Division of Angles and Arcs

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being used—written on the fourth page of the sheet. The tearing is apparently due to the breaking of the seal when the letter was opened.

EDWARD M. LANGLEY.

Can any reader of the *Gazette* identify "Dear Tom"? A Newcome was elected master of St. John's in 1735, but his name is given in the calendar as *John Newcome*.

DIVISION OF ANGLES AND ARCS.

THE rule which is usually given in works on Geometrical Drawing for constructing a regular polygon of n sides is as follows: *Take a circle with diameter AB. On AB describe an equilateral triangle ABC. Divide AB into n equal parts. Let Q be the second point of section reckoning from A. If CQ cut the circle at P, then AP will be the n^{th} part of the circumference.*

The accuracy of this rule must be that of the theorem that if ABC be an equilateral triangle described on the base AB of a semicircle, then straight lines through C divide the base and the curve in the same ratio.

To verify this proposition, consider a straight line through C which divides the base at Q in the ratio $90-x:90+x$ and the curve at P in the ratio $90-y:90+y$ so that the angle DOP is y° and the ratio OQ:OA is $x:90$.

The relation between x and y is

$$x=90 \frac{\sqrt{3} \sin y^\circ}{\sqrt{3} + \cos y^\circ}.$$

It is evident that this equation reduces to $x=y$ in five cases, viz. $y=0, \pm 30, \pm 90$. Calculation shews that the statement is not far from the truth for intermediate values of y . The table of corresponding values of x and y calculated for every 5° is worth examination.

y	0	5	10	15	20	25	30	35	40	45
x	0	5 -	10 -	15 -	20 -	25 -	30	35 +	40 +	45 +
$x-y$	0	-0212	-0366	-0457	-0446	-0301	0	0470	1108	1906
y	50	55	60	65	70	75	80	85	90	
x	50 +	55 +	60 +	65 +	70 +	75 +	80 +	85 +	90	
$x-y$	2832	3832	4837	5689	6258	6317	5564	3594	0	

In the graph Fig. III. the scale for $x-y$ ordinates is 100 times the scale for y abscissae.

It will be seen that $x \approx y$ does not exceed '65 at any point, and that between 0° and 30° this error is less than '05". The accuracy attainable in Geometrical Drawing is not closer than two tenths of a degree, so the theorem which was enunciated above is proved to be accurate for all practical purposes when the ratio of the segments lies between 1:2 and 2:1, but the error may be appreciable when the ratio lies outside these limits.

It appears from the table that if we were to apply the conventional rule to the construction of the regular 20-gon, the error in the arc of 18° (72° in the table) would be as much as '63". Multiplying the error by 20, we find

that the 20 arcs would exceed a complete circumference by 18° . If on the other hand we were to use the arc of 18° measured from the middle of the arc AB , then the error in one arc would be $\cdot 045$ and the 20 arcs would not fall short of a complete circumference by more than 1° .

This comparison leads us to suggest as an improved form of the rule for constructing the polygon of n sides when $n > 6$ viz.: Take a circle with centre O and diameter AB . On AB describe an equilateral triangle ABC . Divide OA at Q in the ratio $4 : n \sim 4$. Produce CO , CQ to meet the circumference. The arc they intercept will be one n^{th} of the circumference.

The problem, to divide an angle into any number of parts, is reduced by the theorem of proportionate division to dividing a finite straight line into the same number of parts. The figure which is drawn to shew the trisection of an angle, shews how simple the method is. The error in theory is quite negligible compared with the inevitable error in draughtsmanship. The method of trial and error remains the only practically useful one for all questions of this type.

T. J. W. WHIPPLE.

CONTRACTED MULTIPLICATION.

I HAVE read with interest the Note on this subject in the January number of the Gazette, and the report of the discussion at the Meeting of the Association. Mr. Lodge suggests figuring the multiplicand. It seems to me, however, less cumbersome to reverse the multiplier at the start. Very fair results have been obtained at Bromsgrove School by the method of multiplication indicated below. Written out at length, the method seems at first sight rather complicated, but in actual practice it is easy to learn.

The method is as follows:

- (1) Arrange the multiplier with its highest figure in the units place.
- (2) Cut off from the multiplicand, by a vertical line, the number of figures to be obtained correct.
- (3) Place the units figure of the multiplier on the right of the line and write the latter in reverse order.
- (4) Each digit of the multiplier is now below the digit of the multiplicand which gives the first figure of the partial product retained. Look, however, one place to the right, and carry from it to the nearest ten.
- (5) Write the first figure of each partial product in the column on the right of the line, and in adding up this column carry to the nearest ten and put nothing down.

Examples.

- I. Find the value of $85\cdot362 \times \cdot 04796$ to the nearest hundredth.

$$\text{Product} = 85362 \times 4796.$$

85,362	
6974	
3414	$4 \times 6, 24$ carry 2, $4 \times 3, 12, 14.$
597	$7 \times 3, 21$ „ 2, $7 \times 5, 53, 37.$
77	$9 \times 5, 45$ „ 5, $9 \times 8, 72, 77.$
5	$6 \times 8, 48$ „ 5.
409	Answer.

$$\text{Rough Check } 85 \times \cdot 05 = 4\cdot 25.$$

The decimal point in the result is always under that in the top line.