mals, man has become the most wonderful living thing on earth, separated by a great gulf from his next of kin, and yet, in spite of his high degree, afflicted with more diseases than any other animal and beset by at least as many tormenting parasites. Inexperienced in his early history, his mind steadily advanced until to-day he contemplates all nature with a yearning to know its mysteries. The changes in the germ-cell sufficing to evolve him are as inscrutable to his reason as the constitution of matter and the interstellar ether, the nature and origin of the cosmical forces and of chemical affinity, the conditions obtaining on other worlds revolving about untold millions of other suns, or the origin, nature and meaning of life itself. But we ardently desire to know these things, to peer out into unfathomable space and to speculate upon the meaning of our existence and the unknowable as we perceive it all about us in the universe. Under such circumstances are we to live but a short time on earth and then be consigned to everlasting oblivion? In contemplating the real significance of the word eternal or everlasting, which must refer to infinity—a duration of time so inconceivable that a number of years expressed in pica type encircling the entire globe would be as naught when compared with it—our reason would appear to answer in the affirmative. But, as a species—*sapiens*—of the genus *Homo,* we can never know. We seem to be but intellectual atoms floating in an infinity of space and time.

THOS. L. CASEY.

ST. LOUIS,
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SPECIAL ARTICLES.

THE SPEARMAN CORRELATION FORMULA.

SOME time ago C. Spearman published a formula for calculating the true correlation by the Pearson formula for observations in themselves variable.[1] This method has been used by several psychologists without a full understanding of the way in which Spearman arrived at the method. Such a use of the method is dangerous since those who apply it can not be accurately informed as to the conditions under which it holds.

Let

T represent the type.

σ represent the variability of the group.

t represent single observations upon individuals.

N the norm of the individual.

v represent the variability of the individual with respect to his norm, including the error of observation.

Assuming that all t's follow the exponential law and representing averages by [ ], we shall find in the long run that

$$[t] = [N] = T.$$

The observed variability of the group may be expressed as

$$\sqrt{\sigma^2 + v^2}$$

in which σ represents the true variability that determines the true correlation in the Pearson formula. The whole problem in practise is to find the value of v.

Spearman says that the true correlation may be obtained by dividing the average correlation for the various trials of the two tests by the square root of the product of the correlations for the successive trials for each test.

Let,

$xy =$ the average product of the deviations for the corresponding single trials in two tests.

$(pq)_1 =$ the same for t_1 and t_2 of the first test.

$(pq)_2 =$ the same for t_1 and t_2 of the second test.

v_1 and $v_2 =$ the true variabilities of individuals in the two tests.

σ_1 and $\sigma_2 =$ the true variabilities as calculated for x and y.

Then by substitution in the formula of Spearman,

$$r^2 = \frac{\dfrac{[xy]^2}{(\sigma_1^2 + v_1^2)\,(\sigma_2^2 + v_2^2)}}{\dfrac{(pq)_1\,(pq)_2}{(\sigma_1^2 + v_1^2)\,(\sigma_2^2 + v_2^2)}}$$

$$r^2 = \frac{[xy]^2}{(pq)_1\,(pq)_2}$$

Hence,

$$\sigma_1^2\sigma_2^2 = (pq)_1\,(pq)_2$$

$$\sigma_1^2 = (pq)_1$$

[1] C. Spearman, *American Journal of Psychology,* January, 1904.

Thus the formula assumes that the successive observations upon the same individual, or t_1 and t_2, are constituents of the same norm in conformity to the exponential law. The method is simply a way of eliminating v from the expression. The assumption upon which the whole rests is that $[pq]$ will be constant no matter what the magnitude of v.

Now, in tests as taken in psychological laboratories it is evident that some change in the norm results with each successive trial. In all tests we find practise, warming-up, etc., as factors leading us to consider the successive t's as ordinates of a curve whose abscissa represents units of time. In practise, at least, we assume a type of curve toward which all individuals tend. The individual curves are variants in the group defined by the type curve. The same conditions are found in growth of stature. It has been shown that individuals tend to the same curve of growth. Also that when an individual varies greatly from the type curve at one period of time he is likely to vary less at another.[2] This implies that the individual tends to fill a space type in a time type so far as his physical growth is concerned. If this is true we should expect to find the same relations with respect to the form of any organ.

I have at hand data for measurements of the alveolar arch with reference to the median plane of the body. These measurements were taken as ordinates of the curve as defined by the teeth. This gives us the same general geometric conditions as were found in curves of growth. Now, before going on with the data let us consider the problem as that of a correlation between the ordinates of individual curves, varying from the type curve according to the exponential law.

We may assume that OM represents any curve. Let t_1 be the first observed trial or ordinate; t_2, the second. Now, no matter what the value of t_2 may be, there must be a correlation between t_1 and t_2 because of their geometric relations. Also, there may be a relation between $(t_1 - t_2)$ and t_2.

Let

$[t_1] - [t_2] = [P]$,

σ_1, σ_2, σ_3, the respective variabilities of these averages,

$x =$ the deviations of t_2 from $[t_2]$,

$y =$ the deviations of t_1 from $[t_1]$,

$y = x + p$.

Then $(x + p)$ is a variable agreeing with a variable y; p is the deviation of $[P]$.

$$[xy] = [x^2] + [xp],$$

$$[y^2] = [x^2] + 2[xp] + [p^2].$$

Also

$$[y^2] = \sigma_1^2,$$

$$\sigma_1^2 = \sigma_2^2 + \sigma_3^2,$$

$$[y^2] = [x^2] + [p^2].$$

Hence, $[xp]$ should be zero, but this can occur only when there is no correlation between t_2 and P.

The value of p may be expressed as

$$[p^2] = [y^2] - 2[xy] + [x^2].$$

Thus, the various terms of these equations can be calculated from the data for the correlation of t_1 and t_2.

I have calculated the following for the successive ordinates of the alveolar arch:

	$[xy]$	$[xp]$	σ_3	r_{xy}	r_{xp}
$t_1\,t_2$	+ 6.08	+ 1.67	2.8	+ 0.71	+ 0.28
$t_1\,t_3$	+ 5.03	+ 0.62	2.3	+ 0.67	+ 0.13
$t_1\,t_4$	+ 3.91	− 0.50	2.8	+ 0.52	− 0.09
$t_1\,t_5$	+ 2.70	− 1.71	2.9	+ 0.38	− 0.27

In this table I have correlated the smallest dimension with each of the successively large dimensions as indicated by t. $[x^2]$ is taken as a constant; σ_3 appears to be constant. The correlation of t_{n-1} and P varies for different points in the curve OM, in such manner that two points may be found for which the correlation is zero. $[xp]$ varies with the magnitude of $[P]$, which in this curve is dependent upon the distance between t_1 and t_2.

I have no good data for a psychophysical test. The only available data, at this writing, are the reaction times in the Columbia University tests. To preserve the form of the preceding table the fifth reaction was taken as the point of departure. Thus:

[2] Franz Boas, Report of the Commissioner of Education for 1896–7.

	$[xy]$	$[xp]$	σ_3
$t_5\,t_4$	$+7.78$	-8.00	4.8
$t_5\,t_1$	$+4.13$	-11.63	4.9

The result is similar to that in the foregoing. $[xp]$ is negative with respect to consecutive trials. I have not calculated the values for all of the five trials because reaction time is not a good case; the distributions being asymmetrical, a disturbing factor is present. This will require special investigation.

These few observations lead to the following hypothesis: When a geometric form is taken as the type of biological activity the correlation between one dimension, taken as fixed, and its variation from another dimension will range indefinitely as positive or negative according to the geometric relations between the points from which the measurements are made. When two dimensions are correlated the degree of correlation will be increased or decreased by virtue of the equalization between the above correlation and the correlation between the parts common to both.

The method used by Spearman to determine the true correlation for psychological tests in which t_1 and t_2 seem to represent ordinates of a similar curve, assumes that $[pq]$ will be constant for the successive trials. Turning to our last formula and substituting

$$[pq] = [x^2] + [xp].$$

we have shown that $[xp]$ is a variable of uncertain range causing $[pq]$ to vary. Thus an unknown variable is introduced by the use of the Spearman formula. There is reason for assuming that $[xp]$ will be negative in many psychological tests, thus reducing $[pq]$, whence the method of Spearman will give correlations artificially increased.

To put it in another way, the formula of Spearman assumes that

$$r_{t_1t_2} = \frac{[pq]}{\sigma_1\sigma_2} = 1$$

or

$$= \frac{[pq]}{\sigma_1{}^2} = 1.$$

It is evident that this can be only when t_1 and t_2 are identical. $[pq]$ will be a constant when t_1 and t_2 are of the same type. We have shown above that the method of observation will sometimes result in a geometrical relation between t_1 and t_2 causing $[pq]$ to vary. Whenever this occurs the method fails.

CLARK WISSLER.

TREATMENT OF SIMPLE HARMONIC MOTION.

THE very great importance of simple harmonic motion in the physical world demands very careful consideration of the method of presenting and treating it for students beginning the work in advanced physics.

From the books on physics which I have at hand, I have selected fourteen which are used by a large portion of American students for their first study of simple harmonic motion. Eleven of these present and define simple harmonic motion merely as the projection on a diameter of uniform circular motion, deriving equations and other definitions by use of this uniform circular motion. Some of them scarcely suggest the question whether there really is such motion; much less, under what conditions or by what law of force it would occur.

Three of the fourteen texts give simple harmonic motion a *dynamic* definition, presenting it as produced by a force acting toward and varying as the distance from a center. But even these three, in treatment, make the *auxiliary circle* very prominent.

An experience of a good many years with large numbers of students leads me to believe that in the minds of very many the *auxiliary circle* with its functions and *circular motion* 'looms larger' than the actual simple harmonic motion. It seems to me highly desirable to dispense with the auxiliary circle in both definition and treatment.

The definition should be dynamic. This dynamic definition should be drawn from experiments.

The treatment should be a problem, a study of the motion caused by a force acting by the law found in the experiments.

I offer the following as an illustration of treating simple harmonic motion as above suggested; and for students not using calculus.

Experiments.—One or more on each, flexure,