MR RUSSELL'S OBJECTIONS TO FREGE'S ANALYSIS OF PROPOSITIONS.¹

ACCORDING to Frege² what a Categorical Affirmative Proposition asserts is: Identity of denotation (or Application—*Bedeutung*) with difference of intension (or connotation or 'meaning'—*Sinn*).—Hence we may say *a is b*, which means *a is identical with b* (identical in denotation that is, for it is clearly *not* identical in connotation or intension). But we cannot say *a is a* because here difference of connotation (*Sinn*) has vanished. And we cannot say *a is not-a* because here identity of denotation is impossible. Mr. Russell objects (p. 485) to this theory of Frege's that it will not solve the following three puzzles

"(1) If *a* is identical with *b*, whatever is true of the one is true of the other, and either may be substituted for the other in any proposition without altering the truth or falsehood of that proposition. Now George IV. wished to know whether Scott was the author of *Waverley*; and in fact Scott *was* the author of *Waverley*. Hence we may substitute Scott for the author of '*Waverley*,' and thereby prove that George IV. wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe."

When George IV. asked whether Scott was the author of *Waverley*, what he wanted to know was, whether the intension ('meaning,' connotation) of *Author of "Waverley"* could be assigned to Scott—i.e., whether identity of denotation could be asserted between Scott and *Author of "Waverley"*. The "first gentleman of Europe" did not want to know whether Scott was Scott—this would have been perhaps more in the style of "Farmer George," with his, "What, what, Young, Young, dead, dead," and his sense of the inexplicability of the fact of the apple being inside the dough in an apple dumpling.

No doubt "if *a* is identical [in denotation that is] with *b*," whatever is true of the thing denoted by *a* is true of the same thing when denoted by *b*³—with the obvious reservation that *a is a* does

¹ MIND, 1906, pp. 483, 485, etc.

² See also my *Elements of Logic*, § VI. p. 46, etc., and § XV. (T. & T. Clark, Edinburgh, 1890), my *General Logic*, Part I. § III., etc. (March, 1892), and MIND, 1893, p. 441, etc.

³ I am not quite sure what is meant by "whatever is true of the one is true of the other". Do *the one* and *the other* mean *the one thing* and *the other thing* (as Jevons has it)? Or do *one* and *other* refer to the symbols *a* and *b*? Neither alternative is satisfactory.

not convey the information that *a* has the intension (or connotation) *b*. If we admit *a* is *a* at all, we must either (1) regard the copula as having a different force in this case from what it has in *a* is *b*, and must point out what this is, or (2) we must hold that in *a* is *a* it is only *denotational* identity of *a* with itself which is asserted—and it seems hard to understand why any one who ‘asserts’ *a* is *a* with this intent, should ever want, or indeed be able, to assert anything else.

On these lines we reach the entirely hopeless view that any

$$S \text{ is } P = \begin{cases} S \text{ is } S \\ P \text{ is } P \\ S \text{ is not } P \end{cases}$$

as Lotze does—i.e., we divest the Categorical Affirmative of the form *S* is *P* of coherence and of meaning. It is, I think, of the essence of Frege's theory (and mine) that *S* is *S* (*a* is *a*) is valueless as a ‘Law’ of Thought or of Logic. That it is so is perhaps sufficiently shown in the ill-success of Lotze's determined, elaborate, protracted, yet absolutely futile effort to exhibit it as a fundamental logical principle of the Logic which uses propositions of the form *S* is *P*.

“(2) By the law of excluded middle, either ‘*A* is *B*’ or ‘*A* is not *B*’ must be true. Hence either ‘the present King of France is bald’ or ‘the present King of France is not bald’ must be true. Yet if we enumerated the existing things that are bald and then the existing things that are not bald, we should not find the present King of France in either list [i.e., The present King of France is not an existing thing that is bald nor an existing thing that is not bald, we cannot ‘identify’ the ‘present King of France’ with any ‘existing’ thing whether bald or not bald]. Hegelians, who love a synthesis, will probably conclude that he wears a wig.”

Certainly nothing of which Existence is predicated can without contradiction be denotationally identified with anything of which Existence (in the same sense) is denied. But if the *A* in *A* is *B* is *the present (A.D. 1909) King of France*, then that *A* is *A* not-*A*, and having admitted it, there can be no further difficulty in the way of asserting of it contradictory attributes—since the complex denoting term *A* not-*A* itself has ‘contradictory’ connotation, and includes elements which together exhaust the universe.

If *the present King of France* is not (1) a self-contradictory term, then (2) it is assigned to some sphere of predication (“universe of discourse”) in which it occurs—which means that the kind of ‘being’ or ‘existence,’¹ is fixed by the context, by certain further determinations of the subject-term; and if this is sufficiently done with reference to any accepted ‘universe’—e.g., of past time, or of supposition, fiction, or prophecy—there need be no difficulty in choosing between any *B* and not-*B*.

“(3) Consider the proposition ‘*A* differs from *B*’. If this is

¹ I use *Existence* and *Being* as equivalent.

true, there is a difference between A and B, which fact may be expressed in the form 'the difference between A and B subsists'. But if it is false that A differs from B, then there is no difference between A and B, which fact may be expressed in the form 'the difference between A and B does not subsist'. But how can a non-entity be the subject of the proposition? 'I think, therefore I am' is no more evident than 'I am the subject of a proposition, therefore I am,' provided 'I am' is taken to assert subsistence or being,¹ not existence. Hence, it would appear, it must always be self-contradictory to deny the being of anything; but we have seen, in connexion with Meinong, that to admit being also sometimes leads to contradictions. Thus if A and B do not differ, to suppose either that there *is*,² or that there is not such an object as 'the difference between A and B' seems equally impossible."

Considerations similar to those applied in (2) seem to apply in the case of the difficulties raised in (3)—it does no doubt appear self-contradictory to deny *all* 'being' of anything of which we think and speak—we must at least *suppose* a thing or object (with certain attributes), in order to be able to talk about it—even to deny it 'existence' or occurrence in some particular sphere—that is, to deny its possession of certain other attributes. A non-entity for thought, is simply that which is not thought about.

We can think of nothing, speak of nothing, without postulating or assuming both application (or denotation) and intension (*Bedeutung* and *Sinn*) in Frege's sense—without these two elements, significant assertion is always and for ever impossible. Whatever we think of or speak of must be thought of as something and as some sort of something—and every term which is used as Subject or Predicate in a Proposition must have both denotation and intension (as Frege, I believe, holds).

If A and B do not differ, "the difference between A and B" of which we think and speak, is still an object with *Sinn* and *Bedeutung*, with 'being' or 'existence,' and attributes, assumed hypothetically—otherwise how could we deny it? And of course we may quite reasonably decide that A and B do not differ, when A and B have just as little Real Existence (however that may be ascertained) as the 'non-existent' difference between them.

"Is there a difference between A and B?" I ask—*e.g.*, between Fair Trade and Free Trade, between Usefulness and Truth, between those two colour-patterns, between this material and that, between my faith and yours?—I ask for information.—The question: Is there *a difference between A and B*? assumes the possibility or thinkability of such a difference, even though A and B be devoid of Real Existence, and has applicability and intension

¹ I use these as synonyms. [Mr. Russell's footnote.]

² What does Mr. Russell mean by *is*, and what is his distinction between Being (or Subsistence) and Existence?

just as truly as *the shining of the sun at this moment* (when it is shining), *the convenience of the house I am planning to build next summer*, *the best measure of social reform of the next Parliament*. The difference between these is not that in some cases there is denotation and connotation (or intension) and not in others, but that the things denoted and connoted belong to different spheres, regions, contexts, or 'Universes of Discourse'. A great deal of our keenest and most absorbing interest is about 'things' of the 'existence' and (sometimes) of the character of which we are doubtful—but in which we could take no interest at all unless they had *Sinn* and *Bedeutung*, what-ness and that-ness, in some region, and as much Reality as that implies. Dr. Cook's finding of the North Pole does not differ from Sir Ernest Shackleton's approximation to the South Pole in *not* having 'denotation' and intension while Shackleton's achievement has both. Dr. Cook's 'discovery' has them just as much, otherwise we could not have thought and talked about it as we have.

I pass to the difficulties which Mr. Russell finds in the relation between 'Meaning' (Frege's *Sinn*, my intension or connotation) and Denotation (Frege's *Bedeutung*, my denotation or application or applicability), pp. 485, etc.

The difficulties are set forth as follows: "When we wish to speak about the *meaning* of a denoting phrase, as opposed to its *denotation*, the natural mode of doing so is by inverted commas. Thus we say:—

The centre of mass of the Solar System is a point, not a denoting complex;

'The centre of mass of the Solar System' is a denoting complex, not a point.

Or again:—

The first line of Gray's 'Elegy' states a proposition.

'The first line of Gray's "Elegy"' does not state a proposition.

Thus taking any denoting phrase, say C, we wish to consider the relation between C and 'C,' where the difference of the two is of the kind exemplified in the above two instances."

"We say, to begin with, that when C occurs it is the *denotation* that we are speaking about; but when 'C' occurs, it is the *meaning*. Now the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation. But the difficulty which confronts us is that we cannot succeed in *both* preserving the connexion of meaning and denotation *and* preventing them from being one and the same¹; also that the meaning cannot be got at except by means of denoting phrases. This happens as follows:—

¹ What precisely is the meaning of 'one and the same'?

"The one phrase C was to have both meaning and denotation. But if we speak of 'the meaning of C' that gives us the meaning (if any) of the denotation."

Now using Denotation as equivalent to Application (Frege's *Bedeutung*) and Intension or Connotation (Frege's *Sinn*) as equivalent to 'Meaning,' it seems to me that the difficulty here may be quite simply explained.

First, every term used as Subject or Predicate of a proposition has both Denotation and Intension. And what the term *quid* denotative denotes, is the thing to which it (the Term) applies. And what the Term *quid* connotative connotes, is those attributes of the thing denoted which are 'meant' by the Term, and which would be given in a Definition of the Term (or description of the thing).

No categorical assertion of the form *S is P* is possible unless *S* and *P* have Denotation and Connotation (or Intension)—identity of Denotation and diversity of Connotation. And if *S* is isolated from its Predicate (which has identical denotation with *S*) or *P* from its Subject, *S* (or *P*) still has both Denotation and Connotation (*Bedeutung* and *Sinn*). That "the meaning [Connotation] cannot be got at except by means of denoting phrases" is no doubt true, but this does not involve any difficulty, for every 'phrase' (word or words) that can be used as a term in a proposition *must* have Denotation (Application, *Bedeutung*).

But to talk of "the meaning denoting the denotation" seems to me a very awkward way of expressing the obvious fact that it is often by means of the connotation (or intension) of a term that we are enabled to see where it should be applied—to find its denotation (I think this must be Mr. Russell's meaning)—and it is surely not the case that "we cannot succeed in *both* preserving the connection of meaning and denotation, *and* preventing them from being one and the same [= ?]" nor that "if we speak of the 'meaning [connotation] of C' that gives us the meaning (if any) of the denotation". If *C* is a Term, it has Connotation (or Intension) and Denotation, and the connotation of *C* is the connotation (not of the denotation of *C* but) of the Term which *quid* denotative, denotes the thing to which *C* applies—the thing of which *C* is the name; the Connotation of *C* implies those attributes of *the thing called C* on account of the possession of which it is called *C*.

Thus:—

(1) The first line of Gray's Elegy (*S*) is identical in denotation with the line which runs: The curfew tolls etc. (*P*).

This line (*S*) is identical in denotation with a proposition (*P*).

(2) The words: *The first line of Gray's Elegy* connote a line occurring in a certain position in a well-known poem.

(3) The words. *The first line of Gray's Elegy* are :—
 { identical in denotation with a Term.
 { not identical in denotation with a proposition.

(1) The centre of mass of the Solar System is a point (*is = denotes, or is identical in denotation with*).

(2) The words: *The centre of mass of the Solar System* connote the place in the system which etc.

(3) The words: *The centre of mass of the Solar System* are a denoting-complex (= Complex-denoting Term).

These examples thus examined seem to me to bring out the strength and inevitability of Frege's analysis—in every case we have denotational identity of Subject and Predicate, in intensional (connotational) diversity, though the things identified are not the same in every case. We are concerned with different Subjects and different Predicates.

In (1) the propositions are taken in their ordinary natural sense.

In (2) the Subject-Term is isolated from its original, intensionally-

diverse, Predicate, for the purpose of determining, by a definitional Predicate, the Subject-term's connotation, intension, or description.

In (3) the Predicate states the logical character of the words composing the Subject-Term.

Compare: (1) the first book on the shelf is (= denotes) Aristotle's *Ethics*;

(2) "The first book on the shelf" connotes *the book which occupies the definite position described*.

(3) "The first book on the shelf" is a complex denoting Term.

It conduces to clearness when in any of the above cases we prefix "The words" to the phrase in quotation marks. The quotation marks simply indicate that the words which they include have to be taken as a name of which the *connotation* (or description, grammatical character, etc.) is predicated, and are an elliptical device.

And all the time (as has been pointed out) our Subject-Term, whatever it is, has both Denotation and Connotation—and so has our Predicate-Term, whatever it is; and our *connotes* is resolvable into *is connoting*—and so on. Whatever conundrums are stated or solved, it is impossible to state them or their solutions in any other way.

The denoting-complex (complex-denoting-term) "the first line of Gray's *Elegy*" denotes the line which consists of the words "The curfew tolls the knell of parting day"—and this denoting-complex denotes the line by means of the connotation of the complex, which indicates a certain place of occurrence. It is this connotation of the phrase: *The first line of Gray's Elegy* which is given when we say: "The first line of Gray's *Elegy*" connotes ('means') the line which occurs at the beginning of a certain poem known under the title given.

The *meaning* (propositional force) of the line (which is a proposition) is, the assigning to that which is denoted by *curfew* of the connotation of the predicate: *tolling the knell of parting day* (denotation of Subject and Predicate identical).

Or take the third case: The first book on the shelf is Aristotle's *Ethics*.—The 'denoting-complex' *the first book on the shelf* denotes the volume which has the given title—it denotes (or applies to) the book in virtue of its (the denoting complex's) connotation, which connotation indicates the position occupied by the book. The connotation of this 'denoting complex' is given by saying: 'The first book on the shelf' connotes the book which occupies the position described. Until I know what the phrase 'the first book on the shelf' connotes ('means'), I cannot use that phrase as *denoting* the book referred to.

And 'the first book on the shelf' denotes Aristotle's *Ethics*:—and the connotation of Aristotle's *Ethics* is given by an explanation of *Aristotle* and of *Ethics*.

There is one thing or object denoted (a copy of Aristotle's *Ethics*):

it is referred to by two different phrases, both of which denote that one volume, but denote it by means of different connotations:—

(1) The first book on the shelf.

(2) Aristotle's *Ethics*.

Both (1) and (2) are complex denoting terms.—When “The first book on the shelf” is called a “denoting complex” it is not the connotation of the Term “The first book, etc.” (i.e. those attributes of the object denoted by the term on account of which the term is applied to it) which is given, but a description of the logical character of the *Term itself*.

It may be noted that in Mr. Russell's use of *Denoting* (or *Denotation*), the term is confined to words or phrases which include some general name—some name having (in Mill's use of the terms) both Denotation and Connotation; and it seems to him to involve difficulty that connotations should be denoted. But in Frege's view as I understand it (and in mine) whatever can be talked about must be denoted—and we are here expressly talking about connotations. As Mr. Russell says: “Frege distinguishes the *two* elements of meaning (connotation or intension), and denotation *on everywhere*” (italics mine).

The use of the word *meaning* as equivalent sometimes to *connotation* and sometimes to the *force or significance of a proposition* naturally leads to some want of clearness as between (1) Connotation (or intension) of a term or phrase and (2) Propositional force of an Assertion. It is impossible to detach the ‘meaning’ of a proposition from the denotation of its terms.

Further, according to Mr. Russell, we do not know whether the denoting phrases which we obtain from propositions denote entities or not, until we know whether the propositions are true (p. 490).

This surely is a very awkward position, for how are we to know (on Mr. Russell's principles see p. 485) whether a proposition is true until we know whether its terms denote actual ‘entities’ or not? Is it not indispensable to have some analysis of propositions of the form *S is P* which can be applied to *all* propositions of that form, and some theory of denotation which can be applied to *all* denotative terms, at whatever stage of investigation or certainty? I think Frege's theory does this, furnishing an analysis of propositions which seems to meet all legitimate requirements, elucidating Education and Deduction as well as Assertion. Does Mr. Russell hold that no analysis of propositions or theory of predication is acceptable or applicable unless it provides a Criterion of Truth and of ‘Real Existence’? Or must the truth of all true propositions have been established before any theory of import of propositions can be arrived at?

E. E. C. JONES.