420. To Find the Centre of Curvature of the Conic $ax^2+by^2=1$ at the Point (x', y')

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intellectual powers there is no question. He was deeply religious—"the whitest soul I ever knew," said Bishop Burnet—strictly just to himself as to others, candid, and affable, but unforgiving anything he deemed unfair. His modesty is well exemplified by his remark, made just before his death, that to himself he seemed to have been only like a boy playing on the seashore, and diverting himself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before him. So Newton stands, one of the great figures in the history of science. W. W. ROUSE BALL.

MATHEMATICAL NOTES.

419. [C. 1. g.] *A Proof of Vandermonde's Theorem and a Generalisation.*

Denote $x(x-1)(x-2)\dots(x-n+1)$ by $x^{(n)}$ and assume, as is obviously possible, that

$$\frac{(x+y)^{(n)}}{n!} = a_0 + a_1x + \frac{a_2x^{(2)}}{2!} + \dots + \frac{a_nx^{(n)}}{n!}, \dots\dots\dots(1)$$

where $a_0, a_1, \dots a_n$ are functions of y only.

Differencing equation (1) r times with respect to x , we obtain

$$\Delta^r \frac{(x+y)^{(n)}}{n!} = \frac{(x+y)^{(n-r)}}{(n-r)!} = a_r + a_{r+1} \frac{x}{1!} + \dots + a_n \frac{x^{(n-r)}}{(n-r)!}, \dots\dots\dots(2)$$

By putting $x=0$, a_r is at once seen to be $\frac{y^{(n-r)}}{(n-r)!}$.

Thus,
$$\frac{(x+y)^{(n)}}{n!} = \frac{y^{(n)}}{n!} + \frac{y^{(n-1)} \cdot x}{(n-1)! 1!} + \frac{y^{(n-2)} x^{(2)}}{(n-2)! 2!} + \dots + \frac{x^{(n)}}{n!}.$$

Similarly, by assuming

$$\frac{(x+y)^{(n)}}{n!} = a_0 + a_1(x+a_1) + \frac{a_2(x+a_2)^{(2)}}{2!} + \dots + \frac{a_n(x+a_n)^{(n)}}{n!},$$

where a_1 is perfectly arbitrary, $a_2 - a_1 = 0$ or 1 , $a_3 - a_2 = 0$ or 1 , and generally $a_{r+1} - a_r = 0$ or 1 for all values of r from 1 to $n-1$, we can prove that

$$\frac{(x+y)^{(n)}}{n!} = \frac{(y-a_1)^{(n)}}{n!} + \frac{(y-a_2)^{(n-1)}(x+a_1)}{(n-1)! 1!} + \frac{(y-a_3)^{(n-2)}(x+a_2)^{(2)}}{(n-2)! 2!} + \dots + \frac{(x+a_n)^{(n)}}{n!}.$$

S. T. SHOVELTON.

420. [L¹. 6. a.] *To find the centre of curvature of the conic $ax^2 + by^2 = 1$ at the point (x', y') .*

If the normals at the extremities of the two chords $lx + my - 1 = 0$ and $l'x + m'y - 1 = 0$ meet in a point (h, k) , then $\frac{ll'}{a} = \frac{mm'}{b} = -1$.

Also, if we take $lx + my - 1 = 0$ as the tangent at (x', y') , the point (h, k) will be the centre of curvature required. Hence

$$ax^2 + by^2 - 1 = (axx' + byy' - 1) \left(\frac{x}{x'} + \frac{y}{y'} + 1 \right)$$

must be the same as $xy(a-b) - akx + bhy = 0$.

The above equation becomes

$$xy \left(\frac{ax'}{y'} + \frac{by'}{x'} \right) + x \left(ax' - \frac{1}{x'} \right) + y \left(by' - \frac{1}{y'} \right) = 0$$

or

$$xy - bxy^3 - axy^3 = 0,$$

whence

$$h = -\frac{a(a-b)}{b}x^3, \quad k = \frac{b(a-b)}{a}y^3.$$

N. M. GIBBINS.