The Strength and Behaviour of Brittle Materials under Combined Stress

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XXXI. *The Strength and Behaviour of Brittle Materials under Combined Stress.* By WALTER A. SCOBLE, A.R.C.Sc., B.Sc., *Whitworth Scholar*.*

[Plate IX.]

1. *Previous Results.*—In a former paper† the writer has given the results of tests on a ductile material under combined stress, the material being steel in the form of turned bars, subjected to bending and twisting. It was found that at the yield-point, selected as the criterion of strength, both the maximum strain and maximum principal stress varied considerably, but the maximum shear was approximately constant. If we consider the great flow which takes place before fracture with ductile materials, the specimen behaving like a viscous fluid, the above result will be understood. It was pointed out that the results of the tests on ductile materials should not be applied to cases in which brittle metals are used. The small yield before fracture, and the positions of the planes of fracture, provide sufficient evidence to indicate that the behaviour is very different from that of ductile metals. Unfortunately there seems to be little information available relating to this subject.

2. *Separation of Metals into Ductile and Brittle.*—To clearly distinguish between ductile and brittle metals, the former may be defined as those which draw out considerably before fracture, fracture taking place across planes approximating to those of greatest shear. A brittle material shows little or no signs of drawing out before fracture, and usually breaks along the plane of principal stress.

3. *The Theory of Elastic Strength.*—Three views are held as to the condition which determines the limit of strength : (1) That the principal stress has a certain value ; (2) The maximum strain must reach a certain amount ; (3) The maximum shear is constant. Hitherto no distinction appears to have been made between brittle and ductile materials when considering the theory of elastic strength. Engineers have

* Read January 25, 1907.

† Proc. Phys. Soc. Lond. vol. xx. ; also Phil. Mag. Dec. 1906.

SCOBLE.

FIG. 2.

Test

accepted one theory to apply to all metals. For ductile specimens the maximum shear has been found to be almost constant at yield, but it is not justifiable to assume that the same holds for brittle materials at the point of fracture.

4. *The Criterion of Strength.*—With ductile materials the point selected as the criterion of strength was the yield-point, but with brittle materials fracture is evidently the best and most convenient. The only other condition available is that at the elastic limit, a point difficult to determine accurately and depending on the history of the specimen. As there is little drawing out there seems to be no important objection to using the point of fracture.

5. *The Method of Loading.*—The most important practical example of combined stresses in a ductile material is that of combined bending and twisting. On this account the tests on steel were made with this kind of loading, the resources of the writer allowing this to be done. Brittle materials would not be subjected in practice to this loading, but it is suitable for determining the conditions which accompany fracture, and it is better for this purpose than that which occurs more frequently. By using the same apparatus the results form a continuation of those obtained by the previous tests, and may be more easily compared with them.

6. *Apparatus and Specimens.*—The apparatus was described in detail in the previous paper, it being sufficient to state here that the bar was constrained at one end against twisting, but was free to take its natural slope under the bending load. At the other end it rested on rollers which formed the support and offered a minimum resistance to twisting. The bending load was applied directly at the centre, and the deflexion measured one inch from this point. The twist was measured at four points along the length of the bar, at three of these by means of mirrors attached to the bar and an arrangement similar to that used with galvanometers, and at the other by a pointer attached to the bar moving over a fixed graduated circle. A wooden pulley fitted over the squared end of the bar, and weights attached to wire ropes supplied the torque. The bars were of cast iron, as uniform as possible, machined all over, being turned to $\frac{3}{4}$ inch diameter and having squared ends. The distance between

the bending supports was 30 inches, and the diameter of the wooden pulley was 29½ inches.

7. *The Tests Grouped.*—The tests may be divided into three groups: the first including those with one kind of loading only; the second those in which a definite fraction of the breaking torque was applied and bending load added to fracture; and in the third a given bending load was first put on and the torque increased until the bar was broken. By plotting the results they checked each other, and tests which did not appear to fit well were repeated. In this way variations due to differences in the material, which are sure to occur even with the most carefully selected specimens, were eliminated so far as possible.

8. *Making the Test.*—During the initial stages the weights added were large, but when the critical load was approached the increments were decreased to one or two pounds. In each case weights were directly applied. After each weight was added the wooden pulley was gently tapped to minimise the constraint due to friction. It may be mentioned that the forces due to friction were measured but found to be negligible, and were therefore neglected when working out the results.

9. *Results of the Tests.*—

Group of Test.	Number of Test.	Bending Moment.	Torque.	Tensile Stress due to Bending.	Shear Stress due to Twisting.	Maximum principal Stress.	Minimum principal Stress.	Maximum Shear.	Remarks.
1.	1	2080	0	50550	0	50550	0	25275	{ Not broken.
	2	0	2700	0	32800	32800	-32800	32800	
	10	0	3140	0	38200	38200	-38200	38200	
	11	2152	0	52300	0	52300	0	26150	
2.	3	2154	681	52350	8270	53505	-1245	27420	{ Not broken.
	4	1957	1363	47550	16550	52695	-5145	28920	
	5	1545	2045	37600	24830	49960	-12360	31160	
3.	6	525	3110	...	...	...	...	...	{ Not broken.
	6	1125	2760	27330	33570	49860	-22540	36200	
	7	1042	2550	25360	31000	46180	-20820	33500	
	8	1568	2136	38300	25950	51250	-12950	32100	
	9	105	2960	2560	36000	37300	-34700	36000	

The stresses are calculated on the assumption that there was no yield.

10. *The Angles of Fracture.*—The fractures were so interesting that a diagram is given, showing the angles (fig. 1)

Fig. 1.

and photographs of the original bars (Pl. IX. fig. 2). The couples applied were left-handed. The part making a small angle with the axis invariably came under the knife-edge. When measuring the angles an attempt was made to determine if in any given bar the slope varied with the position referred to the bending stresses, it being thought possible that it might be different at the bottom of the bar, which was under tension, to that at the neutral axis. No difference could be detected. In certain cases the angle at the bottom was the greater, but in others the smaller. The slight irregularities in the fracture prevent a very accurate measurement over a small length.

11. *Examination of the Results.*—It will be noticed that

the maximum principal stress and maximum shear stress both vary considerably, even after allowing that certain bars were probably exceptionally strong. In fig. 3 the bending and twisting moments are plotted together, showing the relation between these and the differences due to experimental errors and inequalities in the bars. The points lie

Fig. 3.

fairly well on an ellipse. A similar result was obtained with steel bars, the conclusion being that the maximum shear stress was approximately constant, but varied slightly on account of differences in the shear strength in different directions. In that case the bending moment was greater

than the twisting moment, 2660 and 2400 lbs. ins. Here the twisting moment is the greater, and the difference is so large that it could not be explained as above without strong supporting evidence. It was therefore necessary to examine the angles of fracture to find if they afforded any clue to the conditions determining fracture.

12. *The Angles of Fracture.*—The main part of the fracture appears at first sight to be along the plane of principal stress. Considering the bottom of the bar, which is in tension, then facing the axis (fig. 4) gives the distribution of stress

Fig. 4.

t being the tensile stress due to bending, s the shear stress due to twisting, and p the maximum principal stress, then

$$t + s \cot \theta = p.$$

$$s = p \cot \theta.$$

$$t/s \cot \theta + \cot^2 \theta = 1.$$

$$\cot \theta = -t/2s \pm \sqrt{1 + t^2/4s^2}.$$

Using this to give the angles of the planes of greatest principal stress, the following values were obtained. The measured angles are also given for comparison.

Number of Test.	Measured Angle to Axis.	Calculated Angle to Axis.
	Degrees.	Degrees.
1.....	90	90
2.....	56	45
3.....	77.5	81
4.....	69.5	72.5
5.....	64.5	63.5
6.....	58	56
7.....	57.5	56
8.....	60.5	63

The measured angles of fracture and those calculated for the planes of principal stress agree so well, considering the difficulty of accurate measurement, that it is justifiable to state that the bars fractured along the planes of greatest principal stress. But this quantity is far from constant, Sec. 9. The angles of the part of the fracture roughly parallel to the axis are not so regular. The writer has not been able to identify these inclinations with those of any

Fig. 5.

other important plane. It seems that this part has no special significance, and merely serves to join the ends of the spiral; nevertheless the steady decrease of the angles with increase of torque, seen in tests 3, 4, and 5, leaves the impression that it is not accidental. When cast iron is tested in tension it fractures normal to the axis, and in compression across planes making about 35 degrees with the axis. So in tension and torsion fracture takes place across the plane of greatest stress, and in compression apparently by shearing modified by friction. This seems to afford no help towards arriving at any conclusion as to the conditions holding here. The

value of the coefficient of friction corresponding to the angle of fracture in compression is about 0.35. In fig. 5 (p. 459) the maximum shear and tension across the plane have been plotted, and there is a rough indication that the line $S + 0.35 T = \text{constant}$ approximates to the points. There is a decided indication that the maximum shear decreases with an increase of tension across the plane. This could be explained if the straight part of the fracture at the top of the bar corresponded to the plane of maximum shear modified by friction, as then it would appear that this was the seat of the break. But there is no such coincidence of the planes.

13. *Further Examination of the Results.*—Either the maximum stress or greatest elongation being constant would account for the fracture coinciding with the principal plane. The maximum principal stress varied considerably, far too much to say it was even approximately constant. The only quantity which remains is the greatest elongation.

One principal stress is a tension and the other a compression. Representing Poisson's ratio by m , t being the tensile stress due to bending and s the shear stress due to twisting, if the greatest elongation is constant,

$$T/2 + \sqrt{S^2 + T^2/4} - m(T/2 - \sqrt{S^2 + T^2/4}) = \text{constant},$$

or

$$(1-m)T/2 + (1+m)\sqrt{S^2 + T^2/4} = \text{constant}.$$

$T/2$ corresponds to T , and $\sqrt{S^2 + T^2/4}$ to S of fig. 5. We may therefore write this as

$$(1-m)T + (1+m)S = \text{constant},$$

or

$$S + \frac{1-m}{1+m}T = \text{constant}.$$

The line which fits the points best is

$$S + 0.45T = \text{constant}.$$

Thus

$$\frac{1-m}{1+m} = .45; \quad m = 0.38.$$

This would be nearly correct for compression, but it is too high for tension. In addition to this it will be seen that such a line is a very poor approximation to the results. If the

maximum and minimum principal stresses are plotted it is found that no such relation holds. The specimens appeared to twist abnormally, the modulus of rigidity being 530,000 lbs./sq. in. There was no relation between the measured twist and deflexion.

14. *The Material assumed to yield.*—So far no explanation has been found to fit the results. The stresses have been calculated on the assumption that Hooke's law was obeyed. No doubt the material yields, but the extent of the yield is doubtful. It has been thought well to tabulate the stresses obtained when it is assumed that the material yields sufficiently to cause the stresses directly due to bending and twisting to be uniform. To convert the tensile stress it is necessary to multiply by $3\pi/16$, and the shear stress by $3/4$. The results are tabulated :—

Test.	Tensile Stress due to Bending.	Shear Stress due to Twisting.	Maximum principal Stress.	Minimum principal Stress.	Maximum Shear.
1 L	29800	0	29800	0	14900
2 L	0	24600	24600	-24600	24600
10 H	0	28650	28650	-28650	28650
11 L	31000	0	31000	0	15500
3 C	30870	6200	32365	-1495	16930
4 C	28040	12400	32730	-4690	18710
5 L	22180	18630	32780	-10600	21690
6 H	16110	25170	34055	-17945	26000
7 L	14950	23230	31900	-16900	24400
8 H	22600	19460	33860	-11200	22500
9 C	1510	27000	27750	-26250	27000

From this it will be seen that the shear stress varies enormously, but the maximum principal stress is nearly constant. The letter against the number of the test indicates whether the load was low, high, or correct, according to fig. 5. As before the units are pounds and inches. The results which are sensibly out of agreement are those in which there was little or no bending. If the principal stresses are plotted against each other no relation can be traced, so there is no explanation of this fact; nor does it appear that the maximum strain was constant. However, keeping in view the low value found for the modulus of

rigidity, this method of checking the maximum strain is open to suspicion. Having new values for the stresses, it is necessary to find if the angles calculated for the principal planes agree with those measured from the fractures. These are given in the table.

Number of Test.	Measured Angle to Axis.	Calculated Angle to Axis.
	Degrees.	Degrees.
1.....	90	90
2.....	56	45
3.....	77.5	79
4.....	69.5	69.2
5.....	64.5	60.3
6.....	58	54.1
7.....	57.5	54.1
8.....	60.5	60

These agree quite as well as those given earlier, and a uniform distribution of stress would account for the constant angle of the fracture.

15. *Conclusions.*—There seems to be no explanation of the results unless it is assumed that the material yields sufficiently to allow a uniform distribution of the stresses directly due to bending and twisting. Allowing this, the principal stress is remarkably constant, especially if differences in the material are allowed for, but the values are slightly low in cases in which there was little or no bending load. The bars fractured across the planes of principal stress, the angles being constant around the bar, a result which supports the view as to yield. Cast iron undoubtedly fractures along the plane of maximum principal stress under tension and torsion, but in compression the plane is that across which $S + 0.35T = \text{constant}$. Apparently the maximum principal stress is constant when it is a tension, but in compression the only possible method of breaking is by shearing. It does not seem possible to condense these two results into one statement.

In conclusion, the writer wishes to thank Lord Blythwood for placing the resources of the laboratory at his disposal for the purpose of the tests.

DISCUSSION.

The SECRETARY read a letter from Dr. MORROW stating that the author had attempted to find a criterion of rupture by examining what he has referred to as the maximum shear and maximum principal stresses in his specimens. To evaluate these stresses, he uses results of the theory of the bending and torsion of perfectly elastic bodies. The values which he thus arrives at for the stresses are far higher than can exist in the material, and are purely fictitious. If a theory of rupture were formulated by this means it would be valueless. The author meets with more success when he treats the stresses due to bending and the shear stress due to twisting as uniform over the section when fracture occurs. This is a convenient guess at what actually takes place; but it may be nearer the truth than the assumption that the rules for perfectly elastic material may be applied to ordinary metal specimens when stressed to the breaking point. With this estimate of the stresses, the author finds that for brittle materials rupture is characterized by a limiting value of the principal stress. The writer had shown, in connexion with Mr. Scoble's previous paper, that for ductile materials also, if the stresses due to bending are estimated in a certain way, rupture may be determined by the value of the maximum principal stress. These papers show the great need of a more reasonable method for finding the stresses set up in a metal bar by bending action. More experiments of the kind described in the paper are required.

Mr. L. BAIRSTOW said that the values of the "maximum principal stress" given in the first table in the paper were very high as compared with the ordinary breaking stress in tension, whilst the corresponding values in the later table were much nearer those usually obtained. It was therefore likely that the plastic deformation of the cast iron had a considerable effect on the distribution of the stresses. It would have been useful if the actual tensile strength had been given, for comparison with the principal stress, for the angles can be approximated to by calculation and this suggests that failure occurs in tension. That the planes of

fracture could be calculated approximately was not surprising, as plastic deformation has less effect on this quantity than on the actual values of the stresses. The discrepancies in the values of the maximum stress had been explained by the author on the assumption that the stresses throughout the material had become uniform, but he did not think this assumption could be made, as the outside must always have the greatest stress quite up to the point of fracture. A possible source of explanation depended upon the different distribution of the stresses due to torsion and those due to bending. In torsion the whole circumference of the bar is in the state of greatest stress and therefore takes the first permanent set, whilst in bending the maximum stresses occur at the top and bottom over a much more restricted area. The ratio of the stress due to torsion to the torque producing it is greater than the ratio of the stress due to bending and the bending moment. If, therefore, both stresses are over-estimated, those due to bending will have a greater error than those due to torsion. Applying this to the actual experiments, it is seen that the principal stresses, calculated on the assumption of Hooke's law, increase as the bending moment increases or as the torque decreases. It seems probable that the cast-iron bars failed in tension, and there is no reason to suppose that the maximum principal stress is different from the ultimate tensile stress of the material.

MR. ROLLO APPELYARD expressed his interest in the paper, and said he appreciated the difficulties of the subject. Some years ago he had made experiments on the hardness of steel, and he exhibited at the meeting several pieces of the fractured material.

THE CHAIRMAN stated that Mr. Scoble did not favour the idea of internal friction, but the results which he had given in the paper did not contradict that theory.

DR. CHREE, being unable to attend the meeting, wrote to say that what the author described, § 3, as the third theory of elastic strength was apparently that ordinarily known to mathematicians, following Sir G. H. Darwin, as the *maximum stress difference* theory; and in his opinion the stress difference was a more convenient quantity than the shearing stress or

strain on which to concentrate attention. The simplest way of describing the stresses at a point was by means of the stress quadric, and the natural thing was to refer the quadric to its principal axes. In the case of simple traction, the three theories were really identical. Supposing T the greatest permissible simple traction, and E Young's modulus, then in the general case if P_1, P_2, P_3 were the three principal stresses at a point in descending (algebraical) order, T was a superior limit on the greatest stress theory to P_1 , and in the stress difference theory to $P_1 - P_3$; while on the remaining theory T/E was a superior limit to the greatest strain. The interrelationships of the three theories were thus very readily seen.

A shearing stress S in two perpendicular planes, both containing say the x axis, gave for the stress quadric $2Syz=1$; which when referred to its principal axes became $S(y^2 - z^2)=1$, showing at once that $2S$ was the maximum stress difference. If the case treated in the author's § 12 were approached from the standpoint of the stress quadric, one obtained at once in the author's notation (in that paragraph) $\tan 2\theta = -2s/t$. This result was equivalent to that given by the author, but much simpler for computation.

He, Dr. Chree, had pointed out many years ago* that none of the three theories mentioned could be universally true. To show this for the stress difference and greatest strain theories, one had only to consider a solid sphere under *uniform* normal forces. Whether these forces were pressures or tensions, the three principal stresses at every point were equal, and hence the stress difference (or shear) zero; if the forces were pressures the strains were all negative. The greatest strain theory thus set no limit to the intensity of the forces when pressures; while the maximum stress difference set no limit whether they were pressures or tensions. It was obvious, however, that the smallness of the strains and the linearity of the relationship between strains and stresses postulated by the ordinary mathematical theory were incompatible with indefinite reduction or increase in the radius of the sphere.

* Phil. Mag. Sept. 1891, pp. 242 &c.

In view of Kelvin & Tait's adoption of m to denote one of the two elastic constants of isotropy, an example followed in many mathematical papers, the use of that letter to denote Poisson's ratio was to be deprecated. The letters in most common use for the purpose were η and σ . The author did not seem to give Poisson's ratio for his material; but his remarks in § 13 seemed to suggest that the values were different for tension and compression. If this were the case, the material did not possess the properties usually assigned to elastic solids. As a matter of fact, several authorities * had described experiments which seemed to show that cast iron did not follow Hooke's law sufficiently closely to be treated mathematically as an elastic solid.

It was difficult to see how any inference could be drawn as to the theories of strength mentioned from the shape of the fractured surface. Even supposing the mathematical equations to hold right up to rupture, it should be remembered that the stresses before rupture varied over the cross section, and that the critical stress, or strain, was first reached at a single point or along a single line. The fracture might be sudden, but would not be strictly instantaneous. On its commencement, at say a definite point on the surface of the bar—where the stresses and strains were greatest,—the instant the material parted company there, a redistribution of stress and strain took place through the still intact material. The commencement of a rift altered the shape of the cross section, and the distribution of stress and strain existing before rupture commenced threw no direct light on what the stresses and strains were at each point on the surface of fracture at the instant when the material in its immediate vicinity parted company. The subject seemed to call for more consideration.

* Todhunter & Pearson's 'History of Elasticity,' vol. i. art. [1411] and elsewhere.