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On the Elements of (33) Polyhymnia, and the mass of Jupiter.

By *Simon Newcomb.*

More than twenty years ago, I called attention to the great value of observations on the planet (33) Polyhymnia for determining the mass of Jupiter (A. N. Bd. 79, S. 245). I hoped by doing so to secure a continuous series of observations at each opposition from that time onward; but this hope has been disappointed. Still, enough of observations have been made to afford a value of the mass which I think is entitled to greater weight than any other single determination.

The work of making this investigation, like several other works in which I am engaged, has been carried on piece-meal, in a quite irregular way, for more than ten years. As far back as 1884-85, the special perturbations of the elements were computed by Professor G. C. Comstock, from the date of discovery up to near the end of 1888; it being deemed necessary to await observations at the opposition of the latter date before completing the work. Mr. Comstock also discussed all the early observations, and made several additional ones at Madison. Last year I was able to have the heaviest part of the computations that yet remained made by Dr. Norbert Herz; and now I am able to report the completion of the work. As it will be some time before I can get the full investigation printed, I communicate herewith such of the numbers and results as seem necessary to enable the work to be prosecuted by others. It seems very desirable in the interest of future astronomy that the perturbations should be continued from the point where I have left them off. But I am unable to undertake this work myself.

The elements which were used to make the final comparison with observations are designated as Elements V, and are shown in the following table. They are taken as osculating at the fundamental epoch 1873, July 17.0, Berlin mean noon, except that the perturbations by the Earth are not taken to vanish at this epoch, but to have values shown in the first table below. These should be applied in order to get osculating elements.

With elements differing only a few seconds from Elements V, and which therefore I do not deem it necessary to publish at present, the special perturbations of the elements by Earth, Mars, Jupiter, Saturn and Uranus were computed from the date of discovery in 1854, to December 5th, 1888, using the following values of the masses:

Jupiter	$1:m = 1047.879$	(Bessel)
Saturn	$1:m = 3501.6$	(Bessel)
Mars	$1:m = 3093500$	(Hall)

In the work as first done I used general perturbations by the Earth, and omitted the action of Uranus entirely. It was found, however, on examination, that decimals enough had not been used in one term of long period in the general perturbations by the Earth, and I deemed it better to reject them entirely, and use special perturbations. At the same time the opportunity of introducing the action of Uranus was taken advantage of.

In order to lessen the labor of computing these small perturbations, the disturbing forces were not computed for intervals of an integral number of days, as usual, but for equi-distant mean anomalies of Polyhymnia (5° in the case of the Earth; 10° in that of Uranus). This rendered it unnecessary to compute the quantities which depended solely on Polyhymnia for more than a single revolution. Having done this for the middle revolution, the same values were used for the three revolutions preceding and following it. It is true that in this way terms of the first order only are included in the perturbations; these however seem to be sufficiently accurate.

The first of the following tables gives, as already remarked, the adopted values of Elements V for the fundamental epoch.

They are followed by the corrections found from all the observations. Applying these corrections to Elements V we obtain Elements VI.

Next is given the constant terms to be added on account of the Earth perturbations, in order to obtain osculating elements for the same epoch.

I give next the reduction of the Elements VI from the original epoch to the epoch 1888, November 8.0, Berlin. The numbers are given, I hope, with sufficient fullness to render easy the detection of any discrepancy or error in the work. In the top line of the table we have the values of Elements VI as referred to the ecliptic and mean equinox of 1870.0. In the next line is given the perturbations by Jupiter, Saturn and Mars from the fundamental epoch up to 1888, November 8.0. The next line gives the similar perturbations by Uranus, and the next those by the Earth; it being understood however that the latter do not start from the value zero at the fundamental epoch, but from the values given above. In the next line is given the correction to the perturbations by Jupiter on account of the increase 0.000505 m of the mass, of which I shall speak presently. In the next line is given the precession from 1870.0 to 1888.0, including the effect of

motion of the ecliptic. The sums of these quantities are the values of Elements VI for the last epoch, referred to the mean ecliptic and equinox of 1888.0.

It will be remarked that I have supposed the corrections applicable to Elements V to be the same at the epoch 1888.0 as at the fundamental epoch. This assumption is not strictly correct, since the perturbations will be different if computed with the final elements. But the error thus introduced is, I think, quite unimportant at present.

The tables which follow, giving the summation of

the perturbations by Jupiter, and then those by Saturn and Mars, do not seem to require much explanation. The dates are those for which the disturbing forces and differential variations of the elements were actually computed. Oppolzer's method and formulae were used in this part of the work. The summation of the elements themselves corresponds to the intermediate dates, while the table of summation of the perturbations of the mean motion is taken to refer to the original dates for which the disturbing forces were computed.

Elements V	Corr. per Obs.	Elements VI	Earth Pert.
$L = 318^{\circ} 53' 30''.0$	$+ 0''.60 \pm 0''.34$	$318^{\circ} 53' 30''.60$	$+ 1''.27$
$\pi = 342^{\circ} 20' 7.5$	$- 0.89 \pm 0.80$	$342^{\circ} 20' 6.61$	$- 6.60$
$\varphi = 19^{\circ} 52' 54.2$	$- 0.77 \pm 0.15$	$19^{\circ} 52' 53.43$	$+ 2.33$
$\Omega = 9^{\circ} 2' 44.9$	$+ 60.9 \pm 10.0$	$9^{\circ} 3' 45.8$	0.00
$i = 1^{\circ} 56' 20.0$	$- 1.08 \pm 0.31$	$1^{\circ} 56' 18.92$	0.00
$\mu = 732''.81750$	$+ 72 \pm 6$	$732''.81822$	0.00

Epoch, 1873 July 17.0, Berlin. Ecliptic and Equinox of 1870.0.

Reduction of Elements VI to 1888 Nov. 8.0, Berlin.

	L	π	φ	Ω	i	μ
Elements VI	$17^{\circ} 24' 18''.28$	$342^{\circ} 20' 6''.61$	$19^{\circ} 52' 53''.43$	$9^{\circ} 3' 45''.8$	$1^{\circ} 56' 18''.92$	$732''.81822$
Pert. Jupiter, Saturn, Mars	$- 2^{\circ} 29' 57.67$	$+ 0^{\circ} 49' 21.95$	$- 0^{\circ} 24' 18.65$	$- 5' 26.5$	$- 1' 22.27$	$- 3.48587$
Pert. Uranus	$- 1.60$	$+ 3.25$	$+ 0.03$	$-$	$-$	$+ 6$
Pert. Earth	$- 4.33$	$+ 9.32$	$+ 1.69$	$-$	$-$	$+ 202$
$\Delta \mathcal{Q}$	$- 4.57$	$+ 1.50$	$- 0.74$	$- 0.15$	$- 0.04$	$- 177$
Prec. to 1888.0	$+ 15^{\circ} 4.70$	$+ 15' 4.70$	$-$	$+ 13' 55.75$	$+ 8.27$	$-$
	$15^{\circ} 9' 15.41$	$343^{\circ} 24' 47.33$	$19^{\circ} 28' 35.76$	$9^{\circ} 12' 14.9$	$1^{\circ} 55' 4.88$	729.33266

Summation of Perturbations by Jupiter.

Date	$\Sigma \Delta \mu$	$\Sigma \Delta L_1$	$\Sigma \Delta \mu$	$\Sigma \Delta \varphi$	$\Sigma \Delta \pi$	$\Sigma \Delta i$	$\Sigma \Delta \Omega$
1873 June 27	$- 0.056$	$+ 0.042$	$- 0.0101$	$+ 0.008$	$- 0.089$	0.000	$- 0.008$
Aug. 6	$- 0.066$	$...$	$...$	$...$	$...$	$...$	$...$
1888 May 19	$- 6301.062$	$- 2139.954$	$- 133.3467$	$- 1482.885$	$+ 2947.323$	$- 80.958$	$- 288.504$
June 28	$- 6434.409$	$- 2138.580$	$- 135.0096$	$- 1477.461$	$+ 2952.083$	$- 81.041$	$- 287.811$
Aug. 7	$- 6569.419$	$- 2138.924$	$- 136.9138$	$- 1471.352$	$+ 2958.074$	$- 81.153$	$- 287.875$
Sept. 16	$- 6706.333$	$- 2140.978$	$- 138.8304$	$- 1465.229$	$+ 2965.195$	$- 81.282$	$- 289.038$
Oct. 26	$- 6845.163$	$- 2144.656$	$- 140.6076$	$- 1459.459$	$+ 2972.754$	$- 81.416$	$- 291.439$
Dec. 5	$- 6985.771$	$- 2149.804$	$- 142.1659$	$- 1454.148$	$+ 2979.988$	$- 81.542$	$- 295.058$

Summation of Perturbations by Saturn and Mars.

1873 June 27	$+ 0.003$	$- 0.003$	$- 0.0037$	$+ 0.008$	$- 0.023$	0.000	$- 0.003$
Aug. 6	$- 0.001$	$...$	$...$	$...$	$...$	$...$	$...$
1888 May 19	$+ 56.686$	$- 22.743$	$+ 0.5257$	$+ 2.952$	$- 8.376$	$- 0.886$	$- 35.485$
June 28	$+ 57.212$	$- 22.906$	$+ 0.6315$	$+ 2.611$	$- 8.822$	$- 0.883$	$- 35.511$
Aug. 7	$+ 57.843$	$- 22.927$	$+ 0.7373$	$+ 2.278$	$- 9.229$	$- 0.881$	$- 35.510$
Sept. 16	$+ 58.580$	$- 22.799$	$+ 0.8237$	$+ 1.993$	$- 9.489$	$- 0.879$	$- 35.495$
Oct. 26	$+ 59.404$	$- 22.530$	$+ 0.8758$	$+ 1.772$	$- 9.458$	$- 0.878$	$- 35.480$
Dec. 5	$+ 60.280$	$- 22.145$	$+ 0.8849$	$+ 1.604$	$- 9.010$	$- 0.878$	$- 35.475$
	$+ 61.165$						

Actual Perturbations by the Earth.

		ΔL	$\Delta \pi$	$\Delta \varphi$	$10^5 \Delta \mu$
1873	July 17	+1".27	-6".60	+2".33	0"
1888	Oct. 6	-5.12	+7.64	+2.41	- 40
	25	-4.70	+8.70	+2.01	+ 76
	Nov. 14	-4.14	+9.56	+1.46	+280

Actual Perturbations by Uranus.

		ΔL	$\Delta \pi$	$\Delta \varphi$	$10^5 \Delta \mu$
1873	July 17	0".00	0".00	0".00	0"
1888	Oct. 1	-1.60	+3.27	+0.03	+3
	26	-1.60	+3.26	+0.03	+5
	Nov. 19	-1.60	+3.24	+0.03	+7

The Mass of Jupiter.

The great suitability of this method of determining the mass of Jupiter is shown by the fact that although the representation of the observations is far from satisfactory, the probable error of an observation of weight unity being ± 2.2 , yet the probable error of the denominator of the mass of Jupiter came out only 0.030, the result being

$$\mu = 1047.34; \text{ m. e. } \pm 0.044.$$

With this determination I have combined all the other determinations which it seemed to me it was desirable to use.

The following table shows these values, and the relative weights to which I have judged each one entitled. I do not deem it necessary at the present time to give in all detail the considerations which led to the adoption of these weights. I may remark, however, that von Haerdtl's excellent result from the perturbations of Winnecke's comet, which has by far the smallest probable error of any determination yet made, has not been assigned a corresponding weight, because of a distrust on my part whether observations on a comet can be considered as having always been made on the centre of gravity of a well defined mass, moving as if its centre were a material point subject to

the gravitation of the sun and planets. This distrust seems to be amply justified by our general experience of the failure of comets to move in exact accordance with their ephemerides.

Mass of Jupiter from

	μ	Wt.
All observations on the satellites	1047.82	1
Action on Faye's comet (Möller)	1047.79	1
Action on Themis (Krueger)	1047.54	5
Action on Saturn (Hill)	1047.38	7
Action on Polyhymnia	1047.34	20
Action on Winnecke's comet (v. Haerdtl)	1047.17	10
Weighted mean	1047.35	
	m. e. ± 0.065	

I propose to regard this mass of Jupiter as a definitive one, to be adopted in my work on the planetary theories.

In the interest of the astronomy of the future it seems very desirable to apply Gill's heliometer method to the continuous observation of a selected number of the minor planets, especially Polyhymnia. I include this planet, although it is to be feared that it can be reached with the heliometer only at oppositions near its perihelion.

Nautical Almanac Office, Washington, 1894 June 1.

S. Newcomb.

Untersuchung über die vormalige Bahn des Cometen 1886 II.

Von Pfarrer A. Thraen zu Dingelstaedt.

Nach Veröffentlichung meiner Bahnbestimmung des Cometen 1886 II in Nr. 3160-61 der Astr. Nachr., welche eine hyperbolische Bahn desselben ergeben hatte, regte Herr Berberich brieflich bei mir den Gedanken an, dass der hyperbolische Character der Bahncurve wohl durch Störungen verursacht sein könne, welche der Comet vor seiner Entdeckung durch Annäherung an einen oberen Planeten, wahrscheinlich Saturn, erlitten haben möge. Anderweitige Beschäftigungen hinderten mich, die erforderliche umfassende Untersuchung damals sofort durchzuführen. Vorläufige Rechnungen erwiesen übrigens, dass die Annäherung an Saturn nicht bedeutend gewesen war, da er im Minimum seiner Distanz von demselben im September 1884 noch um 4.7 Erdbahnhalbmesser von ihm fern geblieben war. Neuen Anlass, die Frage nach dem Ursprung dieser Hyperbel zu untersuchen, bekam ich, als mir im März der französische Astronom Louis Fabry von Marseille sein Werk »Étude sur la probabilité des comètes hyperboliques et l'origine des

comètes» zusandte, an dessen Schluss der Verfasser seine endgültige Ueberzeugung also ausspricht:

»On ne peut donc pas alléguer la disposition des éléments en faveur de l'opinion que les comètes viennent des espaces interstellaires et comme d'après les deux premiers chapitres, cette opinion ne peut pas rendre compte de l'absence des orbites hyperboliques, il faut admettre que les comètes sont des membres permanents du Système solaire, et par conséquent que ce Système s'étend bien au delà des planètes que nous connaissons. Ce sera notre conclusion finale.«

Mit dieser Schlussfolgerung stand genannter Comet im Widerspruch, da dessen Bahn zur Zeit seiner Sichtbarkeit zweifellos als Hyperbel erwiesen war. Es hatte daher für mich hohes Interesse, durch Berechnung der Störungen, welche dieser Comet innerhalb unseres Planetensystems erlitten habe, die Frage zu entscheiden, ob dessen Bahn eine Widerlegung oder Bestätigung der Theorie des Herrn Fabry