225. The Line at Infinity, Etc.
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Source: *The Mathematical Gazette*, Vol. 4, No. 61 (Jan., 1907), pp. 14-15
Published by: Mathematical Association
Stable URL: http://www.jstor.org/stable/3604267

is 'really difficult'? Apart from details his proof is that which I was taught years ago—due, I believe, to Tannery (if not much older). And does Mr. Picken mean to imply that his proof is not 'really difficult.'?

(2) Mr Picken seems to me to follow Prof. Chrystal in a certain vagueness as to the distinction between a value for $x=1$ and a limit for $x=1$. The function $\frac{x^2-1}{x-1}$ has *no* value for $x=1$; for $x=1$, $\frac{x^2-1}{x-1}$ is strictly and absolutely meaningless. The fact that its limit for $x=1$ is 2 is entirely irrelevant. The functions $\frac{x^2-1}{x-1}$ and $x+1$ are different functions. They are equal when x is *not* equal to 1. Similarly the function $y=\frac{x}{x}$ is $=1$ when $x\neq 0$ and undefined for $x=0$. To calculate $f(x)$ for $x=0$ we must put $x=0$ in the expression of $f(x)$ and perform the arithmetical operations which the form of the function prescribes, and this we cannot do in this case.

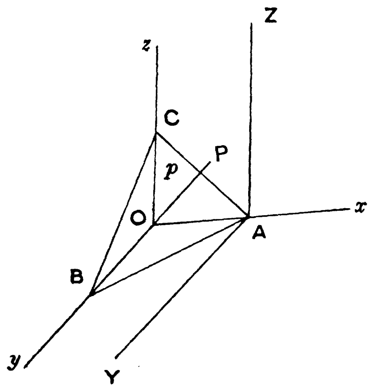
Whether Mr. Picken agrees with me here I cannot say. I lay stress on the point because his language is not quite clear. Thus he says (p. 330) that 'a function of x may have a value for a given value a of the argument, although the expression $f(x)$ fails to provide a value when a is substituted for it'—and I might quote other sentences which I cannot regard as entirely satisfactory. G. H. HARDY.

224. [M[1]. 8. g.] *A curious imaginary curve.*

The curve
$$(x+iy)^2=\lambda(x-iy)$$
is (i) a parabola, (ii) a rectangular hyperbola, and (iii) an equiangular spiral. The first two statements are evidently true. The polar equation is
$$r=\lambda e^{-3i\theta},$$
the equation of an equiangular spiral. The intrinsic equation is easily found to be $\rho=3is$.

It is instructive (i) to show that the equation of any curve which is both a parabola and a rectangular hyperbola can be put in the form given above, or in the form
$$(x+iy)^2=x \text{ (or } y),$$
and (ii) to determine the intrinsic equation directly from one of the latter forms of the Cartesian equation. G. H. HARDY.

225. [L[1]. 1. a.] *The line at infinity, etc.*



Can anyone tell me of an English book which contains a clear and intelligible account of the 'line at infinity'? Such accounts as are contained in the ordinary books on Conics, or in Miss Scott's *Modern Analytical Geometry*, appear to me confusing in the highest degree.

Most undergraduates seem to believe that there really are points at infinity, and that they really do lie on a line, and that if you could get there you would find that $1=0$. The fault lies in the books, which persist in treating conventions as if they were sober statements of fact.

I have found the following construction useful (see figure). Project p in the plane $x+y+z=1$ into P in the plane $x=1$. Taking axes AY, AZ, as shown, in the latter plane we find

$$Y=\frac{y}{x},\ Z=\frac{z}{x},$$

x, y, z being the Cartesian coordinates of p—or (what is the same thing) the areal coordinates of P in the plane $x+y+z=1$, referred to the triangle ABC—and Y, Z the Cartesian coordinates of P in the plane AYZ.

Thus if the locus of P is $f(1,\ Y,\ Z)=0$, that of p is $f(x, y, z)=0$.

G. H. HARDY.

226. [L[1]. 2. b.] *Morley's Problem.*

Let ab, bc, cd, de, ea, ac, bd, ce, da, eb be denoted by A, B, C, D, E, S, T, P, Q, R.

Let U, V be the conics $ABCDE$, $PQRST$, and X, Y those that touch $abcde$ and PQ, QR, RS, ST, TP respectively. Then the collineation transforms A, B, C, D, E, U, X into P, Q, R, S, T, V, Y respectively.

Now PCD, QDE, REA, SAB, TBC are triangles described about X, and each has two vertices on U. Hence in each case the third vertex lies on a fixed conic having a common self-polar triangle with X, U. This conic must be V, since it goes through P, Q, R, S, T.

Also PQR, QRS, RST, STP, TPQ are triangles inscribed in V, and two sides of each touch Y. Hence the third sides touch a fixed conic having a common self-polar triangle with V, Y. This conic must be X, since it touches a, b, c, d, e. Hence U, V, X, Y have a common self-polar triangle, which is therefore unchanged by the collineation.

This self-polar triangle must therefore be the fixed triangle of the collineation unless the collineation has the period two or three, that is, can be reduced by projection to a rotation through 180° or 120°. It seems evident that this does not generally happen, and in fact it never happens. It is not hard to work out the actual transformation when ABC is the triangle of reference, and the multipliers λ, μ, ν are then found to satisfy the condition $\Sigma\lambda\,.\,\Sigma 1/\lambda+1=0$. In the two cases mentioned we have respectively

$$\lambda^2=\mu^2=\nu^2 \quad\text{and}\quad \lambda^3=\mu^3=\nu^3,$$

which are inconsistent with this condition.

Since U, V may be written

$$\lambda^2x^2+\mu^2y^2+\nu^2z^2 \quad\text{and}\quad x^2+y^2+z^2,$$

it is easy to find the relation connecting their invariants, namely,

$$125\Delta^2\Delta'^2+10\Delta\Delta'\Theta\Theta'-4\Delta\Theta'^3-4\Delta'\Theta^3+\Theta^2\Theta'^2=0.$$

A. C. DIXON.

227. [I.] In a review (*Gazette*, March, 1906, vol. iii., pp. 302-304), I have stated that the name of the publisher of a certain pamphlet was not given. I now learn from a correspondent that the title page (missing in my copy) was:—"The power of the Continuum. Inaugural Dissertation zur Erlangung der Doktorwürde genehmigt von der hohen philosophischen Fakultät der Landesuniversität Rostock. Von Harold A. P. Pittard-Bullock aus Guernsey, Dezember, 1905. Referent: Professor Dr. Otto Staude, Druck von E. Ebering, Berlin, N.W." In view of the nature of the pamphlet, this must cause some surprise.

PHILIP E. B. JOURDAIN.

228. [X. 6.] *A mechanical construction of curves representing the movement of a pendulum.*

Consider a right circular cylinder of radius r, and a point O on its axis. Let a sphere of radius b, and centre at any point C in the circular section through O, intersect the cylinder in a curve.