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XLVII. The luminiferous Æther

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XLVII. *The Luminiferous Æther.* By DE VOLSON WOOD, C.E., M.A., Professor of Mechanical Engineering in Stevens Institute of Technology, Hoboken, N.J.*

TWO properties of the luminiferous æther appear to be known and measurable with a high degree of accuracy. One is its ability to transmit light at the rate of 186,300 miles per second†, and the other its ability to transmit from the sun to the earth a definite amount of heat-energy.

In regard to the latter, Herschel found, from a series of experiments, that the direct heat of the sun, received on a body at the earth capable of absorbing and retaining it, is competent to melt an inch in thickness of ice every two hours and thirteen minutes‡. This is equivalent to nearly 71 foot-pounds of energy per second.

In 1838 M. Pouillet found that the heat-energy transmitted from the sun to the earth would, if none were absorbed by our atmosphere, raise 1.76 gramme of water 1° C. in one minute on each square centimetre of the earth normally exposed to the rays of the sun§.

This is equivalent to 83.5 foot-pounds of energy per second,

* Communicated by the Author.

† Professor Michelson found the velocity of light to be 299,740 metres per second in air, and 299,828 metres in a vacuum, giving an index of refraction of 1,000,265. *Journ. of Arts and Science*, 1879, vol. xviii. p. 390.

‡ 'Familiar Lectures on Scientific Subjects,' by Sir John Herschel, p. 65.

§ *Comptes Rendus*, 1838, tom. vii. pp. 24-26.

and is the value used by Sir William Thomson in determining the probable density of the æther*. Later determinations of the value of the solar constant by MM. Soret, Crova, and Violle have made it as high as 2·2 to 2·5 calories. But the most recent, as well as the most reliable, determination is by Professor S. P. Langley, who brought to his service the most refined apparatus yet used for this purpose, and secured his data under favourable conditions; from which the value is found to be $2\cdot8 \pm$ calories† with some uncertainty still remaining in regard to the first figure of the decimal. We will consider it as exactly 2·8 in this analysis, according to which, there being 7000 grains in a pound and 15·432 grains in a gramme, we have for the equivalent energy

$$\frac{2\cdot8 \times 15\cdot432}{7000} \times \frac{9}{5} \times \frac{772 \times 144}{0\cdot155 \times 60} = 133 \text{ foot-pounds}$$

per second for each square foot of surface normally exposed to the sun's rays, which value we will use. Beyond these facts, no progress can be made without an assumption. Computations have been made of the density, and also of the elasticity, of the æther founded on the most arbitrary, and in some cases the most extravagant, hypotheses. Thus, Herschel estimated the stress (elasticity) to exceed

$$17 \times 10^9 = (17,000,000,000) \text{ pounds per square inch} \ddagger;$$

and this high authority has doubtless caused it to be widely accepted as approximately correct. But his analysis was founded upon the *assumption* that the density of the æther was the same as that of air at sea-level, which is not only arbitrary, but so contrary to what we should expect from its non-resisting qualities, as to leave his conclusion of no value. That author also erred in assuming that the tensions of gases were as the wave-velocities in each, instead of the mean square of the velocity of the molecules of a self-agitated gas; but this is unimportant, as it happens to be a matter of quality rather than of quantity. Herschel adds, "Considered according to any hypothesis, it is impossible to escape the conclusion that the æther is under great stress." We hope to show that this conclusion is not warranted; that a great stress necessitates a great density; but that both may be exceedingly small. A great density of the æther not only presents great physical difficulties, but, as we hope to show,

* Trans. Roy. Soc. Edinburgh, vol. xxi. part 1.

† Am. Journ. of Arts and Science, March 1883, p. 195. Also *Comptes Rendus*.

‡ 'Familiar Lectures,' p. 282.

is inconsistent with the uniform elasticity and density of the æther which it is believed to possess; and every consideration would lead one to accept the lowest density consistent with those qualities which would enable it to perform functions producing known results.

In a work on the 'Physics of Æther,' by S. Tolver Preston, it is estimated that the probable inferior limit of the tension of the æther is 500 tons per square inch, a very small value compared with that of Herschel's. But the hypothesis upon which this author founded his analysis was—The tension of the æther exceeds the force necessary to separate the atoms of oxygen and hydrogen in a molecule of water; as if the atoms were forced together by the pressure of the æther, as two Magdeburg hemispheres are forced together by the external air when there is a vacuum between them. This assumption is also gratuitous, and is rejected for want of a rational foundation.

Young remarks:—"The luminiferous æther pervading all space is not only highly elastic, but absolutely solid."* We are not certain in what sense this author considered it as solid; but if it be in the sense that the particles retain their relative positions, and do not perform excursions as they do in liquids, it is a mere hypothesis, which may or may not have a real existence. If it be in the sense that the particles suffer less resistance to a transverse than to a longitudinal movement, there are some grounds for the statement, as shown in circularly-polarized light. Bars of solids are more easily twisted than elongated, and generally the shearing-resistance is less than for a direct stress. It certainly cannot be claimed that the compressibility of the æther (in case we could capture a quantity of it) is less than that of solids.

Sir William Thomson, in making a probable estimate of the density of the æther, made a more plausible hypothesis, by assuming that "the maximum displacement of the molecules of the æther in the transmission of heat-energy was $\frac{1}{50}$ of a wave-length of light, the average of which may be taken as $\frac{1}{50,000}$ of an inch." Hence the displacement was assumed to be $\frac{1}{2,500,000}$ of an inch; by means of which he found the weight of a cubic foot to be $\frac{2}{3} \times 10^{-20}$ of a pound†. We also notice that one Belli estimated the density of the æther to be $\frac{1}{2} \times 10^{-13}$ of a pound‡; but M. Herwitz, assuming this value to be too small and Thomson's as too large, arbitrarily assumed it as

* Young's Works, vol. i. p. 415.

† Phil. Mag. 1855 [4] ix. p. 39.

‡ Cf. *Fortschritte der Physik*, 1859.

10^{-18} of a pound per cubic foot; but arbitrary values are of small account unless checked by actual results.

We propose to treat the æther as if it conformed to the Kinetic Theory of Gases, and determine its several properties on the conditions that it shall transmit a wave with the velocity of 186,300 miles per second, and also transmit 133 foot-pounds of energy per second per square foot. This is equivalent to considering it as gaseous in its nature, and at once compels us to consider it as molecular; and, indeed, it is difficult to conceive of a medium transmitting light and energy without being molecular. The Electromagnetic Theory of Light suggested by Maxwell, as well as the views of Newton, Thomson, Herschel, Preston, and others, are all in keeping with the molecular hypothesis. If the properties which we find by this analysis are not those of the æther, we shall at least have determined the properties of a substance which might be substituted for the æther, and secure the two results already named. It may be asked, Can the Kinetic theory, which is applicable to gases in which waves are propagated by a to-and-fro motion of the particles, be applicable to a medium in which the particles have a transverse movement, whether rectilinear, circular, elliptical, or irregular? In favour of such an application, it may be stated that the general formulæ of analysis by which wave-motion in general, and refraction, reflection, and polarization in particular, are discussed, are fundamentally the same; and in the establishment of the equations the only hypothesis in regard to the path of a particle is—It will move along the path of least resistance. The expression $V^2 \propto e + \delta$ is generally true for all elastic media, regardless of the path of the individual molecules. Indeed, granting the molecular constitution of the æther, is it not probable that the Kinetic theory applies more rigidly to the æther than to the most perfect of the known gases?*

The 133 foot-pounds of energy per second is the solar-heat energy in a prism whose base is 1 square foot and altitude 186,300 miles, the distance passed over by a ray in one second; hence the energy in 1 cubic foot will be

$$\frac{133}{186,300 \times 5280} = \frac{4}{3 \times 10^7} \text{ foot-pounds.} \quad . \quad . \quad (1)$$

Where results are given in tenth-units of high order, as in the last expression, it seems an unnecessary refinement to

* See also remarks by G. J. Stoney, *Phil. Mag.* 1868 [4] xxxvi. pp. 132, 133.

retain more than two or three figures to the left hand of the *tens*; and we will write such expressions as if they were the exact results of the computations.

If V be the velocity of a wave in an elastic medium whose coefficient of elasticity, or, in other words, its tension, is e and density δ , both for the same unit, we have the well-known relation

$$V = \sqrt{\frac{de}{d\delta}}.$$

And for gases we have

$$e = \delta \gamma,$$

where $\gamma = 1.4$; and the differential of the latter substituted in the former gives

$$V = \sqrt{\frac{\gamma e}{\delta}}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The tension of a gas varies directly as the kinetic energy of its molecules per unit of volume. If v^2 be the mean square of the molecules of a self-agitated gas, we have

$$e \propto \delta v^2, \text{ or } v^2 = x \frac{e}{\delta}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where x is a factor to be determined. Equations (2) and (3) give

$$v^2 = \frac{x}{\gamma} V^2. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Assuming, with Clausius, that the heat-energy of a molecule due to the action of its constituent atoms, whether of rotation or otherwise, is a multiple of its energy of translation, we have for the energy in a unit of volume producing heat,

$$\frac{1}{2} y \delta v^2,$$

where y is a factor to be determined. If c be the specific heat of a gas, w its weight per cubic foot at the place where $g = 32.2$, J Joule's mechanical equivalent, τ its absolute temperature; then the essential energy of a cubic foot of the medium will be $c w \tau J$; and observing that $w = g \delta$, we have

$$\frac{1}{2} y \delta v^2 = c g \delta \tau J, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which, reduced by (4), gives

$$xy = \frac{2 c g \gamma \tau J}{V^2}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

the second member of which is constant for a given gas. To

find its value, we have

	Hydrogen.	Air.	Oxygen.
Specific heat*	3.4093	0.2375	0.2175
Velocity of sound, feet per } second at $\tau = 493^{\circ}.2$. }	4163	1090	1040

and $g = 32.2$, $\gamma = 1.4$, $J = 772$. These, substituted in the second member of (6), give

xy for hydrogen	6.599
„ air	6.706
„ oxygen	6.596
	<hr/>
	3)19.901
Mean	6.63

This value, which is nearly constant for the more perfect gases, we propose to call *the modulus of the gas*, and represent it by μ ; and for the purposes of this paper we will use

$$\mu = 6.6.$$

This relation of the product xy being a constant, has, so far as we are informed, been overlooked by physicists, and is worthy of special notice, since it determines the value of one of the factors when the other has been found. Krönig, Clausius†, and Maxwell give for x the constant number 3, but variable values for y ‡.

We are confident that the value of x is not strictly constant; or if it is, it exceeds 3, since the effect of the viscosity of a gas would necessitate a larger velocity to produce a given tension than if it were perfectly free from internal friction. For our purpose it will be unnecessary to find the separate values of x and y ; but if we have occasion to use the former in making general illustrations, we will call it 3, as others have done heretofore. If the correct value of x exceeds 3, it will follow that the velocity of the molecules exceeds the values heretofore computed§. According to Thomson, Stokes showed that in the case of circularly polarized light the energy was

* Stewart on 'Heat,' p. 229.

† Phil. Mag. 1857 [4] xiv. p. 123.

‡ 'Theory of Heat,' pp. 314 and 317. Maxwell states that the value for y is probably equal to 1.634 for air and several of the perfect gases. This would make $x = 4$ nearly.

§ Maxwell gives for the mean square of the velocities (or, in other words, the velocity whose square is the mean of the squares of the actual velocities) of the molecules, in feet per second at $493^{\circ}.2$ F. above absolute zero, hydrogen 6232, oxygen 1572, carbonic oxide 1672, carbonic acid 1570. Phil. Mag. 1873, p. 68. Our equation (4) gives for air 1593.

half potential and half kinetic*; in which case $y=2$, and therefore $x=3\cdot3$.

The energy in a cubic foot of the æther at the earth being given by (1) and (5), we have, by the aid of (4),

$$\frac{1}{2}y\delta v^2 = \frac{1}{2}\mu \frac{\delta}{\gamma} V^2 = \frac{4}{3 \times 10^7}; \quad \dots \dots (8)$$

$$\therefore \delta = \frac{4 \times 1\cdot4 \times 2}{3 \times 10^7 \times 6\cdot6 \times (186,300 \times 5280)^2} = \frac{2}{35 \times 10^{24}} \text{ lb.,} \quad (9)$$

which is the mass of a cubic foot of the æther at the earth, and which would weigh at the place where $g=32\cdot2$ about

$$w = \frac{2}{10^{24}} \text{ of a pound,} \quad \dots \dots (10)$$

compared with which Thomson's value is less than 4000 times this value. Thomson remarked that the density could hardly be 100,000 times as small—a limit so generous as to include far within it the value given in (9). According to equation (10), a quantity of the æther whose volume equals that of the earth would weigh about $\frac{1}{20}$ of a pound. If a particle describes the circumference of a circle in the same time that a ray passes over a wave-length λ , the radius of the circle will be, using equation (4),

$$r = \frac{vt}{2\pi} = \sqrt{\frac{x}{\gamma}} \cdot V \cdot \frac{\lambda}{2\pi V} = \frac{10}{43} \lambda,$$

or the displacement from its normal position will be about $\frac{10}{43}$ of a wave-length, or about $\frac{1}{215,000}$ of an inch at the earth.

Eliminating V between (2) and (8) gives

$$e = \frac{8}{3\mu \times 10^7} = \frac{4}{10^8} \quad \dots \dots (11)$$

for the tension of the æther per square foot at the earth, and is equivalent to about 1·1 pound on a square mile. The tension of the atmosphere at sea-level is more than 30,000,000,000 times this value. It somewhat exceeds the tension of the most perfect vacuum yet produced by artificial means, so far as we are informed. Crookes produced a vacuum of '02 millionth of an atmosphere† without reaching

* Phil. Mag. 1855 [4] ix. p. 37.

† "On the Viscosity of Gases at High Exhaustions," by William Crookes, F.R.S., Phil. Trans. Roy. Soc. part ii. (1881) p. 400: "Going up to an exhaustion of '02 millionth of an atmosphere, the highest point to which I have carried the measurements, although by no means the highest exhaustion of which the pump is capable."

the limit of the capacity of the pumps ; and Professor Rood produced one of $\frac{1}{390,000,000}$ of an atmosphere* without passing the limit of action of his apparatus. The latter gives a pressure per square foot of $\frac{14.7 \times 144}{390,000,000} = \frac{1}{180,000}$ of a pound.

This, in round numbers, is 140 times the value given in equation (11). Even at this great rarity of the atmosphere, the quantity of matter in a cubic foot of the air would be some 200 million million times the quantity in a cubic foot of the æther—such is the exceeding levity of the æther.

Admitting that the æther is subject to attraction according to the Newtonian law, and of compression according to the law of Mariotte, we propose to find *the relation between the density of the æther at the surface of an attracting sphere and that at any other point in space*, providing that the sphere be cold and the only attracting body, and the gas considered the only one involved.

Let δ_0, e_0, w_0 be respectively the density, elasticity, and weight of a unit of the medium, whether æther, air, or any other gas, at the surface of the sphere; δ, e, w , the corresponding quantities at a distance z from the surface of the sphere; r the radius of the sphere, g_0 the acceleration due to gravity at its surface, and g that at distance $r+z$ from the centre of the sphere. Then

$$\frac{\delta}{\delta_0} = \frac{e}{e_0} = \frac{w}{g} \div \frac{w_0}{g_0}$$

and

$$g = g_0 \frac{r^2}{(r+z)^2};$$

$$\therefore e = \frac{e_0}{w_0} \cdot \frac{g_0}{g} w = \frac{e_0}{w_0} \frac{(r+z)^2}{r^2} w. \quad . \quad . \quad . \quad (12)$$

But

$$de = -w dz = -g \delta dz; \quad . \quad . \quad . \quad (13)$$

$$\therefore \frac{de}{e} = -\frac{g_0 \delta_0}{e_0} \cdot \frac{r^2}{(r+z)^2} dz.$$

Integrating between e and $e_0, r+z$ and r , we have

$$e = e_0 \epsilon^{-\frac{g_0 \delta_0}{e_0} \cdot \frac{r^2}{r+z}}, \quad . \quad . \quad . \quad . \quad (14)$$

$$\delta = \delta_0 \epsilon^{-\frac{g_0 \delta_0}{e_0} \cdot \frac{r^2}{r+z}}. \quad . \quad . \quad . \quad . \quad (15)$$

Neglecting the attraction of the earth for the æther, and

* Journ. of Arts and Science, 1881, vol. xxii. p. 90.

considering the sun as the only attracting body, we have g_0 at the sun 28.6×32.2 , and at the earth $z=210$ $r=441,000$ miles, the sun's radius; $\delta = \frac{2}{35} \times 10^{-24}$, equation (9), and $e = \frac{4}{33} \times 10^{-6}$; and these, in (14) and (15), give

$$e = e_0 \epsilon^{\frac{28.6 \times 32.2 \times 2 \times 33 \times 10^6}{4 \times 35 \times 10^{24}} \times \frac{210}{211} \times 441,000 \times 5280}$$

$$= e_0 \epsilon^{\frac{1}{1,000,000}} \text{ nearly, } \dots \dots \dots (16)$$

and

$$\delta = \delta_0 \epsilon^{\frac{1}{1,000,000}} \text{ nearly, } \dots \dots \dots (16')$$

for the tension and density of the æther at the surface of the sun under the conditions imposed. But the millionth root of ϵ is practically unity; hence the elasticity and density at the sun is practically the same as at the earth.

Now, starting at the sun with this result, and finding the density at a distance z from it, then making z infinite, we shall get the 995,000th root of ϵ , the value of which is also sensibly equal to unity; hence the density at infinity would be sensibly the same as at the surface of the sun, the difference in the densities at the sun and at infinity being less than $\frac{1}{1,000,000}$ part of that at the sun. In order to make the density vary sensibly with the distance, the attraction of the central body must be something like a million times as great as that of the sun, or have a diameter a million times as large; but there being no such known body, therefore *the density and tension of the æther may be considered uniform throughout space.* Such has been our conception of it; and it is an agreeable surprise to find it so fully confirmed by analysis.

If the density were uniform, the weight of a given volume of it would vary as the force of gravity. At the surface of the sun a cubic foot would weigh [equation (10) multiplied by 28.6, or] 57×10^{-24} ; hence, for a height h it would weigh

$$\frac{57}{10^{24}} \int_0^h \frac{r^2}{(r+z)^3} dz = \frac{57}{10^{24}} \cdot \frac{rh}{r+h}, \dots \dots \dots (17)$$

which for $h=\infty$ becomes $\frac{13}{10^{14}}$ of a pound, which is the pressure upon a square foot of the sun of a column of infinite height under the conditions imposed. This would compress the first foot of the column about $\frac{1}{1,000,000}$ of its length, and would cause a corresponding increase in the density, the value of which, after this compression, will be found by multiplying the value given in equation (9) by $\frac{999999}{1,000,000}$, which will leave the result sensibly the same as before. Hence, from this standpoint, we

again conclude that the density of the æther may be considered as sensibly uniform throughout space, providing its temperature be essentially uniform.

If we assume that the law of the resistance by which the æther opposes the motion of a body varies as the square of the velocity of the body, we are still unable to assign the coefficient which will give the numerical value; but it is safe to assume that the entire mass of the æther occupying the path of a body moving through it, will not have a velocity imparted to it exceeding that of the body; but to be on the safe side, we will assume that it imparts a velocity equal to itself. The energy thus imparted will be lost to the body. To simplify the case, consider a planet moving in a circular orbit: r the radius of the planet, d its distance from the sun, D its specific gravity compared with water as unity, v_1 the velocity in its orbit; then the mass of æther occupying the place of the planet during one revolution about the sun will be, using equation (9),

$$\frac{2}{35 \times 10^{24}} \pi r^2 \times 2\pi d,$$

which, multiplied by $\frac{1}{2}v_1^2$, will give the energy imparted to it. The kinetic energy of a planet, neglecting its rotation, will be

$$\frac{4}{3} \pi r^3 \times 62\frac{1}{2} D \times \frac{v_1^2}{2g}.$$

Dividing the former, after multiplying it by $\frac{1}{2}v_1^2$, by the latter gives

$$\frac{1}{7 \times 10^{24}} \cdot \frac{d}{rD} \cdot \cdot \cdot \cdot \cdot \cdot (18)$$

for the fraction of the energy lost during one revolution about the sun. Applying this to the earth, we have

$$d \div rD = 93,000,000 \div 3912 \times 5\frac{1}{2} = 43000,$$

and (18) becomes

$$\frac{6}{10^{22}} \text{ nearly, } \cdot \cdot \cdot \cdot \cdot \cdot (19)$$

for the fraction of the energy lost in one year; and hence *at this rate* would require more than 1,666,000 trillion (1,666,000,000,000,000,000,000) years to bring it to rest.

Equation (18) is not applicable to the resistance offered to a comet, on account of the elongated orbit of the latter; but some idea of the effect of the resistance of the æther to the movement of a comet may be found by considering what

it would be if the orbit were circular, having for its radius the perihelion distance. According to Professor Morrison, the perihelion distance of the great comet (6), 1882*, was 716,200 miles, its aphelion distance will be 5,000,000,000 miles, the diameter of its nucleus shortly before disappearing on the solar disc was 7600 miles, the velocity at perihelion 295 miles per second and at aphelion 75 feet per second. But little is known in regard to the density of comets; but to be on the safe side we will assume it as $\frac{1}{1000}$ that of water. This data will reduce (18) to 13×10^{-18} for the fraction of energy lost during one of its revolutions about the sun; and as it would make a revolution in, say, 20 hours, it would lose in one of our years about 57×10^{-16} of its energy, at which rate it would go on for 170 trillions of years. Similarly, at its aphelion its rate of loss would be less than $\frac{1}{6} \times 10^{-15}$ of its energy in more than 2000 years—the time of one revolution in its orbit.

The most careful observations and calculations have failed to detect any effect due to the resistance of matter in space; and the above analysis shows that, within historic times, it has in any case scarcely amounted to an infinitesimal, certainly not sufficient to be measured. And when we consider that our assumptions have been very largely on the unfavourable side, and, further, that the energy imparted to the æther may, partly at least, be restored to the body, we assume that its resistance never can be measured. Laplace, when he found that the force of gravitation, if propagated by an elastic medium, must have a velocity exceeding 100 million times that of light, concluded that astronomers might continue to consider its action as instantaneous (*Mécanique Céleste*, B. x. ch. viii. p. 22, 9035); so may we, with as much confidence, continue to consider the resistance of the æther as *nil*.

Equation (6) gives

$$c\tau = \frac{6 \cdot 6(186300 \times 5280)^2}{2 \times 32 \cdot 2 \times 1 \cdot 4 \times 772} = 92 \times 10^{12}, \quad . \quad . \quad (20)$$

from which the specific heat of the æther may be found if its temperature were known. M. Fourier, the first to assign a value to the temperature of space, assumed it to be somewhat inferior to the temperature at the poles of the earth, or about 50° C. to 60° C. below zero†. M. Pouillet, considering the

* 'Monthly Notices of the Royal Astronomical Society,' vol. xlv. 2, p. 54.

† *Ann. der Chemie*, tome xvii. p. 155.

atmosphere as a diathermanous medium, capable of absorbing in different degrees the radiant heat from the sun and the dark heat from the earth, deduced for the heat of space—or, as he and Fourier called it, the stellar heat—approximately -142°C.^* (-287°F.), which is about 174°F. above absolute zero. It is well known that Pouillet's data were imperfect, several important elements being neglected, notably that of the humidity of the air; still it is not only the first, but, so far as we know, the only attempt to formulate this relation. It served to show, what has since been indicated by more direct experiments, that the temperature of space is very low. The delicate experiments of Professor Langley, before referred to, show a great difference in the degree of absorption by our atmosphere of different wave-lengths. The mean of the values for nine different wave-lengths, treated by M. Pouillet's formula, gives 139°F. above absolute zero, and the smallest value of absorption, which was for the infra-red, gives only 71°F. above absolute zero for the heat of space.

The heat of space may be considered as composed of three parts:—(1) stellar heat, (2) the heat contained in the dark matter of space, (3) the essential heat of the æther.

1. By the stellar heat we mean the heat received directly from the stars. It is a matter of easy calculation that, if the 50,000,000 of stars supposed to be visible with the most powerful telescopes were all at the distance of the nearest fixed star (α Centauri), or 221,000 astronomical units from the earth, and if each radiated the same amount of heat as our sun, the intensity varying as the inverse squares of the distances, the earth would receive from them all less than $\frac{1}{1000}$ as much heat as it now receives from the sun. And when we consider that only a very few stars are within measurable distances, and that the remote ones may be, when compared with these, well-nigh infinitely distant, it is evident that the amount of heat received from the stars is insignificant and may be discarded at the earth.

2. It is certain that there is a large amount of dark matter in space, since the meteoric dust and meteorites must come from

* *Comptes Rendus*, 1838, vol. vii. p. 61. Pouillet's formula is

$$a'' = 1.235 \frac{2-b}{2-b'} - 0.489,$$

in which b' = the absorptive power by the atmosphere of the sun's heat,

b = the absorptive power of terrestrial heat,

t' = the temperature of the stellar heat,

$a = 1.0077$.

If $b = 1$, its maximum, $b' = 0.2$, we find $t' = -235^{\circ}\text{C.}$ (-391°F.), or 71°F. above absolute zero.

beyond our atmosphere. The zodiacal light is supposed to be an evidence of meteoric matter between the earth and sun. The tails of comets are visible by some action of light upon some kind of matter. Matter in space not exposed to the rays of the sun will be at about the same temperature as the æther; but if in the rays of the sun and destitute of an atmosphere at the distance of the earth from the sun, its temperature would be very low. If present laws can be extended so far, and the earth were without an atmosphere, and the heat received were not conducted away, it has been computed that the mean temperature at the equator would be about -70°C. (-94°F.); and at the poles -221°C.^* , or 114°F. above absolute zero. The last result is obtained on the supposition that the poles receive heat directly from the sun a part of the year; it is further shown that if the poles were never exposed to the rays of the sun, the temperature would fall to that of the æther of space. But the data is not uniform, and there is too large an extension of empirical formulas to satisfy one that the above numerical results are reliable; still they point more and more strongly to a temperature not many degrees above absolute zero.

3. By the essential heat of the æther we mean the temperature which would be indicated by a thermometer graduated from absolute zero in a room located in space beyond our atmosphere whose walls were impervious to the passage of external heat. It is the heat due to the self-agitated æther, just as air has a temperature when not exposed to the rays of the sun. If the æther be perfectly diathermanous to the sun's rays, it will receive no heat on account of the heat of the sun flowing through it, though it may be heated from other sources. As direct evidence of an extremely low temperature of space, we cite the facts in regard to the meteorite which fell at Dharmasalla, India, July 14, 1860 †. "The most remarkable thing about it was, while the mass had been inflamed and melted at the surface, the fragments gathered immediately after the fall and held for an instant were *so cold that the fingers were chilled*. This extraordinary assertion, which is contained in the report with no expression of doubt, indicates that the mass of the meteorite retained in its interior the intense cold of the interplanetary space, while the surface was ignited in passing through the terrestrial atmosphere." Since this body had been exposed to the rays of the sun, its temperature must have exceeded that of the space through which it

* Professional Papers of the Signal Service, U. S. A.: Washington, D. C. 1884, No. xii. p. 54.

† *Comptes Rendus*, 1861, tome liii. p. 1018.

passed, as well as been warmed by the heat developed at its surface, from which it may be inferred that it had been *intensely* cold. Direct investigations, given above, indicate that this temperature is less than 200° F. above absolute zero; and we cannot assert that it is not less than 100° F. above, or even much less.

But, however low be the temperature of the æther, it cannot be absolutely cold, or, in other words, it must have a temperature above absolute zero, for otherwise it would be destitute of elasticity, and hence incapable of transmitting a wave. This is shown by eliminating V between equations (2) and (6), giving

$$c\tau = \frac{\mu}{2g\delta J}e,^* \quad . \quad . \quad . \quad . \quad . \quad (21)$$

in which if $\tau=0$, e will be zero, all the other factors being finite, and if $e=0$, then $V=0$ in (2). Indeed, this principle is so well recognized in physics, that a proof in this place seems superfluous. Being unable, in the present state of our knowledge, to do more than assign the probable superior limit of the temperature, we will, for the purposes of this analysis, assume $\tau=20^{\circ}$ F., absolute, being confident that the actual value is between $\frac{1}{10}$ of and 10 times this value. This value in equation (20) gives

$$c=46 \times 10^{11}=4,600,000,000,000 \quad . \quad . \quad . \quad (22)$$

for the specific heat of the æther, that of water being unity. This number so vastly—we might say infinitely—exceeds that for any known gas, as to justify one, at first thought, in looking with suspicion upon the applicability of the above analysis to this medium. Assumptions in regard to the absolute temperature will scarcely improve the appearance of this number. If it be assumed that the absolute temperature be only one degree, the number in equation (22) would be only twenty times as large; and if the absolute temperature be assumed at $1,000,000^{\circ}$ F., the resulting specific heat would still be more than a million times as large as for hydrogen. A few considerations of other properties of the æther may aid one in being reconciled to this paradoxical result. Is the result any more incredible than the fact, everywhere admitted, that every particle of the æther, in transmitting a wave of light, continually

* We note that this equation shows that the specific heat for different gases under the same tension, e , and temperature, τ , varies inversely as the density; and for the same temperature and density the specific heats c will be directly as the tension e . The more perfect gases, as hydrogen, oxygen, and air, conform nearly to this law.

makes 590,000,000,000,000 (6×10^{14} nearly) complete cycles of movements every second, for a wave-length of $\frac{1}{50,000}$ of an inch? The number of such complete movements in air for the fundamental c is only 264; and hence the ratio of the former to the latter of these numbers is nearly 2×10^{12} . The ratio of the specific heat given in (22) to that of hydrogen is nearly $1\frac{1}{2} \times 10^{12}$, which is not so different from that just given for the ratio of cyclical movements in a second of the *æther* and air. The velocity of sound in air at 493° F. above absolute zero is about 1090 feet per second; but if the temperature could be reduced to 20° F., absolute, the law being extended so far, the velocity would be only

$$V = 109 \sqrt{\frac{20}{493}} = 217 \text{ feet ;}$$

but the velocity of light is 982,000,000 feet per second, a number about $4\frac{1}{2}$ million times the former, and near a million of times that of the velocity in air under ordinary conditions. The ratio of the mass of air in a cubic foot at sea-level to that of a cubic foot of the *æther* as computed, far exceeds any of these ratios. The fact is, the known qualities of the *æther* in transmitting light and heat so far transcend those of any known terrestrial substance, that we might anticipate the fact that, in regard to magnitude, all its properties will be extremely exceptional when compared with such substances. We must accept substantially the number in equation (22), or subject this medium to different laws than those of gases.

We may deduce this result by another process; thus, since the specific heats of different gases are as the squares of the wave-velocities in the respective substances, the other elements being the same, if the specific heat of air be 0.23, we should have for the specific heat of the *æther*

$$c = 0.23 \left(\frac{186,300 \times 5280}{217} \right)^2 = 46 \times 10^{11},$$

as before. The correct value of the specific heat of air, 0.2375, would give over 47×10^{11} , and nearly 48×10^{11} ; but these differences are quite immaterial in this connection, the object being to check the former result, and find chiefly qualitative values.

On the other hand, in order that common air might be able to transmit a wave of the known velocity of light, its specific heat being taken constantly at 0.23, its temperature would be,

according to equation (20),

$$\tau = \frac{92 \times 10^{12}}{0.23} = 4 \times 10^{14} \text{ degrees F.} \\ (= 400,000,000,000,000^\circ \text{ F.).}$$

If the sun were composed of a substance having such specific heat, it could radiate heat at its present rate for more than a hundred millions of centuries without its temperature being reduced 1° F. , exclusive of any supply from external sources, or from a contraction of its volume. We know only such substances in the sun as we are able to experiment with in the laboratory; and if there be an exceptional substance in it, we have no means at present of determining its physical properties. It is, moreover, a question whether the æther constitutes an essential part of bodies. We conceive of it only as the great agent for transmitting light and heat throughout the universe.

On account of the enormous value of the specific heat, it will require an inconceivably large amount of heat (mechanically measured) to increase the temperature of one pound of it perceptibly. Thus, if heat from the sun, by passing through a pound of water at the earth, would raise the temperature 100° F. and maintain it at, say, 600° F. , absolute, it would, under similar conditions, raise the temperature of one pound of the æther, if its power of absorption be the same as that of water, $\frac{1}{48,000,000,000}$ of a degree*.

The distance of the earth from the sun being 210 times the radius of the latter, the amount of heat passing a square foot of spherical surface at the sun will be about 45,000 times the heat received on a square foot at the earth normally exposed to its rays, so that, under the conditions imposed, the temperature would not be a billionth of a degree F. higher at the sun than at the earth. This, then, is a condition favourable to a sensibly uniform temperature even if heated by the sun's rays. We are now inclined to admit that the æther is not perfectly diathermanous to the sun's rays, but that its temperature, however small, may be due directly to the absorption of the heat of central suns; for we begin to realize the fact that the æther may possess many of the qualities of gases, such as a molecular constitution, and hence also mass, elasticity, specific heat, compressibility, and expansibility, although the magnitude of these properties is anomalous. We have already considered its

* This illustration is rather crude, since it discards the relative volumes occupied by a pound of the respective substances.

compressibility at the surface of the sun, due to the weight of an infinite column, and found it to be exceedingly small ; now it may be possible that the expansion due to the excess of temperature of a small fraction of one degree at the surface of the sun over that at remote distances will diminish the density as much, or about as much, as pressure increased it, thereby making the density even more exactly uniform than it otherwise would be. According to what we know of refraction, it is impossible for a ray of light to be refracted in passing through the æther only,—at least, not by a measurable amount ; for not only are the density and elasticity practically uniform, but their ratio is, if possible, even more constant as shown by equations (16) and (16'). But the freedom of the æther-molecules may be constrained, or their velocity impeded, by their entanglement with gross matter, such as the gases and transparent solids ; in which case refraction may be produced in a ray passing obliquely through strata of varying densities. Neither is it believed that the æther does or can reflect light ; for if it did, the entire sky would be more nearly luminous. The rays in free space move in right lines.

The masses of the molecules in different gases being inversely as their specific heats, and as the specific heat of hydrogen is 3·4, and the computed mass of one of its molecules $\frac{11}{18} \times 10^{-29}$ * of a pound, we have for the computed mass of a molecule of the luminiferous æther,

$$m = \frac{11}{18 \times 10^{29}} \times \frac{3 \cdot 4}{46 \times 10^{11}} = \frac{1}{22 \times 10^{40}} \text{ lb.} \quad (23)$$

* Stoney concludes that "it is therefore probable that there are not fewer than something like a unit eighteen (10^{18}) of molecules in a cubic millimetre of a gas at ordinary temperature and pressure" (Phil. Mag. 1868 [4] xxxvi. p. 141). According to the Kinetic theory the number of molecules in a given volume under the same pressure and temperature is the same for all gases. The weight of a cubic foot of hydrogen at the temperature of melting ice and under constant pressure being 0·005592 of a pound, and as a cubic foot equals 28,315,000 cubic millimetres, the probable mass of a molecule of hydrogen will be

$$\frac{0 \cdot 005592}{32\frac{1}{2} \times 28,315,000 \times 10^{25}} = \frac{11}{18 \times 10^{29}} \text{ lb.}$$

Maxwell gives $\frac{46}{10^{25}}$ of a gramme = $\frac{3}{7 \times 10^{25}}$ lb., which is about $\frac{2}{3}$ the value given above (Phil. Mag. 1873 [4] xlv. p. 468).

The difference in these results arises chiefly from the calculated number of molecules in a cubic foot of gas under ordinary conditions. Thomson gives as the approximate probable number 17×10^{25} , which is about $\frac{2}{3}$ the value given by Stoney. Thomson's value would make the mass of a molecule of æther about $\frac{1}{13} \times 10^{-40}$ of a pound, which is not much different from that found above.

The mass of a cubic foot of the æther, equation (9), divided by the mass of a molecule, gives the number of molecules in a cubic foot, which will be

$$n = \frac{2}{35 \times 10^{24}} \times \frac{22 \times 10^{40}}{1} = \frac{44}{35} \times 10^{16}, \quad . \quad . \quad (24)$$

which call 10^{16} . This number, though large, is greatly exceeded by the estimated number of molecules in a cubic foot of air under standard conditions, which, according to Thomson, does not exceed 17×10^{25} , a number nearly 17,000,000,000 times as large as that in equation (24); and yet, at moderate heights, the number of molecules in a given volume of air will be less than that of the æther.

Assuming that air is compressed according to Boyle's law, and is subjected to the attraction of the earth, equation (15) will give the law of decrease of the density. Taking the density of air at sea-level at $\frac{1}{400}$ of a pound per cubic foot, $e_0 = 14.7$ lb. per square inch, $r = 20,687,000$ feet, equation (15) becomes

$$\delta = \frac{1}{400} \times 10^{-345 \frac{z}{r+z}}. \quad . \quad . \quad . \quad (25)$$

If $z = \infty$, $\delta = \frac{1}{400} \times 10^{-345}$, which would be the limit of the density, and it is a novel coincidence that this limit is nearly identical with the value found for the density at the height of one radius of the earth according to the ordinary exponential law wherein gravity is considered uniform*.

If the number of molecules in a cubic foot follows the same law, then at the height z there will be

$$17 \times 10^{-345 \frac{z}{r+z} + 25} \quad . \quad . \quad . \quad (26)$$

molecules per cubic foot. Similarly, the value of the length of the mean free path would be †

$$2 \times 10^{345 \frac{z}{r+z} - 6} \text{ inches.} \quad . \quad . \quad . \quad (27)$$

By means of these values, the following table may be formed:—

* The ordinary exponential law results from dropping $\frac{z}{r}$ compared with unity in equation (15), giving

$$\delta = \delta_0 e^{-\frac{z}{26221}} = \delta_0 10^{-\frac{z \text{ ft.}}{60387}} = \frac{1}{416} \times 10^{-\frac{z \text{ miles}}{11.44}},$$

in the last of which, if $z = 3956$, the exponent becomes 345.

† Phil. Mag. 1873 [4] xlv. p. 468.

Height.		Density or tension, that at the earth being unity.	Number of molecules in a cubic foot.	Length of the mean free path.
Fractional parts of earth's radius.	Approximate in miles.			
0	0	1	17×10^{25}	2×10^{-6} inch
$\frac{1}{2}$	50	$10^{-4.3}$	$17 \times 10^{20.7}$	2×10^{-17} „
$\frac{1}{3}$	100	$10^{-8.4}$	$17 \times 10^{16.6}$	$2 \times 10^{2.4}$ „
$\frac{1}{4}$	200	$10^{-16.4}$	$17 \times 10^{8.6}$	792,000 miles
$\frac{1}{5}$	282	10^{-23}	17×10^2	31×10^{11} „
$\frac{1}{6}$	395	10^{-31}	17×10^{-6}	31×10^{19} „
$\frac{1}{8}$	800	10^{-57}	17×10^{-32}	31×10^{45} „
1	3956	10^{-172}	17×10^{-147}	31×10^{160} „
2	7912	10^{-230}	17×10^{-205}	31×10^{218} „
∞	∞	10^{-345}	17×10^{-320}	31×10^{333} „

The numbers in the third column multiplied by $\frac{1}{400}$ will give the density (or mass per cubic foot) at the respective altitudes; and the same numbers multiplied by 15 (or more accurately 14.7) will give the tension per square inch. According to this law, at an elevation of 300 miles the density of the atmosphere will be somewhat less than the density of the æther as given by equation (9).

To find the height at which the tension of the atmosphere, according to the above law, will be the same as that of the æther, we have, by means of equations (11) and (25), substituting in the latter 2116 for $\frac{1}{400}$,

$$2116 \times 10^{-345} \frac{z}{r+z} = \frac{4}{10^8},$$

which solved gives

$$z = \frac{r}{31.24} = 126.6 \text{ miles,}$$

so that at the height of 127 miles the tension would be less than that of the æther, the temperature being uniform.

The mean free path according to the above law, in which gravity varies as the inverse squares, is less, and for great heights much less, than would be found according to the ordinary exponential law. Thus Crookes states that the mean free path of a molecule at the height of 200 miles is about

10,000,000 miles*; but according to the above law it becomes about 792,000 miles.

If a cubic inch of air at sea-level were carried to the height of $\frac{1}{7}$ the radius of the earth, and then allowed to expand freely, so as to become of the computed density of the atmosphere at that point, it would fill a space of $4 \times 10^{28.12}$ cubic miles, or a sphere whose radius is 2,398,000,000 miles, which is nearly equal to the distance of the planet Neptune from the sun; and there would be less than one molecule to the mile. Such are some of the results of extending a law to extreme cases regardless of physical limitations, or of the imperfection of the data on which it is founded. For instance, a uniform temperature is assumed, and, impliedly, an unlimited divisibility of the molecules. The latter is necessary in order to maintain a law of continuity. But modern investigations show that not only air, but all the gases, are composed of molecules of definite magnitudes whose dimensions can be approximately determined; and hence, if there be only a few molecules in a cubic foot, and much less if there be but one molecule in a cubic mile, it cannot be claimed that the gas will be governed by the same laws as at the surface of the earth.

To find the Height of the Atmosphere.—The atmosphere will terminate at that height where the vertical repulsive force equals the weight of the particles in the topmost layer. As a first approximation, conceive that the molecules are arranged in horizontal layers and vertical columns, in a prism whose base is one square foot, and whose height extends to the height of the atmosphere; the base of each column of molecules being one of the molecules in the base of the prism. Considering the number of molecules in a cubic foot of air at standard conditions as 17×10^{25} , and the weight of the same as .08 of a pound, we have for

$$\text{the weight of one molecule of air} = \frac{8}{17 \times 10^{27}} \text{†}. \quad (28)$$

The number of molecules along one edge of the bottom layer will be $\sqrt[3]{17 \times 10^{25}}$ nearly; and the number in the bottom layer the square of this number, or $170^{\frac{2}{3}} \times 10^{16}$, which, according to the hypothesis, will be the number in

* Phil. Trans. Roy. Soc. London, 1881, Part II. p. 389.

† This may be used as a unit for measuring the mass of a cubic foot of the æther. Thus, dividing the value in equation (10) by that in (28) gives 4250; or the mass of æther in a cubic foot is 4250 times the mass of one molecule of air.

the top layer ; and this multiplied by the weight of one molecule will give e , the weight in the top layer ; and equation (14) will give (the temperature of the column being considered uniform)

$$14.7 \times 144 \times 10^{-345 \frac{z}{\tau+z}} = \frac{170^{\frac{3}{2}} \times 10^{16} \times 8}{17 \times 10^{27}} ;$$

$$\therefore z = \frac{\tau}{23.35} = 169 \text{ miles.} \quad . \quad . \quad . \quad (29)$$

But the temperature is far from being uniform. In regard to a definite mass of a gas, we have the well-known relation

$$\frac{e}{\delta \tau} = \frac{e_0}{\delta_0 \tau_0} = \text{a constant} = \frac{pv}{\tau}, \quad . \quad . \quad . \quad (29')$$

where $p=e$ =the pressure on the base of a prism, and v =the volume.

The value of δ from this equation substituted in (13) gives

$$\frac{de}{e} = -g \frac{\delta_0}{\delta_0 e_0} \cdot \frac{\tau_0}{\tau} dz. \quad . \quad . \quad . \quad (30)$$

But with τ an unknown variable this cannot be integrated. If $\tau=\tau_0$ we at once have equation (14). The relation between τ and z is unknown, if indeed there be any algebraic relation between them. It is, however, known that, as a general fact, the temperature decreases with the elevation ; although local causes and air-currents often cause this law to be reversed for moderate heights. The best that can be done, in this case, is to find an expression that will represent, approximately, the mean values of the temperature. It is usually assumed that the average temperature at the earth is about 59° F. or 60° F., and that for latitudes of, say, 40° N. to 40° S. the perpetual frost-line is from 14,000 to 16,000 feet above sea-level ; and observations indicate that the *rate* of decrease of temperature decreases with the height. The last fact is suggestive of an exponential law ; hence assuming

$$\tau = \tau_0 e^{-\frac{z}{a}}, \quad . \quad . \quad . \quad (31)$$

and making $\tau=493^\circ$ F., absolute, at the height $z=15,840$ feet and $\tau_0=520^\circ$ F., absolute, we find $a=296,000$ (or 56 if z be in miles), and our equation becomes

$$\tau = 520 e^{-\frac{z \text{ miles}}{56}}. \quad . \quad . \quad . \quad (32)$$

This gives

Height, miles.	τ , absolute.	Fahr. scale.	Glaisher's observations*.
0	520° F.	59° F.	59° F.
1-5	518	57	
2-5	515	54	
3-5	513	52	
4-5	512	51	
1	510	49	41
2	501	40	32
3	493	32	18
4	484	23	8
5	475	14	- 2
6	467	6	
7	458	- 3	-11.8
50	212	-249	
75	136	-325	
100	87	-374	
120	65	-396	
150	36	-425	
224	9	-452	

The temperatures given in twenty-five or more reports of balloon ascensions, not only give values the mean of which is fairly represented by the celebrated seven-mile ascent of Mr. Glaisher, but his figures, given in the fourth column of the table, represent a more uniform law than is common in such reports. Our computed values exceed his observed values at all points except at the surface of the earth, where they agree. In this ascent he reached the point of freezing at the height of two miles, which is lower than the average, as determined by many observations; and therefore it appears that equation (31) probably represents the general law better than this single set of observations. The effect, however, of the exponential law is scarcely perceptible within the limits of observation; for the exponent of ϵ is so small for elevations under seven miles, that it makes the law of decrease of temperature nearly uniform with equal increments of elevation. Thus, omitting fractions, the computed decrease for the first mile is 10°, and the average for seven miles is nearly 9°; but to assume a uniform decrease throughout the column limits the height of the column independently of pressure or other conditions, for it could not extend beyond the point of absolute zero. There is no objection to applying such a law

* 'Travels in the Air,' by James Glaisher, F.R.S., p. 50.

provided it can be shown to be true—a condition which, at present, is not accepted.

Substituting τ from (31) in (30), and integrating between the limits of z and $z=0$, gives

$$e = e_0 e^{-\frac{ag\delta_0}{e_0} (e^{\frac{z}{56}} - 1)}; \quad (33)$$

which ultimately will equal the weight of the molecules in the top layer. Hence, substituting numbers, we have

$$2116 = \frac{8 \times 170^{\frac{8}{3}} \times 10^{16}}{17 \times 10^{27}} e^{-\frac{296000 \times 0.8}{2116} (e^{\frac{z}{56}} - 1)};$$

which gives

$$z = 86 \text{ miles.} \quad (34)$$

It is evident that the very low temperature of the higher portions of the column will shorten very much the hypothetical column of uniform temperature; but there are other conditions which will modify the preceding analysis. The assumptions in regard to layers and columns would not be realized even under statical conditions, and much less for the conditions in nature. Statically, the molecules would arrange themselves more like shot in a pile, each being over the space between the molecules in the layer below, instead of over a molecule. This arrangement would give a less number in the horizontal layers than assumed above. But the hypothesis of constancy in the number of molecules in the layers is open to greater objections. For the distance between the molecules will increase with the elevation on account of the diminution of the pressure of that part of the column above the point considered, and the elastic force will be correspondingly diminished; while, horizontally, in the plane of a layer of the molecules, the elastic force would remain constant. In other words, in the medium arranged as assumed the tension would not be the same in all directions, and hence would be in unstable equilibrium. As a refinement, we notice that in every heavy fluid the downward pressure at every point exceeds the upward by the weight of a molecule.

Considering, now, that the molecules in the hypothetical layers are distributed uniformly throughout the spaces immediately beneath them, the number in the new top layer will be less than in the former case, and the column will rise to a greater height, and hence will exceed 86 miles; and, in turn, the new column would need another correction, and so on. Assuming that the number in the top layer is 10^{10} , and that the vertical component of the elastic forces follows the law of

equation (33), we find

$$z = 95 \text{ miles ;}$$

and if the number in the top layer be 10^4 , we find $z = 104$ miles, and for *one* molecule, $z = 110$ miles. In a similar manner it would be legitimate to assume that the column was capped by a fraction of a molecule, for that would be equivalent to one molecule at the top of a column having a base of several square feet. We are unable to determine where this process would end in nature; and hence this analysis fails to fix definitely the extreme height of the atmosphere, even for statical conditions.

Assuming that the distance between the contiguous molecules would be inversely as the third root of the densities of the medium, as they would be with sufficient accuracy where the number of molecules in a cubic foot is immense, we have, after substituting e , equation (33), and τ , equation (31), in (29'),

$$\frac{\delta}{\delta_0} = e^{\left[\frac{z}{a} - \frac{ag\delta_0}{e_0} (e^{\frac{z}{a}} - 1) \right]} = \frac{d_0^3}{d^3},$$

where d_0 is the distance between contiguous molecules at sea-level and d the corresponding distance at the height z . Hence

$$d = d_0 \left[e^{\frac{ag\delta_0}{e_0} (e^{\frac{z}{a}} - 1) - \frac{z}{a}} \right]^{\frac{1}{3}} (35)$$

If $d_0 = \frac{18}{\sqrt[3]{170 \times 10^8}}$, $\frac{ag\delta_0}{e_0} = 11.19$; we have

for $z = 86$ miles,	$d = \frac{1}{95}$ of an inch,
„ $z = 95$ „	$d = 4.5$ inches,
„ $z = 104$ „	$d = 11.4$ „

These values of d are greatly in excess of the distances between contiguous molecules in the horizontal layers, according to assumed conditions. Thus, at the height of 104 miles, it was *assumed* that there were 10^2 molecules on the side of a square foot, in which case the distance between contiguous molecules would be about $\frac{1}{8}$ of an inch instead of 11 inches as above. These results ought not to agree exactly, for one analysis assumes that the atmosphere terminates with each assumed number of molecules, while the other assumes that the law is continuous to any height. It is apparent that the laws represented by equations (33) and (35) both become practically discontinuous at a height at or less than 95 miles. For the sake of giving definiteness to the following remarks, we will assume that the mean height for statical conditions is

95 miles. But the conditions in nature are not statical. The changes in temperature in the column will be continually increasing or decreasing its height; the air-currents also operate to change it, first by increasing or decreasing the temperature from the mean at considerable heights, and, secondly, by operating dynamically to push the top of the column upward; the aerial tides may operate to raise the column still higher, and the molecules themselves are supposed to be flying with great rapidity in all directions. An increase of temperature of one tenth the mean value, which at the earth's surface would be about 49° F., would elongate the column about ten miles, and a corresponding decrease would shorten it about the same amount, making it 105 miles in the former case and 85 miles in the latter. The effect of air-currents and aerial tides cannot be so definitely calculated; but it is safe to assume that they may produce a much greater increase of height above the mean than they will depression below the mean; just as in a highly agitated sea, the depressions below the mean surface-level may be small compared with the height above the same level to which the spray from the top of a wave may be thrown. It seems possible, therefore, that when the temperature, air-currents, and aerial tides conspire to depress the column, the extreme height of the atmosphere may be reduced to less than 85 miles; and when they conspire to elevate it, it may possibly rise to a height exceeding 120 miles.

If it be certain, as is assumed, that the meteors are rendered incandescent by atmospheric friction, and the extreme height at which they are visible could be determined by direct observation, it would fix a height less than the extreme height of the atmosphere, independent of other physical considerations; but the movement of these bodies is so extremely rapid that it is impossible to determine their height with astronomical precision. Still computations by Professor Herschel give a height of about 118 miles*, and Professor Newcomb estimates it to be about 100 miles†. It is possible that a meteor would sometimes become inflamed by penetrating the atmosphere

* Professor A. S. Herschel gives the height of twenty meteors varying from 40 to 118 miles. 'Nature,' vol. iv. p. 504.

† Newcomb says:—"The lightning-like rapidity with which the meteors darted through their course rendered it impossible to observe them with astronomical precision; but the general result was that they were first seen at an average height of 75 miles and disappeared at a height of 55 miles. There was no positive evidence that any meteor commenced at a height greater than 100 miles. These phenomena seem to indicate that our atmosphere really extends to a height of between 100 and 110 miles." 'Popular Astronomy,' 1878, p. 389.

only a few miles, for although the atmosphere in the upper regions is extremely rare, yet the actual number of molecules in a cubic foot is large. Thus, according to our analysis, for statical conditions, the topmost cubic foot of the 104-mile column would contain about 1,000,000 molecules; and at the height of 95 miles it would contain about 1,000,000,000,000,000 molecules; so that if the relative velocities of the meteor and air be 20 miles per second, the meteor would encounter an *enormous* number in the twentieth or even the hundredth part of a second, after first entering the atmosphere.

The height of the auroral arch—supposed to be within our atmosphere—has been computed to be from 33 to 1000 miles (see article “Aurora,” *Encyc. Brit.*). But it has been shown by experiment, that a vacuum may be produced through which an electrical discharge cannot be passed, and yet the atmosphere at the height of 150 miles under the most favourable condition, that of uniform temperature, is vastly more rare than the most perfect vacuum ever produced by the most perfect Sprengel pump; and at the height of 200 miles under the same conditions the vacuum would be some 10,000,000 times as great as the most perfect vacuum yet made; while, according to the probable law of the decrease of temperature with the elevation, and in accordance with the probable mass of a molecule of air, the extreme height falls far short of 150 miles. It is evident, therefore, that the assumed determination of the height of the atmosphere by means of the auroral arch is, to say the least, unreliable*.

We have pursued this digression in regard to the atmosphere partly for its own sake and partly to show, by way of contrast and accumulative evidence, that the æther is a substance entirely distinct from that of the atmosphere,—that the former cannot be considered as the latter greatly rarefied, as some have supposed. Admitting the validity of the preceding discussion, some of the distinctive properties are:—

1. The different modes of the movements of the molecules in the two substances in the propagation of a wave; in one the motion being a to-and-fro movement and in the other a transverse movement. These are distinctions recognized by the best writers upon the subject, and are especially noticed by Maxwell in an article on Ether in the *Encyclopædia Britannica*.

2. It is impossible for a wave to be transmitted in air with the known velocity of light, unless its temperature be increased

* Some writers incline to the view that the aurora is due to a cosmic rather than a terrestrial origin. ‘Science,’ 1885, p. 395.

millions of millions of degrees Fahrenheit above the standard temperature; but such a wave is transmitted in the *æther* although its temperature is far less than has ever been produced by artificial means.

3. The ratio of the elasticity to the density in the *æther* is exceedingly large compared with the same ratio in air. The temperature of air being taken at 60° F. and the *æther* at 20° F., absolute, the ratio is, with sufficient accuracy,

$$\left(\frac{980,000,000}{1090}\right)^2 = 8 \times 10^{11} \text{ nearly.}$$

4. The specific heat of the *æther* is, at least, many million times that of air, or of any other known gas.

5. The atmosphere is of variable density, elasticity, and temperature, while the *æther* is well-nigh isometric throughout space in regard to each of these elements.

6. A molecule of *æther* is well-nigh infinitesimal compared with one of air.

7. Air is attracted to a planet with such a relative force, that its extreme height is only a few miles.

8. The ratio of the density to the elasticity of the *æther* is constant; but in the atmosphere, on account of the decrease of temperature with the elevation, the density decreases less rapidly than the elasticity, as may be seen by comparing the first part of equation (35) with equation (33): we have

$$\frac{\delta}{\delta_0} = \epsilon^{\frac{z}{a}} \cdot \frac{e}{e_0}.$$

On this account a wave would be propagated with less velocity in the higher regions of the atmosphere than in the lower, while a wave in the *æther* has a sensibly uniform velocity throughout space.

The question may arise, May not the resistance of the *æther* drag away the remote molecules of the atmosphere, and so scatter them in space along the path of the earth's orbit? Assuming that the atmosphere is moving with the earth through space at the rate of 20 miles per second (which exceeds the actual velocity), and that the resistance of the *æther* is measured in the same manner as for fluids, we have for the resistance $R = k w a \frac{v^2}{2g}$, where v is the velocity of a molecule of air, a is meridian section, w the weight of a unit of volume of the *æther*, and k a coefficient depending upon the form of the body. Making $k=1$, which is greater than its

actual value, and $a = \frac{1}{10^{17}}$ feet, which, again, is in excess of the true area, w the value in equation (10), we find that

$$R = \frac{2}{10^{24}} \times \frac{1}{10^{17}} \times (20 \times 5280)^2 \cdot \frac{1}{64 \cdot 4} = \frac{1}{10^{33}} \text{ of a pound nearly.}$$

The attractive force of the earth for a molecule of air is given in equation (28), and hence the attraction of the earth for a molecule of air will exceed 500,000 times the resistance of the *æther*; hence the molecules accompany the earth in its orbit as certainly as does the moon, and are more rigidly bound to it than is its satellite.

The Kinetic energy of a molecule of air at standard conditions is about

$$\frac{1}{2} \cdot \frac{8}{32 \cdot 2 \times 17 \times 10^{27}} 1600^2 = \frac{2}{10^{23}} \text{ foot-pound;}$$

and of the *æther*, according to our results, about

$$\frac{1}{2} \cdot \frac{1}{22 \times 10^{40}} (273,000 \times 5280)^2 = \frac{1}{2 \times 10^{23}} \text{ foot-pound;}$$

which results are nearly the same; but in a pound of the *æther* there is some 100,000,000,000 times the Kinetic energy of a pound of air.

Considering the terrestrial atmosphere as equivalent to one of uniform density and $5\frac{1}{2}$ miles high, each of whose molecules has a mean square velocity of 1600 feet per second, and the *æther* of uniform density each of whose molecules has the mean square velocity of 286,000 miles per second, a rough approximation shows that the Kinetic energy of the *æther* in a sphere whose radius is 92,000,000 miles (nearly the distance of the earth from the sun) will be only about 100,000 times that in our atmosphere.

The mean free path of a molecule of gas as given by Loschmidt is

$$l = \frac{\text{combined volume of the molecules}}{\text{volume of the gas} \times \frac{1}{8} \text{ the diameter of a molecule'}}$$

and by Maxwell,

$$l = \frac{\mu}{\rho} \cdot \frac{1}{v} = 3 \frac{\mu}{\rho} \cdot \frac{z}{\gamma V};$$

(the last member of which we have added), in which ρ is the density of the gas, μ the coefficient of internal friction, and v the velocity whose square is the mean of the squares of the actual velocities of the molecules. In regard to the *æther*, these equations contain at least three unknown quantities, l , μ , and the diameter of a molecule, and hence they cannot be

completely solved. Comparative results, however, may be found by assuming that the density of the molecules of æther equals those of hydrogen, or is any multiple thereof; for then the diameter of a molecule of the æther might be found (that of hydrogen being 5.6×10^{-10} of a metre); and the combined volume in a cubic foot will equal the number of molecules in a cubic foot multiplied by the volume of one molecule, and hence will be found the length of the mean free path and the coefficient of internal friction.

We conclude, then, that a medium whose density is such that a volume of it equal to about twenty volumes of the earth would weigh one pound, and whose tension is such that the pressure on a square mile would be about one pound, and whose specific heat is such that it would require as much heat to raise the temperature of one pound of it 1° F. as it would to raise about 2,300,000,000 tons of water the same amount, will satisfy the requirements of nature in being able to transmit a wave of light or heat 186,300 miles per second, and transmit 133 foot-pounds of heat-energy from the sun to the earth each second per square foot of surface normally exposed, and also be everywhere practically non-resisting and sensibly uniform in temperature, density, and elasticity. This medium we call the Luminiferous Æther.

XLVIII. *A Determination of the Ohm.*

By Prof. F. HIMSTEDT*.

THROUGH the means placed at my disposal by the Government of the Grand Duchy of Baden, for the determination of absolute resistance, I have been enabled to carry out a determination of the Ohm according to the method recently published by me†; and beg permission to lay an abstract of my results in the following paper before the Royal Berlin Academy.

According to the above-mentioned method, the constant divergences of the magnet in the same galvanometer are observed—produced in the one case by means of induction-currents passing in the same direction through the galvanometer at the rate of n per second; in the other case by means of a constant current whose strength is a known fraction of the inducing current. Let us call the observed angles of divergence α_1 and α_2 , the required resistance r will be found

* Translated from the *Sitzungsberichte der k. Pr. Akad. d. Wissens. Berlin*, July 23, 1885.

† Wied. Ann. Bd. xxii. S. 281 (1884).