MATHEMATICS IN THE ELEMENTARY SCHOOLS.¹

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You may wonder that a high school teacher should have been chosen to speak on elementary school work, and the fact that such a teacher was found to accept the invitation is perhaps only another manifestation of the inalienable right of the higher school to assume a wise attitude toward the work of the lower. However this may be the subject of the elementary school curriculum is more important now than it has ever been before; and this relative importance is likely to increase rather than diminish so long as the present economic and social conditions exist. It is undoubtedly true that the influx of foreign peoples from the lower classes of the nations from which they come has materially affected the average of mental and moral development of the children in our schools. The crowding of people together in cities and large towns, the tendency toward the apartment house or flat, both have tended to change materially the average conditions of life and have brought their results in the enriched course which has received so much criticism. It has been said that the schools are now teaching what should have been taught in the homes, and this is no doubt true; but the fact is that certain things that were taught in the home a generation ago, or less, are not so taught now and it is a question whether they shall go altogether untaught or receive some share of attention in school. Personally I believe that the elementary school curriculum of the present day is an attempt to make formal education what it should be, a supplement to other educational forces that are less completely under our control, with a view of bringing about a more complete development of the mental, moral, and physical characteristics of the pupils.

Another condition that is adding to the importance of the work of the lower grades is the fact that the number of pupils who leave school as soon as the law will allow is continually on the increase. I have no data on the subject but I believe it is true that the percentage of pupils who leave school at an early age is greater now than it was five years ago. I do not see how this can fail to be true. None of us here are rich in this world's goods, perhaps, but we all know that the purchas-

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ing power of a dollar is decreasing to an extent that is alarming to all and distressing to the poorer classes of society. Consequently, it is certain that more and more people are living close to their income, and more pupils are leaving school from force of necessity than ever before. The elementary school, therefore, is their only hope, until part time and continuation schools have reached a higher stage of development than they have at the present time.

However, this does not help us to solve our problems, but only serves to emphasize the need of making every moment of time in the lower grades count for something worth while in preparing the great body of pupils to take their places in the world and lead lives that are something more than mere existence. I do not believe that this necessarily means that the lower grade work must be made purely utilitarian. This would be an unfortunate starting point from which to plan out the subject-matter of any school course, but I do think that some of the work we are now attempting to do is too difficult for the pupils at the time it is attempted, and that the emphasis should be placed at different points than is the practice at the present time so far as I am acquainted with that practice.

I have referred to the enriched course in the elementary schools and to the fact that this enrichment has received bitter criticism from many quarters, but I seriously doubt if any superintendent would dare to brave the storm that would be produced by attempting to return to the old curriculum, even if he thought it sound educational policy. These new subjects are here to stay and we may as well face this fact squarely. But there is another fact brought home to us on every possible occasion by all sorts of people, and that is that the results of our elementary school work are not satisfactory. The president of a bank said recently, at a meeting of school men, that he had yet to find the grammar school graduate who could read intelligently, write legibly, spell correctly, or add a column of figures with any degree of accuracy, and although this statement of the case may be somewhat exaggerated it is typical of the opinion held by a large number of business men. High school teachers, I am obliged to confess, are sometimes equally outspoken in their criticism and offer as an excuse for their own failures the fact that pupils are not properly prepared, as though preparation for the high school were the object of the elementary school course. Sooner or later the community is

going to rise in arms against the wholesale failure of pupils in the early part of the high school course and insist that teachers shall base their requirements on the ability of pupils to meet them and not upon their own standards of what these requirements ought to be.

But to return to the point about which I was speaking, it is a conservative estimate that twenty per cent of the time spent in the elementary schools is spent on music, drawing, manual training, sewing, cooking, nature study, physical exercises, and similar frills, while the total number of hours spent in school is no greater than before these subjects were introduced. It might be mentioned also in this connection that the teaching of the English language to foreign-born children merely as a vehicle of communication is consuming considerable time at the expense of other subjects, and that the average of intelligence among pupils in our schools is undoubtedly lower than it was a few years ago.

We have, then, the problem of trying to do more teaching in less time, and there is a limit to the possibility of so doing without seriously reducing the efficiency of the teaching. It is the old question of two bodies occupying the same place at the same time.

Now under the conditions which I have just described one of two things must happen. Either we must reduce the content and improve the teaching of the subjects which we are attempting to teach, or we must expect the results to be less satisfactory than formerly, and it is particularly in regard to arithmetic that I am supposed to speak to-day, although similar suggestions might be made in regard to technical grammar, political geography and history.

I have said that either the content of the course in arithmetic must be reduced or the results of our teaching must suffer, and it requires little argument to convince the average business man that the latter, at least, is an accomplished fact. Improvements and modifications in methods of teaching have not made up for lack of the time which is absolutely essential for drill, and it is probably true that the average boy now does not compare favorably with the average boy of a generation ago in ability to use arithmetic or in accuracy in the mechanical processes involved.

We are brought, then, to the necessity of reducing the con-

tent of the subject as one means of saving time and there is one other suggestion that I wish to make later that would, I think, relieve the situation to a considerable extent. But do not anticipate that I am going to suggest that any person could live a complete and satisfying existence without the ability to work out the length of time it would take a sum of \$87.45 to amount to \$110.15 if placed at compound interest at $5\frac{1}{2}$ per cent or to find the number of square inches in a ten acre lot. Such ability is of little value to the ordinary individual, it is true, but if there is a class of pupils to whom these things are of value, let them be taken up in the high school, when the time so spent will not be taken from other things that the great bulk of the children can ill afford to lose. I have for a long time maintained that it is worse than useless to attempt the kind of algebra that is usually taught in the first year of the high school, and that at least half of the years should be spent on arithmetic, special attention being given to the use of the equation in solving types of problems that are already familiar.

But to return to the subject and to come down to particulars, the modification of the elementary school course in mathematics that I have suggested is that the elementary schools should omit a considerable part of what is now being taught in arithmetic and confine themselves largely to those things that are likely to be of use to the greater number of children who enter the unskilled vocations, and let the high schools take care of the other pupils. If a further study of arithmetic seems desirable for high schools, either for its vocational value or as a preparation for higher mathematics, why not let those schools teach what they will and as they will? This would at least have the advantage of enabling the high schools to specialize to some extent on arithmetic, the pupils in a commercial course putting special emphasis on interest, stocks and bonds, commission, etc., those in the mechanical course on mensuration of surfaces and solids and the use of formulae, while all would bring to the study the interest accompanying the consideration of new material.

But before going into further detail in regard to the content of the course, there is another way in which our time is not used to best advantage, and it seems to me that the greatest possibility of improvement is through a reallocation of time to written and oral work. A little consideration will convince you, I think, of the immensely greater value of the ability to do

simple problems mentally than the more difficult ones with pencil and paper. I have made careful inquiry among all sorts of people in regard to the use of arithmetic in their everyday lives, and find that surprisingly few have occasion from one week's end to another to resort to written arithmetic while all are doing mental problems many times a day. The housewife, for example, is buying her provisions, cloth, etc., and making use of yards and pounds. The carpenter is concerned with feet and inches, the machinist with inches and parts of inches, but the problems involved are for the most part those that can and must be done mentally and quickly. Hence I believe that a great deal of time that we are now spending on written work is wasted and might better be employed in oral drill, on problems of the same sort, perhaps, but involving simpler relations and smaller numbers. Think, for example, of a class spending thirty minutes or more every day on eight or ten examples like the following:

1. Change $\frac{33}{53}$ to 212ths.
2. Reduce to lowest terms $\frac{856}{936}$.
3. Reduce to an integer or mixed number $\frac{64721}{375}$.
4. Add $32\frac{1}{4}$, $18\frac{1}{8}$ and $45\frac{3}{8}$.
5. Multiply $18\frac{3}{8}$ by 37.
6. Divide $\frac{5}{8}$ of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{4}$ of $6\frac{1}{2}$ by $\frac{1}{4}$ of $\frac{3}{8}$ of $\frac{1}{8}$ of $7\frac{1}{2}$.
7. Change 641,558 inches to miles.
8. Change $\frac{3}{8}$ ft. to a fraction of a mile.

I wonder how many oral examples involving the same processes but smaller quantities could be done in the same length of time? If I were to guess I should say at least three times as many. It is not a question here of "what knowledge is worth while," for it is all more or less valuable, but what knowledge is *most* worth while considering that we cannot do all that we might desire.

This, then, is my first specific recommendation—to reduce greatly the amount of time given to written work and to increase the time spent in oral drill correspondingly. Moreover, I would reduce the size of the numbers used in written work and confine them to those of common occurrence. I would use smaller numbers and give more problems rather than obscure the process with numbers that are unnecessarily large. For example, in dealing with fractions I would use as a rule halves, thirds, quarters, eighths, tenths, sixteenths, and thirty-

seconds, avoiding sevenths, elevenths, thirteenths, etc., and restrict all denominators to two figures. It goes without saying that fractions whose denominators end in zero should be treated as decimals whenever possible on account of the saving of time and labor, and all use of fractions should be discouraged for the same reason. The subject of complex fractions should be omitted altogether from the elementary course. In both multiplication and division of fractions it should be considered a serious offense to omit any possible cancellation.

Before leaving the subject of fractions I wish to call your attention to the blessing that would be ours if we did not have to deal with fractions at all, except in mental work, and how nearly this would be the case if our system of denominate numbers were anything but the abomination that it is. I have made it a business for a long time to see that every pupil who comes under my charge thoroughly realizes this fact; and if every public school teacher would do the same it would not be as difficult in the next generation as it seems to be in this to bring about a reform that has already been delayed too long. This bit of instruction I am obliged to give in connection with work in the metric system in the early part of the course in science, and it takes the form of several examples like the following:

1. (a) How far would six tables extend if placed end to end, each table being 3 feet $10\frac{1}{2}$ inches long?
(b) How far would the same tables extend, each one being three meters $10\frac{1}{2}$ centimeters long?
2. (a) Find the weight of 10 cubic inches of water.
(b) Find the weight of 10 cubic centimeters of water.

These examples are done in columns on the same sheet of paper, part (b) of each example being opposite part (a), and it does not take many of them to convince pupils that a decimal system is a great saving of time. I might add, also, that the adoption of such a system in this country would mean a saving in the science course in the high school, where considerable time is always necessary in teaching the equivalents and in making changes from one system to the other. The mathematics of thermometry, for instance, would be entirely unnecessary under the metric system and the term specific gravity would go out of use.

For a long time it was thought that there could be nothing new in regard to teaching the fundamental operations, but the method of additive subtraction which has come into recent use

has practically reduced the number of such operations from four to three. This method of subtraction is probably so familiar to you that it needs no explanation, and consists simply in determining what number added to the subtrahend will give the minuend. For example, the pupil is taught to ask himself what number added to five will give eight, not how much is five from eight.

In teaching decimals I have only to recommend that method of division which consists in moving the decimal point as many places in both dividend and divisor as will make the latter a whole number, writing the quotient above the dividend. This brings the decimal point in the quotient in a vertical line with that of the dividend, and although it is still a mechanical device it has the advantage over the older method of greater simplicity and therefore it is an economizer of teaching time.

A similar method might be employed in the multiplication of decimals but I have never heard it suggested, and it lacks some of the advantages of the method just suggested in connection with division. In multiplying 34.7 by 1.87, for instance, we might change the decimal point in the multiplier in such a way as to make the latter a whole number, at the same time changing the point in the multiplicand just as many places in the opposite direction. The decimal point in the product would then come in a vertical line with that in the multiplicand.

$$\begin{array}{r}
 34.7 \\
 \times 1.87 \\
 \hline
 2389 \\
 2776 \\
 347 \\
 \hline
 64.849
 \end{array}$$

In teaching denominate numbers there has already been a considerable reduction in the content of the subject, but I think there is opportunity for still further pruning. Due perhaps to the unconscious realization of the difficulties of our system, or lack of system, there has been going on in the past few years a process of evolution which is tending to eliminate some of our units of measure altogether, and to change others so that they will be a little less difficult to handle. As illustrations of this let me call your attention to the growing tendency to allow the unit of length called the rod to fall into disuse. Comparatively few people now think in rods, and I for one am obliged to go through the troublesome mental calculation of changing

to feet whenever rods are used. The same is true to a less extent with the yard, and I confess that the distance 100 yards means little to me until I translate it into feet. I am disposed to omit the rod entirely from the table of linear measure and place it in a footnote along with the fathom, furlong, league and other units once in common use. Another unit which is going through the same process is the gill. The dozen and the gross are now not nearly as common as formerly, dozens giving place to tens and gross to hundreds, and if a name were invented for a group of ten this process would no doubt be greatly accelerated. Many commercial articles are now sold in tens instead of dozens, and I am obliged to acknowledge that sometimes there is no appreciable difference in the price to the consumer. In the stationer's table the same movement is going on, the quire of twenty-four sheets giving place to one of twenty-five, and the ream of 480 sheets to one of 500. Articles that were formerly sold in units of dry measure are now sold by weight and the apothecary's pound is being merged into that of the grocer.

In all these steps I am in favor of aiding the process so far as possible and would suggest that little time be spent on those units that I have mentioned, as we are now doing with some others that have practically disappeared.

Mensuration might well be confined to rectangles and triangles. The trapezoid, which only occurs in board measure, can be treated as a rectangle, taking the average width of the board as the width of the rectangle. Carpeting and papering could be omitted from the elementary course and left for the high school, as well as such subjects as longitude and time, present worth and true discount, commission, duties and customs, stocks and bonds, and exchange. These subjects all offer difficulties that are too serious for the immature pupils of the lower grade and belong properly in a higher course.

In square root where only comparatively small numbers should be used the method based on the square of the binomial should not be attempted. The method of resolving into prime factors is easily understood and is applicable to many cases, but for numbers that are not easily factorable the method of repeated trial may be used.¹ This involves the teaching of no new process and is easy to explain, although not economical of figures in some cases.

¹This method is described by Mr. George W. Evans in his recent monograph, *The Teaching of High School Mathematics* (35 cents), published by Houghton, Mifflin & Co.

I have tried to suggest thus far a few ways in which time might be saved from the written work in arithmetic in order that it might be devoted to the everlasting drill that is necessary if we are to secure any degree of accuracy. For there is no royal road to accuracy and speed, and the only way to secure them is to establish a condition of mind that produces an instant and automatic reaction in the presence of certain conditions without the intervention of any act of judgment. This does not mean that the pupils are to be taught not to think, but that they should be relieved at the place where thinking is of no use in order that they may use their powers to better advantage in situations that require them. And this reminds me that there is one time in the solution of every problem when most children consider that their thinking is done, and that is when they have arrived at a result. It matters little what the result may be so long as it may be labeled *ANS.* and placed in a selected spot on the paper or board.

Now, next in importance to securing accuracy is the ability to detect inaccuracy, and contrary to a common conception among children this ability should be developed in the pupil and not in the teacher. Consequently I believe we might profitably spend more time in teaching the pupil how to test his results, by accurate or by rough checking, the latter always and the former whenever possible. We are all familiar with the pupil who will unblushingly offer the information that a company of men must be formed with $12\frac{1}{2}$ men in the platoon, or that it will cost \$64.79 for stock to build a pine box to hold his pencil, eraser, and penholder. It has been said that the most valuable man in a shop is the one who can guess within ten per cent, and this is undoubtedly true, but to be of most value the guess should precede the formal solution, not occasionally but always. That is, the pupil should spend considerable time in thinking over the condition of a problem and make a rough estimate of what the result of his calculations should be before he is allowed to touch pencil and paper. Then having reached a result he should be taught to ask himself, "Is it possible?" and "Is it reasonable?" This is a method or procedure that I have never seen regularly and systematically carried out, but I believe it would be well worth the time required.

In this connection let me call your attention also to that time saving device called the slide rule, not because I believe in teach-

ing its use to elementary school children, but because it would be upon every teacher's desk if its advantages were once realized. It can be used in many sorts of calculations involving only multiplication and division, the result being read directly from the scale, and contrary to the common belief its manipulation is easily learned.

Having eliminated certain parts of the subject in the lower grades there are others that could be emphasized to the advantage of those who leave school early, especially in classes of boys. There are few indeed who do not stand in need of a knowledge of bills and accounts, and many are at one time or another concerned with notes, coöperative and savings banks, life insurance, taxes, and mortgages. Among foreigners, especially in the smaller cities and towns, there is a well-defined tendency to buy small pieces of land at a little distance from the center of population, the capital being secured by means of notes or mortgages, and through their ignorance these people are often defrauded of what to them are large sums by unscrupulous speculators. Therefore it seems that the simpler transactions of banks and real estate dealers deserve some consideration in the latter part of the elementary school course. Interest, both simple and compound, should be emphasized. The former should be taught by the two months method, and the latter by means of tables after some simple preliminary work to bring out the principle involved.

As an introduction to work in percentage some instruction should be given in handling the simple equations, as many of the problems involving base, rate, percentage, and the amount are much simplified by the use of symbols, these symbols consisting of the initial letters of the words for which they stand. For instance, it is entirely within the comprehension of a seventh or eighth grade pupil that $P=B \times R$ is only a shorter way of saying that percentage is equal to the base multiplied by the rate, and having once learned this and the fact that both members of an equation may be multiplied or divided by the same quantity, he does not need to puzzle his brains to think whether the rate is equal to the percentage multiplied by the base, or the percentage divided by the base. He can pass easily from the statement $P=B \times R$ to the others $P/B=R$, and $P/R=B$. Similarly since $P=B \times R$ and $A=B+P$ then he can be made to see that $A=B+B \times R$ and $A=B(1+R)$. If this method of approach is too difficult for the class in hand then the latter

part of the above should be postponed until a later date, but I do not believe that this will be the case.

In regard to board measure and the areas of surfaces when the latter are expressed in feet and inches, I wish to call your attention to a short method of calculation which is still used as a rule of thumb by some carpenters of the older generation but which has entirely disappeared from the arithmetics of the present day. Suppose for instance that we wish to find the area of a surface that is eight feet four inches long and six feet ten inches wide. The usual method is to reduce both dimensions to inches, multiplying the resulting equivalents and divide by 144, thus:

$$\begin{aligned} 7 \text{ ft. } 3 \text{ in.} &= 87 \text{ sq. in.} \\ 6 \text{ ft. } 10 \text{ in.} &= 82 \text{ sq. in.} \\ 87 \text{ in.} \times 82 \text{ in.} &= 7,134 \text{ sq. in.} \\ 7,134 \text{ sq. in.} \div 144 \text{ in.} &= 49 \text{ sq. ft. } 78 \text{ sq. in.} \end{aligned}$$

Now one of the carpenters mentioned would proceed as follows:

7 ft.	3 in.
6 ft.	10 in.
70 parts 30 sq. in.	
42 sq. ft. 18	"
42 sq. ft. 88 parts 30 sq. in.	
7 sq. ft.	48 sq. in.
49 sq. ft.	78 sq. in.

If he reasoned at all about the matter he would say, "When I multiply inches by inches I get square inches. When I multiply feet by inches or inches by feet I get a unit which is one-twelfth of a square foot or 12 square inches. When I multiply feet by feet I get square feet. Now the first and last units are the only ones in common use so I will change the third into square feet and square inches." You will notice that this involves only multiplication and division which can be done mentally and a little addition, an improvement I think over the first method; the chances for mistakes being in the ratio of one to five at the very outside. This process need not of course be confined to cases involving feet and inches, but can be adapted to the mensuration of any surface whose dimensions are expressed in the same two units, if care be taken to determine the relation of these units to the third which it becomes necessary to use. If for instance we wish to find the area of a surface seven yards two feet long by four yards one and one

half feet wide, it can be done in exactly the same way. Thus:

<i>yd.</i>		<i>ft.</i>
7		2
4		$1\frac{1}{2}$
<hr/>		
	$10\frac{1}{2}$	3
28	8	
<hr/>		
28	$18\frac{1}{2}$	3
6		$1\frac{1}{2}$
<hr/>		
34		$4\frac{1}{2}$

There are a few short cuts in the longer operations of multiplication and division that deserve some time in the latter part of the course but they are very few, and among them may be mentioned these, which are valuable particularly in rough checking:

1.
$$\begin{array}{r} 784 \\ 213 \\ \hline 2352 \\ 16464 \\ \hline 166992 \end{array}$$
2. $634 \times 98 = 63400 - 1268 = 62132.$
3. $217 \times 25 = 21700 \div 4 = 5425.$
4. $14.8 \div 12\frac{1}{2} = .1480 \times 8 = 1.184.$
5. $347 \times 11 = 3817.$
6. $(27)^2 = (25+2)^2 = 625 + 100 + 4 = 729.$

In conclusion I wish to suggest the addition of one topic to the subject in the elementary grades—one for which there was little need ten years ago, but one which now demands some notice. I refer to the graphical representation of the relation of two quantities sometimes in the form of two areas, and sometimes that of a curve. The comparative sizes of armies are often shown by men dressed in the national costumes, or the per capita wealth of countries by bags of gold, and the weather man shows by isobars and isotherms the variations in pressure and temperature throughout the country. Furthermore, the Harvard-Yale football game is reported in diagram in the Sunday paper, and what could be more important than we should all be able to follow every play? But seriously I have had pupils show more interest in finding the cause in the sudden change in the slope of a graph representing the growth of population in the United States since 1789 than they ever did in a formal history lesson, and incidentally I think they learned more history. This is a kind of work that appeals to younger people and might I believe be begun in the elementary schools.

There is one feature of the matter, however, which I confess gives me some uneasiness, and that is the persistency with which pupils speak of the process as "grafting." I try to get around this by giving them a piece of coördinate paper and telling them that anything that is plotted on the square cannot be grafting.

To conclude this rather haphazard collection of suggestions let me say that my work for the past ten years has been in a high school for boys and consequently my intimate knowledge of grammar school methods may be considerably out of date. It may be that all the suggestions I have made that are worthy of attention have long since been adopted and that many others of which I never dreamed have been put into practice.

CONCERNING LOCI.

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That the subject of loci has gained greatly both in prominence and in real importance in the last twenty years is evident; that this gain is likely to continue for some time to come would probably be granted by most people who are in touch with the mathematical work of secondary schools and colleges.

Perhaps it is safe to assume that in view of these facts not a few teachers in secondary schools share my own dissatisfaction with the usual treatment of the subject. It seems to stand by itself, not an integral part of elementary geometry but a side issue; an edge tool that wounds the user as often as it helps him. We feel that it should be a most fruitful conception, something that relates more closely and securely many other conceptions and at the same time leads on to the discovery of still other relations both interesting and useful. Instead we find it too often only one more fagot laid on burdened shoulders and always in danger of toppling off!

Of course, if the student goes on, these difficulties tend to disappear, and sooner or later he comes to a full, and a grateful, appreciation of this method. But are the difficulties unavoidable? Is there no way to reach firm ground except through the Slough of Despond? And what of those who do not go on?

My own solution of the problem has been worked out gradually in the course of the last ten years of teaching, and has now been sufficiently tested to justify my offering it to others whom it may concern.