XLVI. On the quaternion expressions of coplanarity and homoconicism

William Spottiswoode M.A.

companied the oxalic acid, I could not determine from want of material. The presence of oxalic acid however is distinctly indicated by the above reactions. They likewise show the presence of chloride of sodium, potash, sulphuric acid and magnesia.

In comparing this secretion of the leaves of the Ice-plant with the fluid in the ascidia of *Nepenthes*, we find a material difference in their respective compositions, as will be seen by the annexed table, which exhibits the composition of both fluids:—

Composition of the fluid in the ascidia of Nepenthes.	*Composition of the watery secretion of the leaves of* Mesembryanthemum crystallinum.
Organic matter, chiefly malic and a little citric acid.	Organic matter (albumen, oxalic acid, &c.).
Chloride of potassium.	Chloride of sodium.
Soda.	Potash.
Lime.	Magnesia.
Magnesia.	Sulphuric acid.

XLVI. *On the Quaternion Expressions of Coplanarity and Homoconicism.* By WILLIAM SPOTTISWOODE, *M.A., of Balliol College, Oxford**.

THE following investigations relate to certain theorems given by Sir W. R. Hamilton in vol. xxix. of this Journal. Adopting the notation of the original papers, the equation

$$S.\alpha\alpha_1\alpha_2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

(where suffixes are used instead of accents) is equivalent to

$$\begin{vmatrix} x, & x_1, & x \\ y, & y_1, & y_2 \\ z, & z_1, & z_2 \end{vmatrix} =0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

which may be replaced by

$$a\alpha+a_1\alpha_1+a_2\alpha_2=0; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

because, there being no linear relation between i, j, k, this last is equivalent to the system

$$\left.\begin{array}{l} ax+a_1x_1+a_2x_2=0 \\ ay+a_1y_1+a_2y_2=0 \\ az+a_1z_1+a_2z_2=0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

and (2.) or (1.) is the result of these. Hence (1.) and (3.) are alike conditions of coplanarity.

Again, if

$$\left.\begin{array}{l} \beta\ =V.V.\alpha\alpha_1.V.\alpha_3\alpha_4 \\ \beta_1=V.V.\alpha_1\alpha_2.V.\alpha_4\alpha_5 \\ \beta_2=V.V.\alpha_2\alpha_3.V.\alpha_5\alpha \end{array}\right\}, \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

* Communicated by the Author.

then

$$\mathrm{V}.\alpha\alpha_1 = \begin{vmatrix} i, & x, & x_1 \\ j, & y, & y_1 \\ k, & z, & z_1 \end{vmatrix}, \quad \ldots \ldots \quad (6.)$$

with similar expressions the other vectors; and consequently

$$\left.\mathrm{V}.\mathrm{V}.\alpha\alpha_1.\mathrm{V}.\alpha_3\alpha_4 = (ix_1+jy_1+kz_1)\begin{vmatrix} x, & x_3, & x_4 \\ y, & y_3, & y_4 \\ z, & z_3, & z_4 \end{vmatrix} - (ix+jy+kz)\begin{vmatrix} x_1, & x_3, & x_4 \\ y_1, & y_3, & y_4 \\ z_1, & z_3, & z_4 \end{vmatrix}\right\} \quad (7.)$$

so that

$$\left.\begin{aligned} \beta &= \alpha_1.\mathrm{S}.\alpha\,\alpha_3\alpha_4 - \alpha.\,\mathrm{S}.\alpha_1\alpha_3\alpha_4 \\ \beta_1 &= \alpha_2.\mathrm{S}.\alpha_1\alpha_4\alpha_5 - \alpha_1.\mathrm{S}.\alpha_2\alpha_4\alpha_5 \\ \beta_2 &= \alpha_3.\mathrm{S}.\alpha_2\alpha_5\alpha - \alpha_2.\mathrm{S}.\alpha_3\alpha_5\alpha \end{aligned}\right\} \quad \ldots \quad (8.)$$

Whence, taking the product, and omitting those terms which have no scalar parts,

$$\left.\begin{aligned} \mathrm{S}\beta\beta_1\beta_2 = \mathrm{S}.\alpha_1\alpha_2\alpha_3.\mathrm{S}.\alpha\,\alpha_3\alpha_4.\mathrm{S}.\alpha_1\alpha_4\alpha_5.\mathrm{S}.\alpha_2\alpha_5\alpha \\ -\mathrm{S}.\alpha\,\alpha_1\alpha_2.\mathrm{S}.\alpha_1\alpha_3\alpha_4.\mathrm{S}.\alpha_2\alpha_4\alpha_5.\mathrm{S}.\alpha_3\alpha_5\alpha \\ +\mathrm{S}.\alpha\,\alpha_1\alpha_3.\mathrm{S}.\alpha_1\alpha_3\alpha_4.\mathrm{S}.\alpha_2\alpha_4\alpha_5.\mathrm{S}.\alpha_2\alpha_5\alpha \\ -\mathrm{S}.\alpha\,\alpha_2\alpha_3.\mathrm{S}.\alpha_1\alpha_3\alpha_4.\mathrm{S}.\alpha_1\alpha_4\alpha_5.\mathrm{S}.\alpha_2\alpha_5\alpha \end{aligned}\right\}, \quad . \quad (9.)$$

which vanishes identically whenever α coincides with any of the vectors $\alpha_1 \ldots \alpha_5$; so that these last five vectors lie on the cone represented by

$$\mathrm{S}.\beta\beta_1\beta_2 = 0,$$

when x, y, z alone are considered as variable. The only case, which is not at once obvious, is $\alpha = \alpha_4$; suppressing the common factors, (9.) then becomes

$$\left.\begin{aligned} &\mathrm{S}.\alpha\alpha_2\alpha_3.\mathrm{S}.\alpha_1\alpha\alpha_5 + \mathrm{S}.\alpha\alpha_3\alpha_1.\mathrm{S}.\alpha_2\alpha\alpha_5 + \mathrm{S}.\alpha\alpha_1\alpha_2.\mathrm{S}.\alpha_3\alpha\alpha_5 \\ &= \begin{vmatrix} x, & x_2, & x_3 \\ y, & y_2, & y_3 \\ z, & z_2, & z_3 \end{vmatrix} \begin{vmatrix} x_1, & x, & x_5 \\ y_1, & y, & y_5 \\ z_1, & z, & z_5 \end{vmatrix} + \begin{vmatrix} x, & x_3, & x_1 \\ y, & y_3, & y_1 \\ z, & z_3, & z_1 \end{vmatrix} \begin{vmatrix} x_2, & x, & x_5 \\ y_2, & y, & y_5 \\ z_2, & z, & z_5 \end{vmatrix} + \begin{vmatrix} x, & x_1, & x_2 \\ y, & y_1, & y_2 \\ z, & z_1, & z_2 \end{vmatrix} \begin{vmatrix} x_3, & x, & x_5 \\ y_3, & y, & y_5 \\ z_3, & z, & z_5 \end{vmatrix} \end{aligned}\right\} (10.)$$

(or writing

$$\lambda = yz_5 - y_5z, \quad \mu = zx_5 - z_5x, \quad \nu = xy_5 - x_5y), \quad . \quad (11.)$$

$$\left.\begin{aligned} &\begin{vmatrix} x, & x_1, & x_2, & x_3, \\ y, & y_1, & y_2, & y_3, \\ z, & z_1, & z_2, & z_3, \\ . & (\lambda x_1+\mu y_1+\nu z_1), & -(\lambda x_2+\mu y_2+\nu z_2), & (\lambda x_3+\mu y_3+\nu z_3) \end{vmatrix} \\ &\quad = (\lambda x+\mu y+\nu z)\begin{vmatrix} x_1, & x_2, & x_3 \\ y_1, & y_2, & y_3 \\ z_1, & z_2, & z_3 \end{vmatrix} = 0 \end{aligned}\right\} (12.)$$

since $$\lambda x + \mu y + \nu z = 0$$
identically.

The expression (9.) when written thus,

$$\left.\begin{aligned} &\mathrm{S}.\alpha_1\alpha_4\alpha_5.\mathrm{S}.\alpha_2\alpha_5\alpha(\mathrm{S}.\alpha_1\alpha_2\alpha_3.\mathrm{S}.\alpha\alpha_3\alpha_4-\mathrm{S}.\alpha\alpha_2\alpha_3.\mathrm{S}.\alpha_1\alpha_3\alpha_4) \\ -&\mathrm{S}.\alpha_1\alpha_3\alpha_4.\mathrm{S}.\alpha_2\alpha_4\alpha_5(\mathrm{S}.\alpha\alpha_1\alpha_2.\mathrm{S}.\alpha\alpha_3\alpha_5-\mathrm{S}.\alpha\alpha_1\alpha_3.\mathrm{S}.\alpha_2\alpha_5\alpha) \end{aligned}\right\} \quad (13.)$$

may without difficulty be transformed into

$$\left.\begin{aligned} &\mathrm{S}.\alpha_1\alpha_4\alpha_5.\mathrm{S}.\alpha_2\alpha_5\alpha.\mathrm{S}.\alpha_2\alpha_4\alpha_3.\mathrm{S}.\alpha_3\alpha_1\alpha \\ +&\mathrm{S}.\alpha_1\alpha_3\alpha_4.\mathrm{S}.\alpha_2\alpha_4\alpha_5.\mathrm{S}.\alpha\alpha_2\alpha_3.\mathrm{S}.\alpha_1\alpha\alpha_5 \end{aligned}\right\}. \quad . \quad . \quad (14.)$$

which, when equated to zero, gives the relation

$$\frac{\mathrm{S}.\alpha_1\alpha_4\alpha_5}{\mathrm{S}.\alpha_2\alpha_4\alpha_5}.\frac{\mathrm{S}.\alpha\alpha_2\alpha_5}{\mathrm{S}.\alpha\alpha_1\alpha_5}=\frac{\mathrm{S}.\alpha_1\alpha_4\alpha_3.\mathrm{S}.\alpha\alpha_2\alpha_3}{\mathrm{S}.\alpha_2\alpha_4\alpha_3.\mathrm{S}.\alpha\alpha_1\alpha_3}. \quad . \quad . \quad (15.)$$

The equation (9.) expresses the property of the *Mystic Hexagram* of Pascal, and (15.) that of the *Anharmonic Ratio* of Chasles, as was explained in vol. xxix. pp. 118, 327 of this Journal.

XLVII. *On the Meteor of November* 5, 1849. *By* James Glaisher, *Esq.*, *F.R.S.*, *F.R.A.S.*, *and of the Royal Observatory*, *Greenwich.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

IN the number of the Philosophical Magazine for February 1850 is a notice of a fine meteor seen by V. Fasel, Esq., F.R.A.S., at Stone, on Nov. 5, 1849. This meteor was also seen by R. L. Jones, Esq., F.R.A.S., and who wrote to me from Chester on Nov. 6, describing it. As it is seldom that a meteor can be so certainly identified as seen at two different places, I beg to send you the following particulars, in the hope that some other gentleman had the good fortune to see it and to note its path. It is possible that the meteor seen by Mr. Lowe at Nottingham on Nov. 5, at $6^h\ 20^m$, also mentioned in the same Number of the Magazine, may be the same meteor.

The following is Mr. Jones's account of the meteor:—

"I first saw it near the Pleiades, and by estimation (as I could not see the time till I got home) at $6^h\ 10^m$ P.M. G.M.T.; it passed close by α Arietis, 5° or 6° below α Andromedæ and β Pegasi, and disappeared about 10° above the four stars in the head of the Dolphin, occupying about 5° in its transit; it had a head composed of seven or eight small bluish-coloured balls, and left a vivid trace of sparks behind it. That these sparks were not the impression on the retina I am sure, as I closed my eyes, looked on the ground, and on raising my eyes again still saw them. They remained in view at least two minutes, and seemed to be attracted together in three or four masses, and the brightest part was near the meridian."