The Method of Constant Rate of Change of Flux as a Standard for Determining Magnetisation Curves of Iron

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XXVIII. *The Method of Constant Rate of Change of Flux as a Standard for Determining Magnetisation Curves of Iron.*
By J. T. MORRIS and T. H. LANGFORD, B.Sc.

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INTRODUCTION.

DOUBTS must have arisen in the minds of many investigators as to how nearly the magnetisation curves of a sample of iron obtained by differing methods would agree with one another.

Discrepancies which have been found from time to time may be attributed to one or more of the following causes :—

(i.) Inaccuracies in experimenting.

(ii.) Lack of exact repetition of previous conditions. In other words, the physical state may differ in different experiments with regard either to hardness, temperature or previous magnetic treatment.

(iii.) Differences inherent in the methods employed.

The present research was instituted with the object of studying the last of these three causes, and obtaining definite information as to the magnitude of these differences (*a*) between the magnetisation curves for a given sample of iron when determined by the older methods, in which the flux is changed suddenly, and (*b*) by a method in which it is changed slowly and at a uniform rate.

Methods in General Use.—The relation between the magnetising force **H** and the flux-density **B** produced by it in a specimen of magnetic material is usually determined by one of the following methods :—

(*a*) *Ballistic Methods.*—The specimen, preferably in the form of a closed ring, is wound with primary and secondary coils, and the change of flux, caused by a sudden change of current in the primary coil, is measured by the extent of the first swing given by the moving system of a ballistic galvanometer in series with the secondary winding.

(*b*) *Alternate-current Methods.*—The specimen is prepared as in (*a*) with primary and secondary coils, and alternating currents of commercial frequency are passed through the primary winding, the maximum currents and the corresponding maximum values of the flux-density being determined.

(c) *The "Fluxmeter" Method.*—The Grassot fluxmeter* is an instrument somewhat similar to a ballistic galvanometer, but the moving coil is suspended by a single cocoon fibre exerting no appreciable controlling torque. The movable coil is connected to the secondary winding of a specimen ring, as in the ballistic method, and it is deflected through an angle which is proportional to the time-integral of the voltage induced in the secondary winding. The deflection is thus proportional to the total change of flux through the ring, and is independent of the time occupied by the change.

(d) *Magnetometric Methods.*—A long thin bar of the material to be tested is wound with a magnetising coil, and the strength of pole produced by a known number of ampere-turns is determined by means of a magnetometer.

(e) *Attraction Permeameter Methods.*—The sample is arranged to close the magnetic circuit of an iron yoke of such dimensions that its reluctance may be neglected; then the mechanical force of attraction between the specimen and the yoke is measured by observing either the force necessary to detach the specimen or the increase of the force required to cause the specimen to slide on the yoke.†

COMPARISON OF METHODS.

Permeameter measurements have the advantage of rapidity, but the accuracy attainable is not very high, owing to the indefinite degree of contact between the specimen and the yoke. This method is generally employed only for workshop tests. The magnetometric method is liable to inaccuracies due to the lack of uniformity in the magnetisation of the sample, including the indeterminate position of the poles and their demagnetising action. Alternate-current methods have the disadvantage that they are indirect, the required values of the current and flux-density having to be deduced from the observations by a complicated method if accuracy is to be ensured. When the greatest accuracy is required, ballistic methods on ring samples are, therefore, those to which resort is usually made. The test is commonly made by one of two methods: In one of these the current through the primary

* "Some New Electrical Instruments," THE ELECTRICIAN, Vol. LVI., p. 560, 1906.

† S. P. Thompson, "The Electromagnet"; W. H. F. Murdoch, "The Magnetic Testing of Iron," I.E.E., Vol. XL., p. 137, 1907.

winding is increased up to a certain value and then suddenly reversed to obtain a reading ; in the other method the current is increased or diminished by a step at a time, and the consequent ballistic reading taken. This difference in the character of the magnetic change to which the iron is subjected is liable to cause differences in the results obtained, partly owing to magnetic "creep" or viscosity.

A method proposed by C. F. Scott, and apparently first described by J. S. Peck,* as applied to large transformers, possesses the marked advantage over ballistic methods that the variation of the flux takes place at a known and constant rate, enabling experiments to be repeated under exactly similar conditions. Rates of change varying from 50 to 250 lines per square centimetre per second were actually employed in the experiments. Further, standardisation can be more readily performed, as a mutual induction is not required. This method has been modified by Dr. D. K. Morris so as to render it practicable for tests on small samples of iron.†

The Method of "Constant Rate of Change of Flux."—Consider an iron ring in which a flux, N , is produced by a current, C , flowing in a primary winding. By varying the current C in a continuous manner the flux N in the ring is continuously varied, and at any instant the E.M.F. E induced in a secondary winding is proportional to the rate of change of the flux N —i.e., to dN/dt —and the total change of flux which has taken place in the ring during any interval of time ($t_2 - t_1$) is $N_2 - N_1$

$$= \int_{t_1}^{t_2} \frac{dN}{dt} \cdot dt.$$

If now the primary current C be varied at such a rate that the induced E.M.F. E is maintained at a constant value, the flux N is known to be varying at a constant rate, $dN/dt = k$.

In this case the total change of flux through the ring is simply proportional to the time during which it has taken place, for

$$\int_{t_1}^{t_2} \frac{dN}{dt} \cdot dt = k \int_{t_1}^{t_2} dt = k(t_2 - t_1).$$

If n_2 be the number of turns on the secondary winding, and

* J. S. Peck, "On Testing Large Transformers," "Electrical World and Engineer," Vol. XXXVII., p. 1083, 1901.

† D. K. Morris and G. A. Lister, "Transformers and Transformer Iron," I.E.E., Vol. XXXVII., pp. 282-294.

E be the value of the constant E.M.F. in volts induced therein, we have

$$E = dN/dt \times n_2 \div 10^8 = kn_2/10^8$$

$$\therefore k = \frac{E \times 10^8}{n_2},$$

and total change of flux

$$(N_2 - N_1) = k(t_2 - t_1) = \frac{E \times 10^8}{n_2} (t_2 - t_1).$$

Hence a magnetisation curve can be deduced from readings of current and time when the current is varied at such a rate that the secondary induced E.M.F. is constant and of known value.

Since the rate at which the flux is varied can be made as slow as desired, any "creep" in the magnetisation due to magnetic viscosity will be recorded. This method, therefore, appears to be preferable to others as the standard method for determining the fundamental magnetic properties of iron.

The following experiments were undertaken to investigate the differences, if any, between the results obtained by this and by other methods.

METHODS EXPERIMENTALLY EXAMINED.

I. *Method of Constant Rate of Change of Flux, or, more shortly, that of "Uniformly-varying Flux"* (or "Constant Induced Voltage," as it has been termed by previous writers).—After the iron has been demagnetised, the flux-density is continuously raised from zero to the maximum value, as above described.

II. *"Slow Cyclic" Hysteresis Loops*.—A similar method to the first, except that the iron is carried through cycles of magnetisation of gradually increasing maximum value, and the maximum values of **B** and **H** are plotted to give the magnetisation curve.

III. *"Step-by-step" Magnetisation Curve*.—The iron is first demagnetised and the current is then raised by small increments, the flux-density produced being deduced from the throws observed on a ballistic galvanometer after each increment of current.

IV. *"Step-by-step" Hysteresis Loops*.—A similar method to the last, except that the iron is carried through cycles of magnetisation of gradually increasing maximum value, and the

maximum values of **B** and **H** are then plotted to give the magnetisation curve.

V. *Method of Reversals*.—The iron is demagnetised, the current raised to a certain value, reversed, and the resulting throw taken on a ballistic galvanometer. This is repeated for currents of gradually increasing value, and the magnetisation curve so obtained plotted.

VI. *Alternating-current Magnetisation Curve*.—The magnetisation curve is deduced from readings of the magnetising current and induced E.M.F. when the winding of the ring is traversed by an alternating current at a frequency of 60 cycles per second.

Before each reading recorded by methods II., IV. and V. the iron was brought into a cyclic condition by carrying it through a sufficient number of cycles of magnetising force of the same maximum value, some 50 cycles being generally sufficient for this purpose.

PRACTICAL DETAILS OF METHOD OF UNIFORMLY VARYING FLUX.

In Fig. 1 is given a diagram of connections. The voltage induced in the secondary winding **S** is balanced by the fall of potential over a known resistance, r_1 , in series with a known resistance, r_2 , across a cell, **B**, of known E.M.F., equality being indicated by a zero reading on a galvanometer, **G**. The battery **B** is closed on to the circuit $r_1 r_2$, and simultaneously the primary current is caused to begin to change by means of the rheostat **A**, and the experimenter has continuously to adjust the rheostat at such a pace as will maintain the galvanometer reading at zero. This adjustment requires some skill, but an average experimenter easily acquires this with a little practice.

In making a hysteresis test the primary current is continuously varied from a maximum positive value to an equal negative value by a specially designed resistance,* in which two sliders, *a*, *b*, connected to the primary winding **P**, are moved in opposite directions across the edges of the zigzag strip resistance **A**. A photograph of the rheostat employed is shown in Fig. 2.

The primary current is measured on a potentiometer by the

* J. T. Morris, R. M. Ellis and F. Stroude, "The Design of a Continuously Adjustable Rheostat," *THE ELECTRICIAN*, Vol. LXI., p. 400, 1908.

voltage drop across a known resistance, r_3 , the exact times at which the current has certain convenient values being recorded by a chronographic arrangement.

FIG. 1.—CONNECTIONS FOR METHOD OF UNIFORMLY VARYING FLUX.

A Morse inker was employed for this purpose, the receiving coil being in series with a contact made at each swing of a

FIG. 2.

pendulum. The arm carrying the inking wheel was arranged to deflect a pencil (which rested on the travelling paper tape) a small distance laterally. A key was also arranged so that, on

depression, the same pencil was deflected laterally double the distance. Timing and observation marks were easily distinguishable. This arrangement was found to be quite satisfactory.

The above electrical connections could be simplified if the galvanometer were directly closed on to the terminals of the secondary winding, but this arrangement has some objections—(1) that it is difficult to allow for the uncertainty in the value of the secondary E.M.F. induced during the time that the galvanometer deflection is attaining its normal magnitude; and (2) that a separate operation has to be performed in order to standardise the galvanometer. A further slight objection is that there is an opposing magnetomotive force due to the current flowing in the secondary winding. This, in general, however, is so small as to be negligible.

MAGNETISATION CURVE BY ALTERNATING CURRENT.

It is important in connection with transformers and other alternating-current apparatus to determine the maximum value of the alternating flux-wave produced in iron by a magnetising current wave of known maximum value.

In order to carry out this determination the following experiments were made. The iron ring was demagnetised by a continuously decreasing alternating current from a generator, G (Fig. 3). This current was then gradually increased, its R.M.S. value being read on an ammeter, A, and the R.M.S. value of the voltage induced in the secondary winding S measured by the deflection produced on a Duddell thermo-galvanometer, TG, in series with a known variable resistance, R_2 . The thermo-galvanometer was calibrated by a secondary cell, B, of known E.M.F. To determine the wave forms of the magnetising current and secondary induced voltage the strips CS and VS of a Duddell oscillograph were connected as shown, the current strip CS being shunted by a variable portion of a resistance, R_1 , and the volt strip VS being connected so as to measure the volts across the primary winding P, the ammeter A and the current strip CS in parallel with its shunt. The resistance of this circuit was measured, and the measured volt wave corrected by subtraction of the corresponding $C \times R$ drop, thus giving the true induced E.M.F. wave. The area of this was then measured by a planimeter, and from this the maximum flux-density was calculated, corresponding to the R.M.S. volts measured on the thermo-galvanometer. This determina-

tion was made for a number of different values of the secondary induced voltage. As a check, the R.M.S. value of the corrected volt wave was determined, and compared with the value obtained by measurement with the thermo-galvanometer on

FIG. 3.—CONNECTIONS FOR MAGNETISATION CURVE BY ALTERNATING CURRENT.

the secondary; also the R.M.S. value of the volts measured by the volt strip VS was read on the thermo-galvanometer and compared with the value obtained from the oscillogram.

FIG. 4.—FLUX WAVE DEDUCED FROM OSCILLOGRAM.

A specimen oscillogram, showing current and induced voltage waves, together with the flux wave deduced therefrom, is shown in Fig. 4.

The value of the maximum flux-density induced may be determined from the R.M.S. value of the induced voltage by the formula

$$B = \frac{E}{Z \times n \times A} \times k \times 10^8,$$

where E = R.M.S. value of induced voltage.

Z = Number of secondary turns.

n = frequency in cycles per second.

A = cross-sectional area of iron ring in square centimetres.

B = maximum flux-density in lines per square centimetre.

k = factor dependent on the wave-form, being $1/4.44 = 0.2252$ for a sine wave of induced voltage.

The values of the factor k were experimentally determined at various points on the magnetisation curve, and are given in Table I.

TABLE I.—Values of Factor “ k ” in Formula $B = \frac{E \times 10^8}{ZnA} \times k$.

R.M.S. volts generated per turn at 1 period per second.	Factor “ k .”
0.000129	0.227
0.000422	0.228
0.00133	0.211
0.00134	0.217
0.00166	0.192
0.00171	0.193

The maximum value of the magnetising current is obtained from its R.M.S. value as measured on the ammeter by multiplying this figure by the form-factor, the experimentally-determined values of which are collected in Table II.

TABLE II.—Current Wave Form Factors.

R.M.S. amperes.	Form Factor.
0.194	1.430
0.405	1.370
1.854	1.904
2.57	1.935
5.05	1.820
8.135	2.070

Data of Specimen used.—The ring was made up of 24 stampings, 6 in. external and 5 in. internal diameter, of Sankey’s best transformer iron, manufactured about the year 1900, of

24 S.W.G. The mean magnetic length of the ring was found by measurement to be 43.93 cm. The sectional area of the ring was determined from its weight and specific gravity, the latter being determined by weighing in air and in carbon tetrachloride. This liquid is preferable for this purpose to water on account of its non-oxidising properties, and also because of its higher specific gravity. The stampings were insulated from one another by thin paper rings, and the complete ring was lapped over with insulating tape and wound with two No. 19 D.C.C. wires laid side by side, giving two windings of 147 turns each. These could be used in series or in parallel as a magnetising winding, or one could be used as a magnetising winding and the other as a secondary. A secondary winding of 50 turns and one of 10 turns were also wound on. Exact data of the stampings are as follow :—

Weight of pack of 24 stampings	= 653.0 grm.
Volume (sp. gr.=7.805)	= 83.67 c.c.
Mean length of path	= 43.93 cm.
Sectional area	= 1.904 sq. cm.
Effective thickness	= 1.470 cm.
Actual thickness of laminæ+paper insulation	= 1.616 cm.

The paper insulation occupies about 5 per cent. of the total thickness, and the roughness of the iron surface accounts for about 4 per cent.

EXPERIMENTAL RESULTS.

The results obtained experimentally by the above-described methods I. to VI. are given in the form of curves in Figs. 5, 6 and 7, and readings taken from the plotted curves are given in Tables III. and IV.

The magnetisation curves are shown in Fig. 5, the initial parts being shown separately to an enlarged scale of H .

In Fig. 6 the results are shown in the form of permeability curves, and in Fig. 7 in the form of reluctivity curves.

In Fig. 8 the differences between the flux-density obtained by method I. and by methods II., III., IV. and V., with the same magnetising force, are plotted to a greatly enlarged scale against B as obtained by method I. The divergence of the magnetisation curve obtained by method VI. from the standard curve obtained by method I. is too great to allow of its being shown in Fig. 8, but is apparent in Figs. 5, 6 and 7.

Fig. 9 shows hysteresis loops taken by methods II. and IV., the maximum value of H being the same in each case.

The above experiments were carried out in the electrical

TABLE III.

Flux Density B by methods :—						
H	I.	II.	III.	IV.	V.	VI.
0.3	360	100	120	100	210	140
0.5	610	220	350	200	400	320
0.7	990	500	750	400	750	610
0.9	1,530	1,080	1,300	900	1,230	950
1.2	2,680	2,500	2,500	2,300	2,400	1,620
1.5	3,840	3,700	3,650	3,600	3,620	2,500
2.0	5,450	5,600	5,100	5,250	5,300	4,000
2.5	6,560	6,750	6,300	6,350	6,550	5,400
3.0	7,480	7,700	7,250	7,200	7,400	6,550
3.5	8,250	8,420	8,020	8,020	8,120	7,650
4.0	8,850	9,080	8,700	8,600	8,700	8,500
4.5	9,440	9,600	9,200	9,060	9,300	9,220
5.0	9,880	10,100	9,620	9,500	9,800	9,800
6.0	10,700	10,920	10,410	10,300	10,500	10,650
7.0	11,330	11,550	11,050	11,040	11,250	11,300
8.0	11,920	12,100	11,600	...	11,850	11,820
9.0	12,400	12,600	12,100	...	12,300	12,300
10.0	12,850	12,960	12,520	...	12,750	12,760
12.0	13,580	13,600	13,110	...	13,400	13,460
14.0	14,200	14,100	13,590	...	13,950	14,200
16.0	14,720	14,510	13,950	...	14,380	14,620
18.0	15,110	14,750	14,300	...	14,750	15,100
20.0	15,400	15,100	14,650	...	15,050	15,450
25.0	16,040	15,510	15,440	...	15,600	16,120
30.0	16,500	16,050	16,010	...	15,830	16,420
35.0	16,840	...	...	...	16,180	16,650
50.0	17,320	...	...	...	16,700	17,020
70.0	17,850	...	...	...	17,270	17,120

TABLE IV.

Permeability by methods :—						
B	I.	II.	III.	IV.	V.	VI.
500	1,210	740	820	660	880	840
1,000	1,430	1,140	1,240	1,070	1,245	1,075
1,500	1,668	1,510	1,580	1,440	1,545	1,300
2,000	1,950	1,810	1,870	1,750	1,820	1,470
2,500	2,175	2,080	2,080	2,015	2,030	1,630
3,000	2,356	2,270	2,270	2,215	2,220	1,755
3,500	2,500	2,420	2,400	2,370	2,380	1,850
4,000	2,660	2,535	2,480	2,490	2,505	1,940
4,500	2,690	2,640	2,520	2,580	2,590	2,000
5,000	2,740	2,730	2,545	2,620	2,650	2,050
5,500	2,740	2,787	2,555	2,620	2,660	2,090
6,000	2,700	2,780	2,550	2,585	2,645	2,130
6,500	2,620	2,740	2,520	2,520	2,620	2,150
7,000	2,565	2,660	2,465	2,440	2,550	2,160
8,000	2,410	2,490	2,295	2,270	2,360	2,156
9,000	2,190	2,255	2,070	2,035	2,120	2,045
10,000	1,930	2,010	1,830	1,810	1,790	1,910
11,000	1,720	1,765	1,590	1,570	1,660	1,720
12,000	1,465	1,510	1,345	...	1,440	1,500
13,000	1,250	1,260	1,100	...	1,210	1,280
14,000	1,060	1,005	860	...	980	1,070
15,000	868	765	660	...	760	880
16,000	650	520	530	...	520	650
17,000	425	...	...	...	280	330

engineering department of the East London College (University of London), and the authors wish to express their best thanks

FIG. 5.—MAGNETISATION CURVES.

FIG. 6.—PERMEABILITY CURVES.

for the practical suggestions and assistance in the experimental work given by Mr. F. Stroude, B.Sc.

DISCUSSION OF RESULTS.

An examination of the divergences shown in Fig. 8 has led the authors to divide, for purposes of consideration, the curves into three parts: (1) From zero to the approximate point of maximum permeability ($B=5,000$); (2) from this point to $B=15,000$; (3) above $B=15,000$.

1. *Curves below $B=5,000$.*—These may be divided into two groups: (a) Those in which the magnetisation curve is determined with the flux always changing in the same sense, the iron having previously been completely demagnetised; and

FIG. 7.—RELUCTIVITY CURVES.

(b) those in which each reading is determined after the iron has been carried through a sufficiently large number of cycles of magnetising force to obtain constancy in the corresponding value of the flux-density.

Considering first those which belong to group (a), we have two cases: (i.) That of the magnetisation curve obtained by the uniformly varying flux method (method I.), and (ii.) the magnetisation curve determined by the step-by-step method (method III.).

It will be noted in Fig. 8 that there is a difference between these two curves of, broadly speaking, some 250 lines per square

centimetre, from $B=1,000$ to $B=12,500$, and that the step-by-step method is deficient by this amount. This difference appears to be due to some flux not being recorded in the portion of the magnetisation curve below $B=500$ in the step-by-step method. It is quite probable that this is due to the known behaviour* of iron under weak magnetising forces. When a magnetising force of, say, $H=0.2$ is suddenly applied, the flux at once changes by a certain amount and afterwards continues to change, though at a diminishing rate; this latter "creep" would not be recorded by an ordinary ballistic galvanometer,

FIG. 8.—CURVE SHOWING DIFFERENCES BETWEEN MAGNETISATION CURVES OBTAINED BY VARIOUS METHODS.

and therefore would not wholly be taken account of in the step-by-step method, but it is included in method I.—that of uniformly varying flux. (The time of one complete swing of the ballistic galvanometer used throughout these experiments was 8.6 seconds.)

With the object of investigating this point the following experiment † was made. With the connections arranged as for

* J. A. Ewing, "Magnetic Induction in Iron and Other Materials," p. 124.

† Hopkinson, "Original Papers," Vol. II., p. 257.

the ballistic test, a sudden change was made in the magnetising current while the circuit of the galvanometer and secondary winding was held open, so that the galvanometer did not receive the "kick" due to the instantaneous change of flux in the ring. The galvanometer circuit was then immediately closed, so that if there were any residual creeping of the flux there would be a steady deflection of the galvanometer.

FIG. 9.—HYSTERESIS LOOPS BY METHODS II. AND IV.

The maximum creep observed occurred when a magnetising force of $H=1.25$ in one direction was suddenly changed to the value corresponding to the maximum instantaneous permeability in the other direction ($dB/dH=6,000$). Under these conditions, the creep was 40 lines per square centimetre per second, about 0.1 second after the change in H , and this creep diminished to 3.3 lines per square centimetre per second after about 1 second, disappearing entirely after about 5 seconds. On completely reversing the current the creep after about 0.1 second was only 4.6 lines per square centimetre per

second, and after 1 second it had decreased to less than 1 line per square centimetre per second. This effect, therefore, although appreciable, does not appear to account wholly for the observed deficiency in the ballistic magnetisation curve. It will be noticed that all the curves in Fig. 8 start with a sharp downward inclination from the axis, showing that during the initial part of the magnetisation curve the highest value of $\mathbf{B}$ for a given value of $\mathbf{H}$ is attained by the method of uniformly varying flux, or, in other words, a higher value of $\mathbf{H}$ is required to produce the same observed value of $\mathbf{B}$ by the other methods. One reason for this may be that when the flux is changed suddenly, as in ballistic methods, an excess magnetising force may be required to counteract the demagnetising force due to eddy-currents caused in the laminations, and this may become considerable when the change of flux is very rapid. In order to investigate this point, an estimate was made of the demagnetising force due to eddy currents, and this was found by calculation to be about $0.00000046 \times d\mathbf{B}/dt$. Now the excess $\mathbf{H}$, for a flux-density of 500, required for the step-by-step method, as compared with the uniformly varying flux method, amounts to about 0.15, and this amount would be accounted for by eddy currents if the change from $\mathbf{B}=0$ to $\mathbf{B}=500$ took place in about 0.0015 second, which does not seem unlikely. In the case, however, of the magnetisation curve drawn through the tops of the hysteresis loops obtained by the uniformly varying flux method another explanation must be sought, as the rate of change of flux, $d\mathbf{B}/dt$, is too small to produce appreciable eddy currents. It may be that, on slowly increasing the magnetising force from zero to a certain value and then carrying it through cycles having this value for a maximum, the value of $\mathbf{B}$ obtained initially may be greater than the value obtained after the iron has attained the cyclic state, owing to the first loops being unsymmetrical about the axis of $\mathbf{H}$.

With regard to group (b), which includes methods II., IV. and V., it is difficult to find any rational explanation for the differences observed. The authors have, however, been impressed with the idea that when a sudden change of considerable magnitude is made in the magnetising force the instantaneous flux variation, as compared with the total flux variation, may depend on the steepness of the magnetisation curve at the point attained instantaneously by the flux; in other words, it might depend not so much on the permeability

at that point as on the rate of change of induction with magnetising force—*i.e.*, on $\frac{dB}{dH}$. One is led to imagine that if $\frac{dB}{dH}$ is great, then the stability of the molecular magnet grouping is correspondingly weak, and, keeping the Ewing model in view, one would predict that, for a sudden large change in the magnetising force, a magnetic creep depending to a certain extent on this factor $\frac{dB}{dH}$ would be expected.

This quantity $\frac{dB}{dH}$, and also the permeability, both calculated from the magnetisation curve obtained by the method of uniformly varying flux (method I.), have been plotted in Fig. 10.

FIG. 10.— μ AND $\frac{dB}{dH}$ CURVES.

It will be observed from Fig. 8 that the maximum deficiency in B as obtained by methods II., IV. and V., occurs at about that part of the magnetisation curve where $B=2,000$, and Fig. 10 indicates that this is the point where the steepest part of the magnetisation curve commences, or, in other words, where $\frac{dB}{dH}$ attains its maximum value. It appears probable, therefore, that the maximum value of B attained when the iron is carried through a sufficiently large number of cycles of H depends to some extent upon the slope of the magnetisation

curve at that point, B having a diminished value where the magnetisation curve is steepest.*

2. *Curves between $B=5,000$ and $B=15,000$.*—From Fig. 8 it will be observed that over this range, whilst the magnetisation curve obtained by the step-by-step method (method III.) maintains its deficit of about 200 lines per square centimetre with respect to the magnetisation curve obtained by the proposed standard method of uniformly varying flux (method I.), the magnetisation curves obtained by methods II., IV. and V., in which the iron is brought into a cyclic state before readings are taken, show a decreased deficit with respect to the standard curve, the curve obtained by the slow cyclic loops method (method II.) actually showing, over this range, an excess of some 200 lines per square centimetre. During experiments by methods I. and II. the iron was subjected to the same kind of magnetic change—i.e., the flux was varied slowly and at a uniform rate. Whilst in method I. the observed value of B for a given value of H is that attained when the iron is first magnetised, the value observed by method II. is that attained after the iron has been carried through about six loops slowly, previous to which the current has been reversed about 100 times. The results appear to indicate that over this range the value of B attained for a given value of H increases as the magnetising force is carried through successive cycles, until finally a steady value of some 200 lines per square centimetre in excess of the initial value is attained.

Curves above $B=15,000$.—As considerable divergencies were obtained in the upper regions of the magnetisation curves by the various methods examined, it was thought that the temperature of the iron might have some bearing on these discrepancies, for in some of the experiments the full magnetising current may have caused a rise of 100°C . in the worst case. To investigate this matter the magnetisation curve of the iron was taken with the specimen at normal temperature and repeated with the ring heated by means of an alternating current to a temperature above that which it was likely to have attained in any of the foregoing experiments (some 150°C .).

* To make this investigation more complete, it would be necessary to devote considerable time and attention to the experimental study of magnetic viscosity, and it was in the hope of being able to do this that the publication of this Paper has been delayed for nearly two years; leisure for this, however, has not been forthcoming, and consequently it is thought advisable to publish the investigation as it stands.

Results were obtained similar to those described by Dr. J. Hopkinson.* For small magnetising forces, say up to $H=5$, the permeability was increased, whilst for strong magnetising forces it was decreased, as a result of the rise of temperature. Portions of the magnetisation curves taken with the iron hot and cold are shown in Fig. 11. On the same diagram are plotted the points obtained as the maximum values reached in the "slow cyclic" hysteresis loops (method II.). It will be noticed that, as the limiting values of these loops increase, so

FIG. 11.—MAGNETISATION CURVES WITH IRON HOT AND COLD.

they gradually pass from the "cold" to the "hot" curve. The authors, therefore, conclude that the results obtained above $B=15,000$ are of little scientific value.

DEGREE OF ACCURACY.

In comparing the accuracy attainable by the various methods it is only necessary to consider the accuracy of the means employed to measure the flux-density B , the measurement of

* Hopkinson, "Original Papers," Vol. II., p. 186.

the magnetising force H resolving itself merely into a measurement of current, which may be made by the same means, whatever be the method used in determining the flux-density, except in the case of the alternating-current method.

Methods I. and II., Uniformly-varying Flux.—The measurement of flux-density by this method involves (1) the opposition of a constant P.D. to the E.M.F. induced in the secondary winding, and the measurement of this opposing P.D.; (2) the maintenance of equality between the generated and opposing voltages; and (3) the measurement of the time during which the flux is being changed.

With regard to (1), the opposing P.D. is obtained from a secondary cell connected to a high resistance, subdivided in a known ratio. Since the current given by the cell is very small, there is no difficulty in attaining constancy for the few minutes required for a test. The P.D. of the cell may be measured accurately by a potentiometer, and the opposing P.D. is then known with great accuracy from the known ratio in which the resistance is divided. As regards (2), the galvanometer used to ascertain the equality between the generated and opposing E.M.F.s may be as sensitive as desired. It is used merely as a "null" instrument, and its calibration curve is of no consequence. Since the secondary winding of the ring carries no current, it may be made of wire as thin as mechanical considerations will allow, and a large number of turns may be wound on. The rate of change of the flux may therefore be made small, without involving a very minute opposing E.M.F., and the change of flux represented by the smallest time measurable by the chronometric device used may be made as small as desired. In the case of (3), the accurate measurement of the time during which the flux changes presents no great difficulty, being of the order of from one to five minutes.

Methods III., IV. and V., Ballistic Methods.—The measurement of flux-density by these methods depends on the observing of the throw given by a ballistic galvanometer, the calibration curve of which has to be determined with the aid of a mutual inductance. The accuracy attainable depends, therefore, on the accuracy of the mutual inductance, the constancy of the zero of the galvanometer, and also to a lesser extent upon the time period of the moving system of the galvanometer (8.6 seconds), the accuracy being greater as the time of swing is greater, owing to the movement of the swinging system being less in the time during which the impulse is being received.

Method VI., Alternating-current Method.—The accuracy of this method depends upon the accuracy with which the R.M.S. value of an alternating current can be measured, and also upon the accuracy with which the wave-forms of current and flux can be determined. By the use of a thermo-galvanometer the R.M.S. value of the magnetising current and of the induced E.M.F. may be measured with great accuracy, but the measurement of oscillograms presents difficulties owing to thickness of the line traced out on the plate by the vibrating beam of light and other causes, whereby the percentage accuracy in the measurement of the form-factors is not so great as that attainable in the measurement of currents.

Observation Errors.—In Tables V. and VI. are set out the differences observed between curves taken on different dates by the same methods in order to give an idea of the accuracy of repetition. In Table VII. are given the collected results expressed in percentage accuracy.

TABLE V.—*Accuracy of Repetition of Magnetisation Curve by method of Uniformly Varying Flux.*

Flux-density B by standard curve taken December 14 and 16, 1908.....	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
Difference in B between this curve and curve repeated by same method February 9, 1909.....	+32	+32	+20	0	−10	+40	+62	+15	+80

TABLE VI.—*Accuracy of Repetition of Magnetisation Curve by Ballistic Methods.*

Flux density B by method of uniformly varying flux (method I.)...	2,000	4,000	6,000	8,000	10,000	12,000	14,000	16,000
Difference in B between curves taken May 5, 1908, and April 30, 1909, both by "step-by-step" method (method III.).....	−40	−230	−140	−55	0	−50	+30	−160
Difference in B between curves taken August 4, 1908, and April 20, 1909, both by "method of reversals" (method V.)	+125	+80	−30	+25	+180	−20	−220	−290

TABLE VII.—*Comparison of Percentage Accuracy.*

—	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
Uniformly varying flux	+3.2	+1.6	+0.7	0.0	−0.2	+0.7	+0.9	+0.2	+0.9
Step-by-step	...	−2.0	...	−5.7	...	−2.3	...	−0.7	...
Reversals	...	+6.2	...	+2.0	...	−0.5	...	+0.3	...

Effect of Intervening Temperature Rises on Accuracy of Repetition.—The authors have, after considerable experience, had forced upon them the fact that if determinations of permeability upon the same sample are to be repeated with an accuracy of even 5 per cent. on the lower portions of the magnetisation curves it is essential that the ring be not subjected to considerable rises of temperature, say of $100^{\circ}\text{C}.$, even for short periods. Effects due to heating have been noted by many experimenters, such as S. R. Roget, in a Royal Society Paper.* Rises of temperature may be brought about by demagnetisation with alternating currents, or by the heat produced by excessive currents used with the object of determining the induction density at high values of H . The discrepancies observed at low values of H between two determinations made at the same temperature, but at different times, with an intervening period of heating, are not to be confused with the previously mentioned discrepancies in the higher regions of the magnetisation curves which are due to the differing temperature of the iron whilst the test is being conducted. If the iron be allowed to regain its original temperature after being heated, tests may be repeated accurately in the higher regions of the magnetisation curve.

TIME REQUIRED FOR TESTS BY DIFFERENT METHODS.

As regards the time required for determinations by the various methods experimentally examined, ballistic methods are undoubtedly the most tedious, particularly when, for great accuracy, a ballistic galvanometer with a long period of swing is employed. The alternating-current method (method VI.) has considerable advantages in this respect, a full set of readings of magnetising current and induced voltage occupying a very short time. When, however, it is necessary to take oscillograms at various points in order to plot curves of form factors, the time required is enormously increased. The method of uniformly-varying flux (method I.) is peculiarly adapted for use where time is a consideration, and at the same time a high degree of accuracy is desired. A complete magnetisation curve may be taken in a very few minutes, and the mean of many such curves obtained in, say, one hour, the iron being demagnetised between each test.

* S. R. Roget, "Effects of Prolonged Heating on the Magnetic Properties of Iron," *THE ELECTRICIAN*, Vol. XLI., 1898, p. 182.

CONCLUSION.

The method of "uniformly varying flux" appears to possess advantages, both scientific and practical, over the older methods in use for the testing of ring samples of magnetic materials. It avoids difficulties due to eddy currents and magnetic viscosity, which effects are themselves due primarily to rapid or irregular changes of flux. It also has the practical advantage that experiments may be carried out with rapidity, accuracy of repetition, and under standard or predetermined conditions of magnetic change.

The authors, therefore, would commend this method to the careful consideration of those interested in the carrying out of magnetic tests, especially where great accuracy under definitely known conditions of experiment are essential.

ABSTRACT.

The research described was instituted with the object of finding what differences there were between the magnetisation curves for a given sample of iron when determined (1) by the older methods in which the flux is changed suddenly, (2) by a method in which it is changed exceedingly slowly and at a uniform rate.

The methods experimentally examined were:—

1. Method of constant rate of change of flux.
2. "Slow cyclic" hysteresis loops by method 1.
3. "Step by step" magnetisation curve.
4. "Step by step" hysteresis loops.
5. Method of reversals.
6. Alternating-current magnetisation curve.

Details are given of the theory and practical working of method 1. In this method the magnetising current is continuously increased through a primary winding by a specially designed resistance at such a varying rate as to maintain a constant voltage generated in a secondary winding. A certain amount of skill is required in operating the resistance, but an average experimenter may easily acquire this with a little practice. The complete change of the current occupied times varying from one up to some five minutes.

Tables are given of the magnetisation curve determined by the six different methods at a number of values from H 0.3 up to 70.0, and of the permeability from B 500 up to 17,000.

At low values of the magnetising force the uniformly varying flux method gives results of some 200 lines per square centimetre in excess of the older methods.

As regards the time required for determinations by the various methods experimentally examined, ballistic methods are undoubtedly the most tedious. The alternating-current method (method 6) has considerable advantages in this respect, a full set of readings of magnetising current and induced voltage occupying a very short time. When, however, it is necessary to take oscillograms at various points in order to plot curves of form factors, the time required is enor-

mously increased. The method of uniformly-varying flux (method 1) is peculiarly adapted for use where time is a consideration, and at the same time a high degree of accuracy is desired. A complete magnetisation curve may be taken in a very few minutes, and the mean of many such curves obtained in, say, one hour, the iron being demagnetised between each test.

The method of "uniformly varying flux" appears to possess advantages, both scientific and practical, over the older methods in use for the testing of ring samples of magnetic materials. It avoids difficulties due to eddy currents and magnetic viscosity, which effects are themselves due primarily to rapid or irregular changes of flux. Besides rapidity of experiment it also has the advantage of accuracy of repetition under standard or predetermined conditions of magnetic change.

The method is, therefore, commended for the carrying out of magnetic tests, especially where great accuracy under definitely known conditions of experiment are essential.

DISCUSSION.

Mr. A. CAMPBELL remarked that it was rather difficult to say whether it was tests made with a slowly varying flux that were required in practice. He thought tests made with cyclical changes were more useful. The differences in the results given by the two methods were important. He pointed out that the method could be used for the determination of the ohm by substituting a known mutual induction for the iron under test.

Dr. A. RUSSELL pointed out that the value of H was different at the inner and outer radii of the iron ring used by the authors.

Dr. S. W. J. SMITH said it would have been interesting and instructive if the authors had performed a similar series of experiments by the different methods upon some material freer from magnetic viscosity than soft iron. Experiments made upon a sample of nearly pure iron in weak fields did not seem to be explicable along the lines suggested by the authors for the iron they used.

Prof. C. H. LEES drew attention to the extraordinary simplicity of the reluctivity curves. It looked as if $1/\mu$ were a linear function of H .

The PRESIDENT remarked that by superposing on the constant magnetising field an alternating field which was gradually reduced to nothing, much larger values of the apparent permeability were obtained. For weak fields it was over 100,000. This effect was due to the residual magnetism left by the alternating field.

Prof. J. T. MORRIS, in reply, stated that they had not tried their method at widely different speeds. In reply to Dr. Russell, he stated that the same iron ring was tested by the different methods. Replying to Mr. Campbell, he had thought of the present method as a means of measuring a mutual induction.