variation of the permeability of the transformer core during each complete cycle, as has been suggested by very competent authority.¹*

1. See Professor Duncan's paper already cited; also H. A. Rowland:—"Notes on the Theory of the Transformer;" Phil. Mag., July, 1892.

A different interpretation of the phenomenon is possible. My experience in this matter favors the view that the upper harmonics are due to the interference between the electromotive forces generated in the alternator and the transformer. I prefer, however, to devote more time to the examination of this exceedingly important subject before venturing to discuss the validity of this possible new interpretation.

Allow me now to sum up briefly the main points of this discourse.

The practical bearing of "Low Frequency Electrical Resonance" in so far as I have worked out the subject seems to rest on the following characteristic features :

It brings out very prominently the purely mechanical character of the phenomena of electricity, and it does that not by referring to carefully prepared delicate experiments of a skilled physicist, but by referring to phenomena which every electrical engineer can observe every day in the testing room, telegraph and telephone stations, and in central stations for power and lighting. A full appreciation of this purely mechanical character of the phenomena of electricity is of the greatest practical importance, for we must remember that the founder of this mechanical view, the immortal Maxwell, in opening this new view of electrical phenomena contributed to the advance of this science as much as, if not more than, any other investigator of this century.

It offers a new, simple, and exceedingly convenient method of transforming electrical energy from low to high potential.

It enables us to observe very accurately the behavior of dielectrics and of iron when under the inductive action of resonant currents, and thus to determine the exact limits within which condensers can be employed in the solution of electrical engineering problems.

It offers a new and exceedingly simple method (the method of harmonic analysis mentioned above) of studying the working of alternating current machinery under various conditions of load.

It points out a new and apparently very promising direction in which the difficult but exceedingly important study of the magnetic properties of iron can be pushed ahead.

In closing my remarks I wish to thank my pupil, Mr. Milton C. Canfield, for the very efficient assistance he has given me in the preparation of this lecture.

DISCUSSION.

THE PRESIDENT:—Gentlemen, you have heard Dr. Pupin's admirable paper. Does the INSTITUTE wish to make any remarks on it?

MR. TESLA:—Gentlemen, I do not know whether I can contribute in any way to the clear and skillful exposition which Prof. Pupin has made of the phenomena of resonance. They have been familiar to me for a long time. In fact, two or three years back I began to work on lines in which an observation of some of the rules advanced by Dr. Pupin is an absolute necessity, and one of the reasons why I have only reluctantly consented to deliver a lecture on some high frequency phenomena, which I have shown on two or three occasions was, that I felt inadequate to the task, because in every experiment one has to depend on certain delicate balances and one cannot always feel sure of succeeding in the experiment, especially in a public lecture. Dr. Pupin has admirably succeeded in performing before an audience a number of difficult experiments. I saw this marvellous experiment with the iron core, performed by him last week, I believe, and I have been much impressed with the importance of the molecular condition of the iron. Any observation relating to the iron must be of interest, as it may lead to valuable results, and since I am called upon, I may mention a few of my own experiences, though they might not be in direct connection with the subject treated by our lecturer. About two years ago I observed a curious effect which I abstained from mentioning so far, because I could not explain it to my satisfaction. It was this:—I had an alternator of high frequency operating an induction coil. The secondary of this induction coil was connected to a Cardew voltmeter, and I had made the windings so as to bring the deflection into the best range of voltmeter. A condenser was connected to the alternator. I varied the amount of iron in the core of the coil and observed the rise in the electromotive force. As iron wires were added the electromotive force would continuously rise up to say 100 volts, Then there was a point at which when I added a few iron wires, the electromotive force fell off, about two volts. A further addition did not make any change whatever. I observed that effect repeatedly and repeatedly, but could not satisfactorily explain it. Now in the light of the experiment of Dr. Pupin and with the knowledge we presently possess, I cannot think else than that there was something of the same nature in the phenomenon. The delicate balance which, as Dr. Pupin has shown can be easily upset by the introduction of a single iron wire in the coil, reminds me of an experience with a machine which was regulated by means of an "auxiliary or third brush," a very interesting device, but owing to the extreme sensitiveness not quite commercially available. The idea is to take a dynamo, having the field magnet in series with the circuit and connect a point of the

field coil or of the external circuit by means of an auxiliary brush with a point of the commutator. The sensitiveness obtainable by this means can be anything one chooses, and so great can it be made that the slightest disturbance on the machine may upset the whole circuit, and if there be arc lamps in the circuit, for instance, one may barely tap a lamp thus causing an imperceptible movement of the carbon rod and all the lamps will go out, if the auxiliary brush is in such a position and the field coil so connected that the shifting of the neutral line is an *accumulative* effect. In order to improve the action of the lamp mechanism I had the machine constructed in such a way as to give periodic impulses of a certain intensity, which produced a noticeable vibration in the field magnet, and so well did I know the machine, operating it day and night (for at that time practically all my life was passed in the laboratory), that I could tell from these vibrations, by holding an iron bar in a certain position to the field magnet whether the machine was working properly. I noted then that sometimes all of a sudden the lights would go out. After this had occurred about a dozen times, I recognized that when I came with the iron bar near to, or touched the field magnet I disturbed the field enough to displace the neutral line and the lights would go out because the machine would discharge itself.

Another observation which I may mention about the iron, came to my attention in 1886. I then made some experiments on a so-called "thermo-magnetic generator. This *nihil bonum* principle has no doubt occupied the time of many inventors. I had attacked the problem in quite a different way, sufficiently promising to justify an engineer to enter into an investigation and see what could be done. The method I followed was to enclose in a stove an iron core which closed the circuit of a magnet, this iron core consisting of a box of iron with small sheets of iron running through offering an enormous cooling surface. I used steam passed through the interstices to cool down the iron. My expectation was that the iron would just cool down a little so as to become magnetized when steam was admitted by means of a valve. The circuit of the magnet would then be closed and the valve was automatically shut, thereupon the iron was again heated and demagnetized. To a certain measure the valve worked well, but the vibration was too slow. It generally so happened that when the steam was admitted it would be decomposed and a greenish flame would shoot out of the stove. This indicated that the temperature of the iron was too high. I would bring the temperature of the iron down by admitting two or three times the steam in pressing upon the valve. Every time I did this, the flame would shoot out of the stove. Then there would be a miss for about two or three times in succession, no flame appearing, the iron being cooled sufficiently, but upon doing it the fourth or fifth time which took about a second or two, the flame would shoot out once more. I could not explain this phe-

nomenon. A little later, I obtained the classical work of Dr. Hopkinson on the recalescence and other properties of iron at high temperatures and plainly saw that it was recalescence. The temperature must have just been varied around the critical point. Still another fact which I recollect to have observed in experiments with high frequency currents is, that an iron rod or wire, when an arc is formed through it, burns and sputters with violence, considerably more so with high than with low frequency or direct currents. It is possible that the effect is solely due to the magnetic properties of the iron, but it might be more or less produced by the increased resistance and consequently more intense heat evolution. With great magnetizing forces an iron wire inserted in a high frequency coil is instantly brought to a high temperature. This I have before pointed out, but Dr. Pupin has observed a similar thing with low frequency currents when a condenser is in circuit and resonating action takes place.

I wish to say in regard to converting to high potential, starting from a low potential dynamo, by simply combining capacity and self-induction with the circuit, that is by resonating action, I have in my paper, which has just gone to print, dwelt specifically upon converting in this manner, and in fact I am delighted to see that Dr. Pupin has taken the views he has here expressed. My usual method was to attach to the alternator a coil consisting of a great number of sections, and to associate adjustable capacity with the coil, and then to vary the number of sections and the capacity until the best resonance was obtained. This obviated the employment of a secondary coil; but I found that in adjusting, it was preferable to convert first the low tension currents to such a high potential that the necessary capacity would become extremely small, so as to be reduced merely to that of two small plates, because of the great difficulty of producing a condenser of large capacity adjustable by extremely small steps, which it is necessary to employ when the potentials are small. If we convert to a potential of say 50,000 volts then the capacity becomes very small, and we can just use a few plates immersed in oil, which permit varying the capacity by extremely small degrees, and without sudden, disturbing breaks. This is one of the reasons why I have always preferred to convert in a secondary circuit. The adjustment by varying self-induction by means of an iron core is, as Dr. Pupin's experiment now plainly shows, not always practicable. I think that the experiments we have seen here demonstrate that we are as yet far from having studied the properties of iron thoroughly, and that much remains to be done in this direction. I agree with Dr. Pupin that the condenser, which I had thought was going to be of great help in alternating work, does not really appear to have a great future in connection with apparatus with large iron cores, as motors or transformers, because of the action of the iron, and for one more reason besides, namely, that we can, especially if we have a high potential or high frequency, always

wind a coil so as to take up just enough capacity to be in resonance with the circuit, in so far as the iron core will permit.

I only have to add, that I sincerely thank Dr. Pupin, (which probably is the wish of all), and that I have profited very much by his lecture, which is a clear and precise exposition of the principle, as well as a most interesting experimental demonstration of electrical resonance with a dynamo machine.

[Adjourned.]

[COMMUNICATED AFTER ADJOURNMENT BY DR. PUPIN.]

Mr. Tesla is very kind and generous with his compliments. I do not think, however, that his statement of facts is free from ambiguity. He states that the phenomena of resonance which I have discussed were familiar to him for a long time. My discussion referred to low frequency resonance, whereas Mr. Tesla has, I think, always worked with high frequency apparatus, so that his familiarity with low frequency resonance, the subject which I discuss, is probably not based upon his own experimental investigations. At the time when the Ferranti effect called forth so much valuable discussion in England, Mr. Tesla¹ contributed some remarks to this discussion, which remarks culminated in the following:—"The writer looked for a case of resonance,² but he was unable to augment the effect by varying the capacity very carefully and gradually, or by changing the speed of the machine. A case of pure resonance he was unable to obtain. When a condenser was connected to the terminals of the machine * * * the capacity which gave the highest E. M. F. corresponded most nearly to that which just counteracted the self induction with the existing frequency."

Mr. Tesla states also in this article that the highest value of the augmentation of the potential which he was able to obtain by connecting a condenser in series to the armature of his high frequency (10^4 periods per second) machine was *only four or five times the impressed E. M. F.* What then could Mr. Tesla infer from this result in regard to resonance effects with low frequency? Certainly nothing that was encouraging.

At no other time did Mr. Tesla, so far as I am aware, approach the subject of resonance any closer than in the article just referred to.

Perhaps Mr. Tesla means that he was acquainted with the phenomena of low frequency resonance which I discussed from the investigations of other men, like Fleming, Kapp, Glazebrook, Swinburne, Lodge, Blakesley, etc.³ I appreciate fully the value of the labors of these men, but still I fail to see a very close re-

1. *Electrical World*, N. Y., Feb. 21st, 1891.

2. The meaning of this term Mr. Tesla did not explain.

3. See *London Electrician* for 1889, 1890 and 1891, also "Fleming's Alternate Current Transformer," vol. ii., p. 401.

lation between their work and mine, excepting in one point, and that is, that they as well as I, have shown theoretically that it is possible to produce a high rise of potential by a combination of self-induction and capacity. *I have shown it experimentally also.* But I think that I have pointed out very clearly in the course of my lecture that I did not regard the resonant rise of potential as an end but simply as a means to an end, the end being harmonic analysis of E. M. F's, behavior of iron and dielectrics under the inductive action of resonant currents of low frequency, weeding out of harmonics by means of resonance, etc. With these phenomena Mr. Tesla does not appear to be familiar, either from his own experimental investigations or from investigations of others, excepting those phenomena which Dr. Duncan had investigated some time before me and which I mentioned in the course of my lecture.