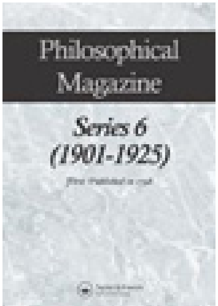

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III. Note on the remarkable case of diffraction spectra described by Prof. Wood

Lord Rayleigh O.M. P.R.S. [a]
[a] Terling Place, Witham
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Using these we have

$$\int \left\{ f^2 + \left(\frac{df}{d\xi}\right)^2 \right\} d\xi = \tfrac{1}{2}\xi\phi'^2(1+\gamma_1^2).$$

5. *General Conclusion.*—By combining the above we obtain the value of

$$\int \left\{ f^2 + \left(\frac{df}{d\xi}\right)^2 \right\} d\xi$$

for a bar under any terminal conditions whatever expressed in terms of ϕ and ϕ', where

$$\phi = \mathrm{A}\cos\gamma_1\xi + \mathrm{B}\sin\gamma_1\xi.$$

Thus the changes needed in the notes, in the form and relations of the normal functions and in the equation determining the influence of a generalized force have been fully dealt with for the case in which rotatory inertia terms are retained. It will be seen at once, that while the analysis is somewhat lengthy, the results reached are not of a very complicated character.

My best thanks are due to Prof. Karl Pearson, both for suggesting the above investigation and for his constant help during the progress of the work.

III. *Note on the remarkable Case of Diffraction Spectra described by Prof. Wood.* *By* Lord Rayleigh, *O.M., P.R.S.**

In the Philosophical Magazine for Sept. 1902 Prof. Wood describes the extraordinary behaviour of a certain grating ruled upon speculum metal which exhibits what may almost be called discontinuities in the distribution of the brightness of its spectra. Thus at a certain angle of incidence this grating will show one of the D-lines of sodium, *and not the other*. In fig. 1, p. 398, Prof. Wood gives ten diagrams fixing the positions (in terms of wave-length) of bright and dark bands in the spectrum at various angles of incidence ranging from 4° 12′ on the same side of the normal as the spectrum to 5° 45′ on the other side. In general there may be said to be two bands which approach one another as the angle of incidence diminishes, coincide when the incidence is normal, and open out again as the angle increases upon the other side. In the tenth diagram there is a third band whose behaviour is different and still more peculiar. In the movement of the two bands the rate of progress along the normal spectrum is the same for each. The above represents the cycle when the grating is in air. "If a piece of plane-parallel glass is cemented to the front of the grating with cedar-oil

* Communicated by the Author.

the cycle is quite different. In this case we have a pair of unsymmetrical shaded bands which move in the same direction as the angle of incidence is changed."

An important observation relates to polarization. "It was found that *the singular anomalies were exhibited only when the direction of vibration (electric vector) was at right angles to the ruling.* On turning the nicol through a right angle all trace of bright and dark bands disappeared. The bands are naturally much more conspicuous when polarized light is employed."

The production of effects changing so suddenly with the wave-length would appear to require the cooperation of a large number of grating-lines. But, as the result of an experiment in which all but about 200 lines were blocked out, Prof. Wood was compelled to refer the matter to the form of the groove. To this cause one would naturally look for an explanation of the difference between this grating and others ruled with the same interval, but it does not appear how the discontinuity itself can have its origin in the form of the groove.

The first step towards an explanation would be the establishment of a relation between the wave-lengths of the bands and the corresponding angles of incidence; and at the time of reading the original paper I was inclined to think that the determining circumstance might perhaps be found in the passing off of a spectrum of higher order. Thus in the spectrum under observation of the first order, an abnormality might be expected at a particular wave-length if in the third order light of this wave-length were just passing out of the field of view, *i. e.* were emerging tangentially to the grating surface. The verification or otherwise of this conjecture requires a knowledge of the grating interval (ϵ). This is not given in the published paper; but on hearing from Prof. Wood that there were 14,438 lines to the inch, I made at once the necessary calculation.

If θ be the angle of incidence for which light of wave-length λ is just passing off in the nth spectrum,

$$\epsilon(1 \pm \sin\theta) = n\lambda. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the first diagram the angle of incidence is 4° 12′ and the wave-lengths of the bands are given as 609 and 517, or in centimetres $6{\cdot}09 \times 10^{-5}$ and $5{\cdot}17 \times 10^{-5}$. Also $\epsilon = 2{\cdot}540/14438$ cm., and $\sin\theta = {\cdot}0732$. Using these data in (1), we find for the larger wave-length $n = 3{\cdot}10$, or $n = 2{\cdot}68$, according as the upper or the lower sign is taken. Again, for the smaller wave-length we find with the upper sign $n = 3{\cdot}65$, and with the lower $n = 3{\cdot}15$. To reconcile these numbers with the suggested relation it is necessary to suppose that 609 is passing off in the third spectrum on the same side as that on which the light is incident, and 517 in the third spectrum upon the other side. But the agreement of 3·10 and 3·15 with the integer 3·00 seemed hardly good enough, and so the matter was put aside until recently, when my attention was recalled to it in reading an article by Prof. Ames * on Rowland's ruling-machines, from which it appeared that gratings have been ruled with three different spaces, viz. 14438, 15020, and 20000 lines to the inch. If we permit ourselves to suppose that the number of lines in the special grating is really 15020 to the inch in place of 14438, the alteration would be in the right direction, 3·10 becoming 2·98 and 3·15 becoming 3·03, so that the mean would be about correct.

In view of this improved agreement it seems worth while to consider how far the position of the bands recorded in the other diagrams would accord with the formula

$$\lambda=\tfrac{1}{3}\epsilon(1\pm\sin\theta), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

taking ϵ to correspond with ruling at the rate of 15020 to the inch. In one respect there is a conspicuous agreement with Prof. Wood's observations. For if λ_1, λ_2 are the two values of λ in (2), we have at once

$$\lambda_1+\lambda_2=\tfrac{2}{3}\epsilon, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that the two bands move equally in opposite directions as θ changes.

The results calculated from (2) for comparison with diagrams (2) .... (10) (fig. 1) are given below.

θ.	Calculation.		Observation.		No.
	λ_1.	λ_2.	λ_1.	λ_2.	
2° 37′......	590	538	589	537	2
0° 15′......	566	561	566	559	3
0° 5′......	564	563	562	561	4
0° 0′......	564	564	561	561	5
0° 5′......	564	563	......	......	6
1° 15′......	576	551	575	549	7
1° 53′......	582	545	581	542	8
2° 38′......	590	538	589	538	9
5° 45′......	620	507	619	506	10

* Johns Hopkins University Circular. Notes from the Physical Laboratory, Ap. 1906.

The numbers headed "observation" are measured from Prof. Wood's diagrams; but owing to the width and unsymmetrical form of some of the bands they are liable to considerable uncertainty. It would appear that (with the exception of the third band in diagram (10)) all the positions are pretty well represented by (2).

As regards the observations when the face of the grating was cemented to glass with cedar-oil, we have in place of (1)

$$\epsilon(1 \pm \sin\theta') = n\lambda',$$

where λ' is the wave-length and θ' the angle of incidence in the oil. Now if μ be the refractive index of the oil,

$$\lambda' = \lambda/\mu, \qquad \sin\theta' = \sin\theta \,. /\mu,$$

so that

$$\epsilon(\mu \pm \sin\theta) = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

if as usual θ and λ are measured in air.

In the diagrams of Prof. Wood's fig. (2) there are four angles of incidence. The bands are markedly unsymmetrical and the numbers entered in the following table are those corresponding to the sharp edge. The values for n are calculated from (4) on the supposition that $\mu = 1{\cdot}5$, the lower sign being chosen if the angles on the first side are regarded as

θ.	λ.	n.
12° 8′.........	541	4·03
7° 8′........	590, 469	3·94, 4·96
3° 53′.........	610, 489	3·97, 4·95
−2° 29′.........	655, 529	3·99, 4·93

positive. The wave-lengths observed correspond pretty well with the passing off of the fourth and fifth spectra on the opposite side to that upon which the light is incident. There seems to be nothing corresponding to the passing off of spectra on the same side. Upon the whole there appears to be confirmation of the idea that the abnormalities are connected with the passing off of higher spectra, especially if the suggested value of ϵ can be admitted.

The argument which led me to think that something peculiar was to be looked for when spectra are passing off may be illustrated from the case of plane waves of sound, incident upon a parallel infinitely thin screen in which are cut apertures small in comparison with λ. The problem for a single aperture was considered in Phil. Mag. xliii. p. 259, 1897 *, from which it appears that corresponding to an incident wave of amplitude unity the wave diverging from the aperture on the further side has the expression

$$\psi = \mathrm{M}\frac{e^{-ikr}}{r}, \quad \ldots \ldots \quad (5)$$

where $k=2\pi/\lambda$, r is the distance from the aperture of the point where the velocity-potential ψ is reckoned, and M represents the electrical *capacity* of a conducting disk having the size and shape of the aperture, and situated at a distance from all other electrical bodies. In the case of a circular aperture of radius a,

$$\mathrm{M} = 2a/\pi. \quad \ldots \ldots \quad (6)$$

The expression (5) applies in general only when the aperture is so small that the distance between any two points of it is but a small fraction of λ. It may, however, be extended to a series of equal small apertures disposed at equal intervals along a straight line, provided that the distance between consecutive members of the series is a multiple of λ. The condition is then satisfied that any two points, whether on the same or on different apertures, are separated by a distance which is very nearly a precise multiple of λ. The expression for the velocity-potential may be written

$$\psi = \mathrm{M}'\frac{e^{-ikr_1}}{r_1} + \mathrm{M}'\frac{e^{-\quad 2}}{r_2} + \ldots, \quad \ldots \quad (7)$$

where r_1, r_2, &c., are the distances of any point on the further side of the screen from the various apertures, and M′ is the electrical capacity of each aperture, now no longer isolated, but subject to the influence of the others similarly charged.

It is not difficult to see that if the series of apertures is infinitely extended, M′ approaches zero. For, if ϵ be the distance between immediate neighbours, and we consider the condition of the system when charged to potential unity, we see that the potential at any member due to the charges on the other members has the value

$$\frac{2\mathrm{M}'}{\epsilon}(1+\tfrac{1}{2}+\tfrac{1}{3}+ \ \ldots \ldots) = \mathrm{M}' \times \infty.$$

Accordingly $\mathrm{M}'=0$, indicating that the efficiency of each aperture in allowing waves to pass to the further side of the screen is destroyed by the cooperative reaction of the series of neighbours. The condition of things now under contemplation is that in which one of the lateral spectra formed by

* Or 'Scientific Papers,' iv. p. 283.

the series of holes (considered as a grating) is in the act of passing off, and it is evident that the peculiar interference is due to this circumstance *. The argument applies even more strongly, if less simply, to an actual grating formed by a series of narrow parallel and equidistant slits cut in an infinite screen.

The case of a reflecting grating differs in some important respects from that above considered. An investigation applicable to light is now nearly completed. It confirms the general conclusion that peculiarities are to be looked for at such angles of incidence that spectra of higher order are just passing off, but (it is especially to be noted) only when the polarization is such that the electric vector is perpendicular to the grating.

Terling Place, Witham, May 4.

P.S. June 5.—In answer to further inquiry Prof. Wood tells me that he thinks the ruling may perhaps be 15020 to the inch, but (the grating being for the time out of his hands) he is not able to speak with certainty.

IV. *On the Effect of Stress on Magnetization and its Reciprocal Relations to the Change of Elastic Constants by Magnetization.* *By* K. Honda *and* T. Terada, *Lecturers on Physics in the Tōkyō Imperial University* †.

[Plates I.-V.]

IN our previous experiments, we investigated in some detail the change of elastic constants of several ferromagnetic metals and alloys caused by magnetization, with special regard to the order of applying the stress and the field, and found that in some cases the change is considerably large, and moreover that it differs more or less for different orders of applying the stress and the field. In order to find the proper explanation of these facts, it will be necessary to investigate with the same specimens as in the previous experiment the change of magnetization by stress, with

* If ϵ be not a precise multiple of λ, the series $1+\frac{1}{2}+\frac{1}{3}+\ldots$ *ad infin.* would be replaced by

$$e^{-ik\epsilon}+\tfrac{1}{2}e^{-2ik\epsilon}+\tfrac{1}{3}e^{-3ik\epsilon}+\ldots,$$

which is equivalent to

$$-\log\{2\sin(\tfrac{1}{2}k\epsilon)\}+\tfrac{1}{2}i(k\epsilon-\pi).$$

† From the Journ. Coll. Sci. Tokyo. Communicated by the Authors.