591. Elementary Solution of Bee Cell Problem

Author(s): E. M. Langley

Source: *The Mathematical Gazette*, Vol. 10, No. 155 (Dec., 1921), pp. 370-371

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3604629>

Accessed: 24-12-2015 01:51 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

Hence, since the circles are equal, and the chords SM' , $P'M'$ are equal, the arcs SM' , $P'M'$ are equal. It follows that the locus of P' coincides with that of a point fixed on a circle equal to and rolling on $SM'X'$.

This proof obtains the epicycloid definition of the cardioid by inversion from the parabola.

A. CLEMENT JONES.

591. [K¹. 14.] *Elementary Solution of Bee Cell Problem.*

Given (i) cubic content to equal that of prism terminated by the two regular hexagons $ABCDEF$, $GHKLMN$, of length l .

(ii) Cell to be closed at one end by 3 equal rhombs $FQBT$, $BRDP$, $DSFP$ meeting at a point P on the axis OZ of the prism.

(iii) The surface to be a minimum.

Let a be a side of one of the hexagons.

Surface $= 6(BPQ + BQGH)$

$$= 6 \left(\frac{1}{2} \cdot \frac{a}{2} \sqrt{3} \cdot 2 \frac{a}{2} \operatorname{cosec} \phi + la - \frac{1}{2} a \frac{a}{2} \cot \phi \right)$$

$$= 6la + \frac{3a^2}{2} (\sqrt{3} \operatorname{cosec} \phi - \cot \phi), \text{ where } \phi = \text{inclination of rhombs to axis.}$$

Put

$$y = \sqrt{3} \operatorname{cosec} \phi - \cot \phi.$$

$$2 \cot^2 \phi - 2y \cot \phi + 3 - y^2 = 0,$$

$$\cot \phi = \frac{y \pm \sqrt{3y^2 - 6}}{2};$$

$$\therefore \text{min. value of } y^2 = 2, \text{ giving } \cot \phi = \frac{1}{\sqrt{2}};$$

$$\therefore AQ = \frac{a}{2\sqrt{2}}; \tan AQB = 2\sqrt{2}; \cos AQB = \frac{1}{3};$$

$$\therefore BQ = \frac{3a}{2\sqrt{2}}; \cos TBQ = \frac{\sqrt{2}}{\sqrt{3}}; \cos PBQ = \frac{4}{3} - 1 = \frac{1}{3} = \cos AQB;$$

$$\therefore \hat{PBQ} = \hat{AQB} \text{ and } \hat{BQF} = \hat{BQG} = \hat{FQG}.$$

Founded on Maclaurin's geometrical proof given at some length in Cresswell's *Maxima and Minima*.
E. M. LANGLEY.

592. [L¹, 5. c.] To find the evolute of the conic in (1) *Cartesians*, (2) *trilinears*.

Eliminant of $\alpha\beta\gamma$ between $L=0$, $L'=0$ and $S=0$ is $\Sigma\Sigma' - \Pi^2 = 0$.

Cartesians. If (x_1y_1) is the point at which the line $lx+my+n=0$ is normal, the tangent is $xX_1+yY_1+Z_1=0$;

$$\therefore lX_1+mY_1 \text{ or } (al+hm)x_1+(hl+bm)y_1+(gl+fm)=0.$$

Calling this $l'x_1+m'y_1+n'=0$, we have to eliminate x_1y_1 between this,

$$lx_1+my_1+n=0 \text{ and } S_1=0.$$

$$\begin{aligned} \text{Now } \Sigma' &= l'(Al'+Hm'+Gn')+\dots+\dots \\ &= l'\Delta l+m'\Delta m = \Delta(al^2+2hlm+bm^2), \end{aligned}$$

$$\text{and } \Pi = l \cdot \Delta l + m \cdot \Delta m = \Delta(l^2+m^2);$$

$$\therefore \text{tangential equation of evolute is } (al^2+2hlm+bm^2)\Sigma = \Delta(l^2+m^2)^2.$$

Trilinears. Here $(\alpha_1\beta_1\gamma_1)$ is the point of contact and $l\alpha+m\beta+n\gamma$ is to be perpendicular to $\alpha X_1+\beta Y_1+\gamma Z_1=0$;

$$\begin{aligned} \therefore lX_1+mY_1+nZ_1-(mZ_1+nY_1)\cos A \\ -(nX_1+lZ_1)\cos B-(lY_1+mX_1)\cos C=0; \end{aligned}$$

$$\text{or } pX_1+qY_1+rZ_1, \text{ where } p=l-m\cos C-n\cos B, \text{ etc.};$$

$$\text{or } l'a_1+m'\beta_1+n'\gamma_1=0,$$

where $l'=ap+hq+gr$.

$$\begin{aligned} \text{Now } \Pi &= l(Al'+Hm'+Gn')+\text{etc.} \\ &= \Delta(pl+qm+rn) = \Delta\Omega, \end{aligned}$$

$$\begin{aligned} \text{and } \Sigma' &= l'(Al'+Hm'+Gn')+\text{etc.} \\ &= (ap+hq+gr)\Delta p+\dots \\ &= \Delta(ap^2+bq^2+cr^2+2fqr+2grp+2hpg) \\ &= \Delta\bar{H}\Omega - \Sigma P + \bar{K}^2, \end{aligned}$$

$$\text{where } \bar{H} = a+b+c-2f\cos A-2g\cos B-2h\cos C,$$

$$P = A\sin^2 A + \dots + 2F\sin B\sin C,$$

$$\bar{K} = \Sigma(Al+Hm+Gn)\sin A;$$

$$\therefore \text{evolute is } \Sigma(\Delta\bar{H}\Omega - \Sigma P + \bar{K}^2) = \Delta^2\Omega^2,$$

$$\text{or } \Delta^2\Omega^2 - \Delta\bar{H}\Sigma\Omega + P\Sigma^2 = \Sigma\bar{K}^2,$$

$$\text{i.e. } [\text{Foci}] = [\text{Conic}][\text{Centre}]^2.$$

N. M. GIBBINS.