CRITICISMS AND DISCUSSIONS.

A LETTER FROM MR. HERBERT SPENCER.

To the Editor of The Monist :

As I feel it a duty to reserve, for other purposes, the very small power of work now left to me, I am obliged to decline entering upon a controversy. I must leave readers to examine for themselves—little hoping, however, that they will do so.

One point only I wish to note. The use of the expression "forms of thought," instead of "forms of intuition," was simply an inadvertence ; as will be manifest on observing that though I have used the wrong expression in the note, I have used the right expression in the text (p. 203), as also throughout my criticism of Kant's doctrine in *The Principles of Psychology*, Part VII, Chapter IV, "The Reasonings of Metaphysicians," § 399.

HERBERT SPENCER.

LOGIC AS RELATION-LORE.

In the French *Revue Philosophique*, in the August and September 1891 numbers of the same, M. George Mouret has an essay entitled "Mathematical Equality" in the course of which and as though subsidiary to his ostensible purpose he discourses upon the topics of relations and concepts and upon the fundamental elements of logic in general. His essay is really more important as a contribution to logical doctrine than as a treatment of mathematical equality.

The scope of his discourse will be seen by reference to his closing paragraphs in which he sums up what he considers to be the results achieved by him in his essay. Therein, he says that he has "treated of the general theory of the composition of concepts and relations and set the foundations of the logic of analysis and the logic of definition."

I. THE SPENCERIAN AXIOM.

The determining factor of every philosophical dissertation is of course some very general supposition which is taken as established and which exercises a controlling influence over all the observations of its author.

In this case this determining supposition is found in what M. Mouret calls his "*axiom of symmetry*." The same is thus stated by him "*Two things which have the same symmetrical relations to a third thing have between them that same relation.*"

M. Mouret is not one of those scholars that lack hospitality for other writings than those of their own nationality. From this fault so noticeable in the work of so many of the French scholars M. Mouret himself seems to be free. Indeed so far as regards the previous work done in the domain he sets himself to examine, he accords almost exclusive esteem to the writings of English thinkers. In fact he declares himself so far as regards his present topic a disciple of Mr. Herbert Spencer and puts his "*axiom of symmetry*" as an adaptation of that maxim of his said master which is of the tenor as follows, viz: "*Things which have a definite relation to the same thing have a definite relation to one another.*" This maxim, as Mr. Spencer tells us, was suggested to him by a remark of the late eminent author who is known to the world under the pseudonym of George Eliot, who herself stated it under the form "*Things that have a constant relation to the same thing have a constant relation to each other.*"

Those who are well acquainted with the psychology of Mr. Spencer will recognise that this maxim of his is made by him the very backbone of all his observations upon reasoning. If it has the validity which he imputes to it, it has an importance which it will be hard to overestimate, but if on the contrary, and as we shall submit, it is in every form in which it has been stated, certainly unsound and misleading, it is high time that its virtue should be brought into question.

The *dicta* of the masters whether they first enounce the same or whether they only give currency thereto by their ratification are always proper subjects for special scrutiny. There is always found a disposition to accept them on their mere *ipse dixit* without any attempt at criticism or independent observation. This is decisively *not* the scientific mood or mode. The spirit of that modern leaven that is currently referred to under the name of Science is characteristically a critical one, and one that is considerably irreverent in regard to the authority of mere personality. In this it is happily distinguished from the spirit that has marked the past history of what may be called the "*regular*" schools of philosophy.

M. Mouret is not alone in his inadvertent esteem for the maxim in question. In the issue of *Mind* for October 1891 Mr. L. T. Hobhouse publishes an article entitled "*Induction and Deduction*," in which he gives an undue appraisal to the worth of the maxim under consideration, even though the author of the article seems to be well aware that said maxim stands in much need of qualification.

We venture to say that this maxim in all its forms has gained whatever currency it has enjoyed in virtue alone of the incompetent comprehension that too generally prevails in regard to the nature and characteristics of that sort of things that are relations.

A notable example of this lack of comprehension is supplied in the logical treatise of Mr. Carveth Read, a work ostensibly founded upon the significance of the

category of relation and yet in which at the very start the author tells us that a relation is something which is indefinable.

II. IMPORTANCE OF RELATION-LORE.

This topic of relations is one that is neglected in a degree that reflects no credit upon the pretensions of those who undertake to instruct others in matters logical and philosophical. The thing itself is in the thinking of every one and the term and its derivations are in universal use. They are used as though they imported an idea that no one was liable to misapply or to misunderstand. The truth, however, is that of all the stock terms in our graver discourse this very word "relation" and its derivatives are the ones that are oftenest heard and read without any lucidity of mind concerning their proper intent as a part of their context. They are used with an assortment of meanings and non-meanings that are quite distracting to try to follow and quite vain to try and reconcile. In particular the difference between *relationship* and *relation*, between the *ground* or *foundation* of the relation and the relation itself, between the plural fact, whether of tendency, interaction, transition, or *status*, that is a co-condition with the relations, and the relations that co-condition that same plural fact, is constantly ignored in thinking and in the expression thereof, to the more or less confusion in, and inconsequence of, the whole discourse delivered.

It is no slight commendation of the perspicacity of M. Mouret to observe that he has discovered that the way towards a resolution of the problems he sets himself to work out lies through what to him appears the altogether unexplored regions of relation-lore, for it is evident that he regards himself as a pioneer in this field.

III. WORKS ON RELATION-LORE IN THE ENGLISH LANGUAGE.

In observing this we cannot but hold M. Mouret unfortunate in not having been put upon better lines of inquiry. He seems to have been wholly unaware of the treasures of investigation in this domain that exist in the English language and that for many years have been available for the student. His case in this respect is seen in the exaltation which he gives to the semi-popular discourses of Mr. Spencer and Mr. Read, as contrasted with the profound researches of DeMorgan and Boole and their disciples. It is evident that he has judged concerning the comparative quality of the various lines of English research not after an examination of his own, but after the current popular renown. For example, he speaks of the work of DeMorgan and Boole as presenting "only a simple mode of representing some of the logical laws" and as being "surrounded with a formidable and complicated apparel which disguises the value of their tentatives."

Since M. Mouret is manifestly an earnest student of the topic of relation-lore this language shows that he has at best only a second or third-hand knowledge of what DeMorgan and Boole really did. He ought to have known that in the recondite field of research in question all really competent treatment of the same would be very far from having any "popular" quality. For a man to discourse of relation-

lore in ignorance of what DeMorgan, the very father of the "Logic of Relatives," accomplished is like discoursing of Darwinism in ignorance of "The Origin of Species."

We opine that when M. Mouret shall have consulted the great memoir of DeMorgan in the tenth volume of the "Cambridge Philosophical Transactions" or better, when he shall have become acquainted with the more developed work of Mr. C. S. Peirce, to whom beyond question relation-lore is most indebted for its present state of progress, he will have a better esteem for the value of the "tentatives" of DeMorgan and Boole and their disciples. Mr. Peirce has published three principal papers on the subject in question. The first of these was published in 1870 in the ninth volume of the "Memoirs of the American Academy of Arts and Sciences." Then in 1880 and in 1884, while Mr. Peirce was lecturer on logic at Johns Hopkins University, he published in the *American Journal of Mathematics* two papers dealing more or less extensively with relation-lore. One of these his "*Hauptwerk*" as it is called by Professor Schroeder of Carlsruhe, appears in the third volume of the journal mentioned and the other in the seventh volume of the same. Mr. Peirce was one of the contributors to our new American "Century Dictionary," and in that work under the definitions of Relation and Relativity there appears a summary treatment of the subject which is as we take it the work of Mr. Peirce and which might have given to M. Mouret hints which he would have appreciated. Also the editor of this magazine in his article "Are there Things in Themselves?" in the January 1892 number thereof incidentally touches upon the topic under consideration in such a way as to correct some of the more inveterate misconceptions.

IV. M. MOURET'S THEORY OF RELATIONS.

The article of M. Mouret is in so many points so excellent a discourse that the chief reflection one is inclined to make is that upon its own principles it ought to have been better. He seems to have been widely awake to the primordial nature of relations as philosophical data. He says: "Every notion or relation is a *function* of relations more directly known and enters as a relational element into other relations less proximate to the common sources," and also that "every concept or notion ought to resolve into a group of relations."

By such tokens as these we naturally look to see M. Mouret making it his very first concern to explain fully the nature of those primordial data that are relations. Indeed he seems himself to be fully aware of this natural expectation for he says: "What then is a relation; what is a concept or notion? To this double question an answer is necessary and a precise answer not consisting in the substitution of one form of words for another form of words bearing the same meaning or no meaning at all."

We cannot however find that he has done this. Instead of it and almost while saying that every concept and notion ought to resolve in a group of relations

he announces, "What I have to examine is the constitution, the structure of these *relation-generating groups*." Thus he starts with a synthesis when what is needed is analysis. He starts with supposing a group of relational elements indeterminate in number and proceeds to inquire as to the conditions that must subsist with regard to them, respectively and in combination, in order that a *definite* relation may subsist as to a pair of the relational elements. These conditions he finds to be four in number. First, he finds that—

"It is necessary that every one of the terms of the group should be connected one to the other by *definite relations*; that between any two terms there must always be intermediate terms that connect them in a continuous way." This he calls the condition of "Solidarity." Secondly, he finds that there must obtain the condition of *Co-Existence*. By co-existence he intends—

"Not a definite co-existence in time, that is to say, a relation of simultaneity or concomitance, nor yet that established co-existence which constitutes the causal relation, but an indefinite co-existence independent of the order of its terms and of all consideration of time or duration."

Having thus supposed his group all well stocked with relations, he proceeds to relegate most of them to the limbo of inconsequence by invoking a *principle* which he calls the principle of *indetermination*. By virtue of this principle in every particular case the *particular determinations* of all the terms become indeterminate and those of the intermediate terms doubly so. Thus the supposed facts of the case become fit for the existence of the *Third* condition, that of *Abstraction*, and for the arising of a general concept or notion.

But corresponding to every concept or notion is its negative or opposite concept or notion. As this negative depends necessarily upon certain *particular determinations* of the same terms that bear the *particular determinations* and which being singly indeterminate admit of the positive concept or notion, there necessarily must obtain two systems of singly indeterminate *particular determinations* relative to but incompatible with one another, and so relative that the negation of one set entails the obtaining of the other set, or in other words either set being negated entails the obtaining of the other set. These facts constitute what M. Mouret calls the *principle of incompatibility* and involve his *Fourth* condition of *Relativity* stated by him as follows:

"All the particular determinations of the extreme terms must not be compatible with the system and the negation of certain relations of the system must entail the negation of the relation which they make between the extreme terms."

V. REFLECTIONS ON M. MOURET'S THEORY.

Now we cannot regard this as a successful attempt to explain the nature and characteristics of relations, or to unfold the involutions of relation-lore.

We fully realise that if every concept or relation resolves into a group of relations we must in some form or other take what are relations in reality as data to be-

gin with, but this does not prevent us from taking our datum terms for our turn of explanation as not requiring at present any recognition other than as relational elements. What is needful as a prime requisite on the very start of any research in relation-lore is to obtain a clear idea of what is meant by a relation. Meanings are primarily matters of mental status. We have to determine the relation that subsists between the mind and the object through the mediating interpretation of a word, and the mental affection lies nearest and logically comes first. It may very well be that the mental affection requires correction, but this cannot take place until its faults are observed, and these cannot become evident until the mental affection is itself duly understood.

The disciples of that school of logic in which De Morgan and Boole, both eminent mathematicians, hold so exalted a rank as discoverers, regard cognition as arising in consequence of brain functioning or *mental operation* and study the results of this operation as yielding their import in dependence upon and only in dependence upon the proper operation in virtue of which they arise.

Now no cognition whatever, even of the most elementary sort, arises except in connection with and in consequence of that operation of the sensibility which is *distinction*. Distinction is of multitudinous and manifold aspects. In all its phases whether it be passive or active it is naught else than the arising or the assigning of relations. The attempt to posit an unqualifiedly absolute—that is, an unqualifiedly unrelated—universe of discourse must be futile and blank, necessarily and insuperably. Any form of notation that pretends to express such a universe of discourse, is only saved, if at all, from being unqualifiedly nonsensical, by standing as antithetical—that is, by being *related* to forms of notation that express relation and nothing else than relation. This rigorously prime operation of distinction is not only pure relation-ing but it is of that sort of relation-ing that is at once a distinguishing and a conjoining. The "One and the Many" are insuperably implicit therein. Distinction having operated to various extents, and thereby various relations having come into view, we become aware of those items of experience that are objects or facts. Each and every one of these objects or facts are in truth distinguished and are therefore in no strict sense *indiscernible* from each other, but since no science can possibly obtain in relation to mere particulars we find it useful to disregard various points of distinction that obtain in respect to various objects and facts and to converge our regard upon the points wherein distinction, not absolutely vanishes, but *tends to vanish*.

By this operation, which is *abstraction*, various objects and facts become in mental regard fit and useful to be taken as copies of one another and as indifferent for use in most of the turns of mental life.

There are indeed various relations, objects, and facts, with respect to which no further operation or operations of distinction than the mere distinctions of the time, the place, or the occasion of their various manifestations have been applied nor can without great difficulty be applied. But we are therefore by no means entitled

to say that such are in truth irresolvable. Contrarily, and reasoning inductively we are justified in concluding that every relation, object, and fact will under analysis of adequate power resolve without limit into other relations, objects, and facts.

We are not yet prepared to see that the ultimate components of relations, objects, and facts resolve into relations, and nothing but relations, because we are not yet prepared with an explicit idea of the nature and characteristics of these elementary objects.

The study of M. Mouret since it starts with relations combining them under the conditions of Solidarity Co-Existence, Abstraction and Relativity, (which are nothing else than other relations or compounds of relations,) does not seem to us to advance us at all in the most fundamental requisite. He says no more than to say that in order for the groups of relations to generate further definite relations the relations thus grouped together must be related to one another and then that most of these relations must be disregarded.

M. Mouret distinguishes, with respect to a relation three factors, the *Matter*, or the relational elements grouped together, the *Form*, or the order in which the relational elements are arrayed, and the *Foundation*.

As his study of the topic of relations is professedly for the purpose of enabling him the better to solve the nature of the relation of mathematical equality, his success may be estimated by reference to his conclusions in regard to that relation. These are as follows: "The relation of equality is formed of undetermined matter, it possesses a binary form, and has for a condition a relation of indiscernibility between the two elements."

Such conclusions appear to us to be impotent not to say erroneous. If two things obtain at all, they obtain as two and not one, in very virtue of being distinguished the one from the other. Except with regard to some more or less arbitrary distinctions, like the distinction between co-incident points, all distinctions obtain only in virtue of some relation that can be nothing else than a point of *discernibility*. Numbers and other mathematical things are taken as not-different not because they are in truth indiscernible but because for the turn in hand their points of difference are irrelevant.

Concerning the much mooted question of the proper field of logic as a science M. Mouret holds it to be the "science of relations and general concepts." Although we hold that logic is particularly concerned with the lore that is more directly related to the phenomena of erroneous thinking and its correction, the view of M. Mouret is not unacceptable. "Reasoning consists in the observation that where certain relations subsist certain others are found," as Mr. Peirce has remarked:

VI. CONCERNING MEANINGS AND EXPLANATIONS.

As preliminary to our account of relations we will make an observation which seems to us of considerable use in connection therewith. It is not without its bear-

ing on the theory of definition or rather upon the broader theory of explanation. With M. Mouret we hold that every concept and relation resolves into a compound of relations. Since relations are data that are absolutely elementary at least so far as we are at present instructed they are of course not subsumable under any other sort of data that are better known. Moreover, whatever explanation we here make must needs be made by means of written words. Thus an important question arises as to what method is to be pursued in this special exigency. The theory of definition leads us to the same difficulty, for although the meanings of many words can be defined in terms that are more proximate to the elementary relations, we will always come at last to terms that admit of no improved explanation by such a method. There is no device of words that can evade or supersede the ultimate recourse to things. Now the significations of words are learned in most cases not so much by definitions and verbal descriptions as by *the observation of the various applications of the words*. Indeed this is the primitive way in which the meanings of words are found out. The child knows nothing of what, say, the word *horse* means until some one shows it an actual horse and may be pointing to it says repeatedly, *horse*, in such a way as to excite the observation of the child to the intended application of the word to the thing. This is because the relation of every general sign to its object subsists only in consequence of a mental association, and until this mental association is created the sign has no meaning. The methods of evoking these mental associations are at present quite unmethodical and do not receive the attention which their importance merits. One feasible method is to present or to state a number of scenes that shall present the object in various ways in connection with the sign thereof, and thus to excite attention to the proper application of the sign. The geometer does this by means of his diagrams without which or their mental counterparts all his mere words would be in vain.

Mr. Edward T. Dixon has lately published a work on the "Foundations of Geometry" in which he would introduce as a fundamental datum what is really an altogether new and exceedingly abstract conception which he calls by an old name, that of *direction*. The old term has never been as yet taken in any abstract universal sense because apart from definite right lines showing it as an attribute any abstract universal meaning is wholly unassignable. But the conception that Mr. Dixon would instal is removed in abstractness from such a universal yet one more step in universality. A three-fold infinity of right lines differing in direction can be drawn in ordinary space to each of which pertain two corresponding universals of direction, one converse to the other. Now the conception of direction that Mr. Dixon proposes for service as an elementary geometrical datum is the universal that subsumes all these lower ranking universals as particulars. Of course he has difficulty in even trying to explain what he means. Realising the impossibility of subsuming it in any way he takes a method which if it were more thoroughly applied, and wholly emancipated from the lingering notion of definition, might have been more successful. As he actually left the matter his real meaning

can only be drawn from close study of the way in which he applies the term in his discourse in general.

Owing to its excessive abstractness his conception is wholly unfit for service in elementary geometry. One has to become a good geometer before the conception can even be approached.

VII. ANOTHER THEORY OF RELATIONS.

We shall proceed to explain what we regard as a true and adequate notion of a relation by stating some scenes that display the same. We do not regard it as needful to state many of them and we take for our first one, the common transaction of making a donation. We have here for relational elements or terms as they are usually called a set of three. Separately, or as not yet brought into relation in virtue of the giving, there may be, say *G* an owner of *W* a watch and *R* the intended beneficiary. The plural fact of the giving is the *relationship* or the *foundation* of the *relations* that arise in virtue of said giving. This *foundation* becomes to be in virtue of the creation of such relations by the giving. Either one of the set of three may be taken as the datum of reference and according to the election in this respect, the relations may differ and the technical names we are about to give will vary in their application. Since simplicity will be gained thereby and also our present turn fully subserved we will take *G* as the datum term of reference. So taking it *G* is called the *relate* and both *W* and *R* are called the *correlates*. For this present turn and in very virtue of the giving and only in virtue thereof *G* becomes related to *W* and *R* in a certain relation one of the names of which is giver. When *W* or *R* are taken as relates certain other relations appear, some of the names of which are respectively *present* and *recipient*. In relation to the relation of giver the relations of present and recipient are named *converse* relations, as are likewise the relations of present and giver to the relation of recipient and the relations of giver and recipient to the relation of present. Here are three distinct relations growing out of the same relationship or foundation. As each relational element has its corresponding negation, the true logical system of a set of three terms involves not less than eight relations.

We take for our second scene the case of a boundary. This might be a surface or a point but we will take the special case of a line on a surface. Here we have again a set of three, the spread on one side, *A*, the opposite spread, *B*, and the line *L*. *A* has a certain relation, say above, to *L* and *B*, *B* has the certain relation, below, to *A* and *L*, which relation is converse to the relation above: and *L* has the certain relation, boundary, to *A* and *B* which relation is converse to the relations above and below.

The two examples now given are cases of the *conjugative* kind. The relationship is a conjugative one and the relations are conjugative relations. The distinguishing characteristic of a conjugative case is the fact essentially involved of the mediation between relational elements by another certain element, or in other

words the bringing of diverse relational elements into relation by the function of another relational element. Without the mediation or function of this conjugating element neither the relationship nor the conjugative relations can exist. There is reason to believe that all conjugative cases can be certified as cases of three relational elements or as compounds of a number of such sets of three. To ordinary uncritical thought which is largely constrained by the trammels of ordinary language the most abundant sort of relations appears to be of that sort that are taken to involve only two relational elements. These are cases of what are called *dual* relationships and the relations that arise out of them are called *dual* relations. Such are those like father, son, husband, wife, etc. Strictly viewed they ought to be regarded as *degenerate* relationships and relations just as a pair of lines is regarded as a degenerate conic.

VIII. CAUTIONS AND APPLICATIONS.

Now besides the error of confounding relations with relationship, it is a very common fault to think and speak of a relation as being *between* two or more terms. This imports into thought the thoroughly misleading idea of an intervening independent existence for relations. Relations are attributive predicates of terms and each one of them pertains strictly to its proper term or combination of terms, in the same sense for this turn (*pro hac vice*) that qualities are held to pertain to their so-called substances. And yet relations so pertain to their proper terms not in virtue of such terms separately but in virtue of their membership in the plural fact which obtains as the *relationship* or *foundation*. The notion of a relation as a "betweenness" has perhaps been fostered by the exact coincidence of relations pertaining to the several members of the same relationship. When on contemplating the connection, say of two points, we observe that the distance of one from the other is apparently indistinguishable from the distance of the other from the one, we naturally overlook the fact that we are truly to regard the connection as the coincidence of two really distinct relations, and regarding the pair of relations as one thing and finding it not attributable to one point more than to the other we dissociate it from both. But when we consider a pair of relations that are converse to one another and that arise out of a dual relationship like that of husband and wife we may see that there is *no* betweenness, no single relation that interlies, but two relations, one the relation of husband and the other the converse relation of wife.

An interaction, say like that of approach under the influence of gravitation, is a relationship. Each body stands in the relation of a *puller* of the other and the mediating term which we find impossible to argue out of the account we call the attraction of gravitation. In this case the relation of action of the one body is not usually distinguished from the relation of action of the other one. Indeed this is the case in all cases of mechanical action and we lay it down as a maxim that action and reaction are equal but they are not alike since their directions are opposite.

Sensation is a relationship, since it is our interaction. The object interacts with the brain. As to the conjugating term we are as yet in the dark and so we are in the habit of regarding this case as a dual relation. The relation of the brain to the object is that of a *knower* and that of the object to the brain that of a *stimulator*. Each character or mark of the object that becomes apparent gives rise to relations and their respective converses each correlative pair of which are respectively so many distinct interactions of detail in the entire interaction. Whatever an object as known to us is, it is in virtue of those relations of brain action and detailed object stimulation, which are relations and always relations. Since consciousness exists only by the arising of relations of distinction, supposably in consequence of internal brain interaction, is it presumptuous to allege that consciousness consists of relations or a complex of relations?

IX. NATURE OF OBJECTS.

With regard to the object no one can prevent whoever may be so disposed, from imputing to it various points of possession that do not and cannot interact with the brain. So far as such imputed points are regarded as merely not yet interacting but possible to interact with knowing substance such points in no wise differ in essential nature from the known attributes. They are potential relations and nothing else. But in so far as they are regarded as essentially impossible of ever interacting with knowing substance in any possible stage of its development such regard is pure nonsense and utterly without any assignable meaning. There is no occasion whatever for such an imputation, for the existence of interaction actual or potential is fully adequate to explain all that will ever present itself to be explained.

At this point let us instruct ourselves with an example of the reasonings of a much and deservedly honored philosopher. He says:

"In the most general predicate which is *determined* Being or *existence*—for all things in the universe are determined beings—we have an evident two-foldness (a composite nature) which allows of a further analysis into pure Being and determination."

We will parallel this analysis. For the sake of simplicity we will take a limited right line. It has the determinations straight and long, not length in the sense of measure, for length is ambiguous in its intent, but length in its qualitative sense—its linearity so to speak. Now separate from it first its straightness without however giving to it any other determination, and then its quality of longness. We have then a *pure* line, that is neither straight, nor long, nor anything else. Such is an example of "Pure Being." We say however that its very being as a line is absolutely dependent on its determination as a long line; that such a determination alone constitutes it a line, is at once its determination and its being, that there is no two-foldness at all but only two names, and that as one-fold its determination as long and its being vanish together. What is true of a line is true of all relations and compounds of relations whatever.

Thus not only all knowledge but all existence so far as that term can ever have any meaning is relative; relative to all intents and for every possible turn.

To those who accept the essentials of this account of relations it will be easy to see what is the nature of an object and that of a concept or notion. An object is a relation or some congeries of relations that usually present themselves as a coherent whole to our sensibility or to consciousness. This is primarily effected in virtue of some efficacy which we cannot appropriate to ourselves, and so we distinguish our own personality from that manifold that we call the objective world. It is pure self-stultification after having made this most useful distinction to try and abolish it. Nothing but an utter abolition of all useful thought can result from so taking the data of experience.

X. NATURE OF A CONCEPT.

But objects are individual and generally found with various points of distinction some of which are irrelevant to most of the turns of mental life. We therefore neglect the irrelevant points and take many objects as copies of one another. This process is *not* the formation of the concept or notion but it suggests and prompts that formation. We cannot but regard it as an error to take a conception as a *sum* of individuals. It seems to us to be rather in the nature of a *locus*. A curve contains an infinity of points and yet the curve is not any sum of points even though it is often allowable to speak of it as the sum of all its points. So any concept, say, man, is not all the men that now live nor yet all the men that eternity both backwards and forwards has contained and will contain. A concept is a manifold and strictly universal and infinite in respect to the particulars it subsumes. We speak of the infinitive mood of a verb because the meaning of the word as thus taken is not put under any modification. In like manner the meaning of any concept though subject to various limitations in its applications is as a concept merely to be regarded as obtaining in a purely infinitive sense. Professor Jevons found a difficulty in classifying what he called *material* terms, such as stone, sand, water, etc. Other logicians have put such terms as singular terms, while still others have classed them as general terms. There is a great variety of such terms. Potatoes, wheat, butter, ice, cattle, water, hydrogen, the names of all the elements, ether, electricity, time, space, love, virtue, etc., are instances. It seems to us that such terms are the normal types of general terms and that the canonical forms of our universal propositions ought to be unquantified not only as regards the predicates but also as regards the subjects. Why not "man is animal" just as "lard is grease" or "man is mortal" just as "butter is cheap"?

Moreover the distinction between a general concept and one that is called singular is only one of degree and not of kind. Every so-called singular term is potentially at least only an individual instance under a possible general or universal concept. A striking example of this potentiality is furnished by the modern generalisations of that formerly singular term, space. If these observations are well

founded, the universals of thought even though arising out of the facts of experience and rigidly beholden to experience for every last element out of which they are constructed, form nevertheless a Formal-Thought-World, and the mind of man in virtue of its powers of imaginative construction and generalisation has a constitution that enables it to subsume the actual objective universe as only one particular of a universality of a higher rank.

Under such a conception of the objective world and the world of thought and their relations the old dispute of realism versus nominalism would take a new aspect. *Universals in re* even though they were admitted to exist would become universals no longer in the higher universe of thought. True universals would only subsist as universals of the world of mind. The laws of Form and Formal Thought would thus become of chief moment in philosophy and no one could be recognised as properly laying claim to the title of philosopher without proficiency therein.

XI. FALLACY OF THE SPENCERIAN AXIOM.

Concerning the "axiom of symmetry" only a few examples of its fallacy are needful. Mutual friendship is certainly a "symmetrical" relation, but *A* and *C* may be mutual friends and *B* and *C* mutual friends also, but it in no wise follows that *A* and *B* are friends. They may be decidedly unfriendly as we often see the case. Take a case of equilibrium the cases of which seem to be favorite ones with M. Mouret. We suppose that planets may be regarded as in a relation of equilibrium with the sun and yet these mere equilibrations with the sun do not make any equilibrium between them. They do not knock together it is true but this is due to their own direct relations and not their relations of equilibrium with the sun.

The distances of points from each other is a "symmetrical" relation and yet point *A* may be from point *C* the very same distance that point *B* is from *C*, but the distance of points *A* and *B* from one another may vary from coincidence to double the distance *A C-B C*.

XII. NATURE OF ARITHMETICAL EQUALITY.

Concerning the relation of "mathematical equality" there is no single relation that obtains throughout mathematics as such. There is numerical equality upon which the equality in service in numeric algebra is founded, and there is geometric equality, the equality of vectors, etc., all different from one another. M. Mouret seems to have only numeric equality in view. He claims this relation to be not only of a very simple nature but that it is the very foundation of the notions of magnitude and quantity. He even declares that mathematics could not exist without this relation. Did he lose sight of the usual proof of Fourier's celebrated theorem?

As we have explained, things that are distinguished are not really alike but only for certain turns taken to be so. This assimilation of things is of various grades. In arithmetic, meaning arithmetic in its most general sense, the only logical comprehension that the various numbers possess is respectively their greater

or less partitionability; m is the same as n means in arithmetic that whatever has the numerical rank of m has also precisely the numerical rank of n no matter what summations or other numerical operations m or n may represent. Identity of this sort is arithmetical equality. It seems a simple relation for the reason that its intervention very decisively simplifies our arithmetical comprehensions. It is however a coincidence of two relations that are converse to one another. These relations are "not less than" and "not greater than." It is universally admitted that the more inclusive a notion or concept is in extension, the more simple and primary it is than any other notion or concept included as an instance under it. Now all equality is "not less than" but not all "not less than" is necessarily equality; hence, "not less than" is a wider and more primary notion than equality. On the same considerations "not more than" is in the same case. Equality is the limiting case between the variable and logically more simple cases of "not less than" and "not more than." The notion of quantity emerges on comparison however vague between any two objects that have size, independently of the notion of equality. If this were not true how could we have the notions of infinitely large and infinitely small.

It is indeed true that without the notion of equality the theory of numbers and the mathematical analysis could subsist in a rudimentary state only, but to say that they would not exist at all is rash and not maintainable. The relations "not less than" "not more than" would still allow of some truly mathematical propositions, operations, and calculations. In that essentially qualitative notation that is ordinary language the relation that corresponds to equality is of very limited range but a relation that is analogous to "not less than," viz., supersumption, is very efficient.

With a theory of numbers and a mathematical analysis using only the relations "not less than" "not more than" in lieu of the relation of equality the fundamental operations, addition and substitution, would find some scope of application and hence the derivative operations, multiplication, powering, etc., and their inversions, subtraction, division, etc., would obtain in some fashion and to some extent. This can readily be seen by any one who is familiar with the way in which expressions of inequality are used in modern mathematical analysis.

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OBSERVATIONS ON SOME POINTS IN JAMES'S PSYCHOLOGY.

II. EMOTION.

Nothing in Professor James's work will be likely to strike the average reader as more paradoxical than his views on the subject of Emotion, which he must be allowed to state in his own words. After premising that he will limit his discussion, in the first instance, to what may be called the coarser emotions, as fear, grief, rage, love, in which every one recognises a strong organic reverberation, he goes on to say :