T H E

AMERICAN JOURNAL OF SCIENCE

[F O U R T H S E R I E S .]

ART. XIII.—*Circular Magnetization and Magnetic Permeability*; by JOHN TROWBRIDGE and E. P. ADAMS.

VERY little quantitative work appears to have been done upon the problem of the intensity of magnetization produced by an oscillatory current in an iron wire. Previous work of this kind has been done with oscillatory currents, either of very much higher frequency or of very much lower intensity than those used in the experiments about to be described.

In 1894, I. Klemencic* gave an account of some experiments on this problem. He studied the effects of the oscillatory currents induced in an iron wire by Hertz waves, reflected and received by parabolic mirrors. By comparing the heat produced in a short iron wire through the action of these oscillatory currents, with the heat produced in a similar non-magnetic wire, he was able to arrive at a value for the effective resistance of the iron wire to oscillatory currents. Then by making use of Lord Rayleigh's formula :

$$R' = \sqrt{\frac{1}{2}\mu l n R}$$

he calculated the value of the permeability of the iron. The heat produced in each case was compared by the use of a thermal junction held near the wire under investigation. In this way he arrived at the value 118 for the permeability of the iron to oscillatory currents, and estimated the maximum value of the field strength at 290. He reasoned that since this very large maximum field strength produced such feeble magnetization, the permeability of iron to oscillatory

* Wiedemann's *Annalen*, liii, p. 707.

currents was a constant. The frequency used by Klemencic was about 9×10^7 oscillations per second.

The experiments described below are with oscillatory currents of frequencies ranging from 600 to 3000, with maximum field strengths of higher values than those used by Klemencic. It appears from the results given below that with oscillatory currents of as low frequencies as this, the permeability is not a constant, but depends upon the strength of the magnetic field; that is, the iron behaves toward oscillatory currents in much the same manner as it does toward steady currents.

Similar results were brought out recently in an article by Marchant.* He showed that the same dependence of permeability to field strength holds in the case of oscillatory discharges as in the case of steady currents. No figures are given in his article by means of which a quantitative idea of this relationship may be obtained.

The arrangement of apparatus used in our investigation was as follows: B is a battery of 10,000 storage cells; C a con-

denser consisting of 300 glass plates, coated on both sides with tin-foil, the coated surface being 43×33 centimeters. The battery circuit was controlled by a switch at E. The discharge circuit, CADC, contained a spark-gap at D, of cadmium terminals. The self-induction of this circuit could be changed by placing coils of copper wire of different dimensions at A.

When the switch at E was closed the condenser was charged; it then discharged itself through the circuit CADC. This latter circuit was so proportioned that the discharge was always of an oscillatory character, that is, approximately, by the well-known theory, R^2 was less than $4L/C$, where R is the resistance, L the self-induction, and C the capacity of the circuit. The self-induction of A was large enough so that the self-induction of the rest of the circuit could be neglected; and the electrostatic capacity of the circuit could also be neglected in comparison with the capacity of the condenser.

* *Nature*, Aug. 30, 1900, p. 413.

FIGURE 2.—Oscillatory Discharges of a Condenser, With periods 600 to 3,000.

In order to study the frequency of oscillation, the revolving mirror method was used. A concave mirror of about 300 centimeters focal distance was attached to the shaft of an electric motor. The image of the spark was reflected by the revolving mirror upon a photographic plate, and when the plate was developed the length of this image could be measured. Some typical photographs are shown in the accompanying cuts, fig. 2. The speed of the motor was obtained by means of a chronograph, the cylinder of which was directly attached to the shaft of the motor.

The formula connecting the periodic time of an oscillatory discharge, T , with the self-induction, L , and the capacity, C , of a circuit is :

$$T = \frac{2\pi}{\left\{ \frac{1}{LC} - \frac{R^2}{4L^2} \right\}^{\frac{1}{2}}}$$

or since $R^2/4L^2$ is very small compared with $1/LC$,

$$T = 2\pi\sqrt{LC}$$

In this formula T is known from the length of the spark on the photographic plate, the distance of the plate from the mirror, and the speed of rotation of the mirror.

C was determined by placing at A a coil of copper wire whose self-induction to steady currents could be accurately calculated. It was then assumed that for the low frequency used, the self-induction to oscillatory currents did not differ appreciably from its calculated value. That this was a legitimate assumption within the limits of experimental error, was shown by the close agreement that existed between this calculated value, and the value obtained by measuring the self-induction by means of Rayleigh's bridge, employing a frequency of about 700. Knowing thus L and T , C could be calculated by the above formula. The value of the capacity obtained for the lowest frequency used, viz: 1.85 micro-farads, agrees very closely with its value measured by comparison with a standard condenser.

Two different coils of copper wire, corresponding to frequencies, when alone in series with the condenser, of 633 and 1423 respectively, were used. For the higher of these two frequencies the capacity was about 5 per cent less than for the lower. Part of this change may have been due to the fact that the change of self-induction of the copper coil was neglected, but this latter was very small.

The iron wire, whose permeability was under investigation, was arranged in the form of a rectangle, around the four sides

of the Jefferson Physical Laboratory—200 feet long and 60 feet wide. The wire was the ordinary soft iron wire, 0.105 inches in diameter. This iron wire was arranged so that it could be put in series with either of the two copper coils, and also so that it could form practically the only self-induction in the discharge circuit.

The general method of procedure was as follows: As stated above, one of the copper coils was placed at A and the capacity of the condenser obtained. Next the iron wire was placed in series with the same copper coil. This of course changed the frequency of the discharge, but since the change was not large, it was assumed that the capacity of the condenser was not sensibly altered. Knowing the periodic time and the capacity, the self-induction of both copper coil and iron wire was given by the formula $T = 2\pi\sqrt{LC}$. Then the self-induction of the iron wire alone was obtained by subtracting from this combined self-induction, the calculated self-induction of the copper coil. Then by means of Lord Rayleigh's formula:

$$L' = l \left(A + \sqrt{\frac{\mu R}{2nl}} \right)$$

the permeability of the iron was obtained.

The same process was repeated using the other copper coil, and the permeability at this different frequency obtained in the same way. Finally the iron was placed alone in the discharge circuit, and the self-induction of this obtained directly from the formula: $T = 2\pi\sqrt{LC}$. In order to get the capacity of the condenser at this highest frequency, it was assumed that it had diminished by 5 per cent. This, of course, can only be regarded as a rough approximation.

The experimental data and calculations follow:

The self-induction of the two coils of copper wire was calculated by means of Maxwell's formula:

$$L = 4\pi n^2 a \left(\log \frac{8a}{R} - 2 \right) + 2l \left(\log \frac{D}{d} + 0.11835 \right)$$

where:

- n number of turns.
- a mean radius of coil.
- R geometric mean distance of cross-section of coil.
- l total length of wire.
- D diameter of insulated wire.
- d diameter of bare wire.

In these coils :

$$\begin{array}{ll} D & 0.3969^{\text{cm}} \\ d & 0.1241^{\text{cm}} \end{array}$$

For the larger coil :

$$\begin{array}{ll} a & 43.5558 \\ R & 2.1292 \\ n & 144 \\ L & 3.421 \times 10^7 \end{array}$$

For the smaller coil :

$$\begin{array}{ll} a & 40.2016 \\ R & 1.4194 \\ n & 64 \\ L & 7.125 \times 10^6 \end{array}$$

The iron wire was in the form of a rectangle, 200 feet long and 60 feet wide, the diameter of the wire being 0.105 inches. Rayleigh's formula for self-induction with alternating currents is :

$$L' = l \left(A + \sqrt{\frac{\mu R}{4\pi nl}} \right)$$

where :

$$\begin{array}{ll} l & \text{length of wire.} \\ R & \text{resistance for steady currents.} \\ n & \text{number of alternations per second.} \\ \mu & \text{permeability.} \end{array}$$

A is a constant, depending upon the geometric form of the circuit, and can be calculated from the self-induction of the circuit with steady currents. For steady currents, Rayleigh's formula becomes :

$$L = l \left(A + \frac{\mu}{2} \right)$$

Using Maxwell's formula for the self-induction of two parallel wires, and applying it to this case, we get for the self-induction of the iron wire to steady currents :

$$L = 4c \log \frac{b}{a} + 4b \log \frac{c}{a} + \mu(b+c)$$

Here

$$\begin{array}{ll} c & \text{length of rectangle.} \\ b & \text{width} \\ a & \text{radius of wire.} \\ l & \text{length of wire.} \end{array}$$

Hence

$$L = 311200 + 15860 \mu / 2$$

and

$$1A = 311200$$

The resistance of the iron wire to steady currents was measured by means of Wheatstone's Bridge, giving,

$$R = 4.3 \times 10^9$$

Thus Rayleigh's formula, for calculating the permeability, when we know the self-induction of the iron wire to alternating currents, becomes:

$$L' = 311200 + \sqrt{\frac{4.3 \times 10^9 \times 15860}{4\pi n}} \mu$$

Discharging the condenser through the larger coil, we had :

Length of half-oscillation	8.98 ^{cm}
Speed of mirror	3.0
Distance plate from mirror	300.9

From this we find the number of complete oscillations per second to be :

$$n = 633$$

Using this value of n , and the calculated value of the self-induction of the coil given above, we find for the capacity of the condenser :

$$C = 1.848 \text{ micro-farads.}$$

Discharging the condenser through the same coil and the iron wire in series we had :

Length of half-oscillation	6.91 ^{cm}
Speed of mirror	2.2
Distance plate from mirror	300.9

giving

$$n = 605$$

This gives for the combined self-induction of iron wire and copper coil :

$$L = 3.745 \times 10^7$$

Subtracting from this the self-induction of the coil, that of the iron wire becomes :

$$L' = 3.24 \times 10^6$$

Substituting this value in Rayleigh's formula we get for the permeability of the iron :

$$\mu = 327$$

Discharging the condenser through the smaller coil alone, we had :

Length of half-oscillation	7.42 ^{cm}
Speed of mirror	5.6
Distance plate from mirror	300.9

giving,

$$n = 1423$$

This value of n , with the calculated value of the self-induction of the coil, gives as the capacity of the condenser at this frequency :

$$C = 1.756 \text{ micro-farads.}$$

Discharging the condenser through the smaller coil and the iron wire in series, we had :

Length of half-oscillation	11.85 ^{cm}
Speed of mirror	8.0
Distance plate from mirror	300.9

giving

$$n = 1280$$

Using 1.756 as the capacity of the condenser, and this value of n , we get for the combined self-induction of coil and iron,

$$L = 8.805 \times 10^6$$

Subtracting the self-induction of the coil, that of the iron wire becomes :

$$L' = 1.68 \times 10^6$$

And by means of Rayleigh's formula,

$$\mu = 443$$

Discharging the condenser through the iron wire alone, we had :

Length of half-oscillation	2.42 ^{cm}
Speed of mirror	4.2
Distance of plate from mirror	300.9

giving

$$n = 3300$$

Assuming the capacity of the condenser to be 1.668 micro-farads, and using this value of the frequency, we get for the self-induction of the iron wire :

$$L' = 1.395 \times 10^6$$

By means of Rayleigh's formula,

$$\mu = 711$$

The values of the permeability so calculated are a kind of mean value for a single discharge of the condenser. The magnetic force is that due to the current in the wire, so that the iron becomes circularly magnetized. This magnetic force is 0 at the axis of the wire, and increases to the value $2i/a$ on the circumference, where i is the current and a the radius of the wire. Thus the magnetic force is not a constant over the cross-section of the wire; at any point in the wire its value is $2\pi ir/a^2$, where r is the distance of the point from the axis of

the wire. The mean value, found by integrating this expression over the cross-section of the wire, is :

$$H_{\text{mean}} = 4i/3a = 2/3 H_{\text{max.}}$$

The following values were obtained by Klemencic* for the susceptibility of iron wire to a steady circular magnetizing force. The last column gives the permeability as calculated from his results :

i (Amperes)	$H_{\text{max.}}$	H_{mean}	k	μ
·12	·23	·15	14·3	180
·23	·43	·29	16·7	210
·46	·87	·58	20·2	255
·85	1·62	1·08	28·7	362
1·26	2·40	1·60	46·8	590
1·45	2·77	1·85	61·2	771
2·39	4·56	3·04	103·5	1304
3·62	6·90	4·60	112·9	1423

These values of the permeability are thus seen not to differ very much from the average values of the permeability with longitudinal magnetization.

From the results of our experiment and the above table, it would seem that the oscillatory currents we employed were equivalent in their magnetizing effects upon the iron wire to steady currents varying in strength from about 0·8 to 1·4 amperes. We can easily get an approximate idea of what the maximum current strength was as follows. The images measured on the photographic plates were in every case the images of the first half-oscillation. Neglecting the damping during this first half-oscillation, which is small, the maximum current is given by the following expression :

$$I_{\text{max.}} = \frac{E}{\sqrt{R'^2 + 4\pi^2 n^2 L'^2}}$$

E is the potential to which the condenser was charged. The e.m.f. of each cell was very approximately 2 volts, giving as the total e.m.f.,

$$E = 20,000 \times 10^8$$

We can thus form the following table, making use of the maximum current strengths thus obtained, and the maximum magnetic field strengths :

$I_{\text{max.}}$ (Amperes)	Frequency.	$H_{\text{max.}}$	Permeability.
140	605	53	327
282	1280	106	443
742	3300	278	711

* Wied. Annalen, lvi, p. 585.

It might be supposed that we could have made a known change in the current by employing one-half the battery to charge the condenser. One-half, however, was not sufficient to charge the condenser to such a degree that sufficient light could be obtained in the spark. We believe that the following inference, however, is correct: the permeability is seen to increase with increasing field strength, in spite of the increase of frequency. But the increase of permeability with increase of field strength is seen to be much less rapid than in the case of steady currents. For, from the table of Klemencic, a permeability of 327 corresponds to a maximum current of about 0.8 amperes, while a permeability of 711 corresponds to a current of about 1.4 amperes. But in the case of an oscillatory discharge, these two values of the permeability correspond to maximum currents of 140 and 742 amperes respectively. This clearly shows the effect of period in diminishing the permeability; for the same change of permeability is produced by increasing the currents in the ratio 8:14 with steady currents as is produced by increasing the currents in the ratio 1:5 in the case of oscillatory currents.

Jefferson Physical Laboratory, Harvard University.