On Curves of Electromagnetic Induction

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XXXVII. *On Curves of Electromagnetic Induction.**By* W. GRANT*.

[Plates XV., XVI., & XVII.]

IN the month of June 1879 I communicated to the Physical Society a series of measurements of the conjugate positions of two equal circular coils† of wire whose axes were parallel to each other—that is to say, measurements of the relative positions of the coils when they were so placed that their coefficient of mutual induction became nothing. I also showed how, by the aid of these measurements, a curve could be drawn such that when one coil remained fixed, and the other was moved with its centre always in the curve and its axis parallel to that of the first coil, the mutual induction between the coils retained the constant value zero. An obvious extension of this investigation was to try to trace out some of the curves of constant positive or negative induction lying on opposite sides of the curve of no induction; and it is the results of experiments made for this purpose, along with those of others which were made for the purpose of tracing some of the curves of constant induction between the coils with their axes perpendicular to each other, which I have now to bring before the Society.

The arrangement of the apparatus and the nature of the experiments will be easily understood by reference to the annexed diagram.

A, B, C, and D are four coils of wire made as nearly identical in all respects as possible. Each coil consists of 182 turns of No. 24 silk-covered copper wire, of 0·065 centim. diameter, wound in a rectangular groove in a flat wooden reel. The inner circumferences of the coils are 21·95 centim.; their outer circumferences are 28·2 centim. The depth of the section of the coils perpendicular to the axis is 0·9947 centim.; the width of the groove parallel to the axis is 1·0 centim.: the section of the coils is therefore approximately one square centimetre. The mean radius is 3·99, or approximately 4·0

* Read June 11, 1881.

† Phil. Mag. for November, 1879.

centim. The resistances do not agree quite so closely; but their mean gives $R=2.4$ ohms at $11^{\circ}.5$ C.

The coils are mounted on straight guides which have grooves in their upper surfaces, along which the coils can be made to slide; and their positions are indicated by scales attached to the guides. The readings are obtained to centimetres and millimetres direct, and tenths of a millimetre by estimation. The coils A and B are mounted with their axes coincident on a straight guide EF, of sufficient length to allow their centres to be separated to a distance of 70 centimetres. The coils C and D are mounted on a pair of guides OP and MN, OP being about half the length of MN, and attached to it at right angles midway between its ends. The axes of the coils are parallel to OP and perpendicular to MN: hence

C can be moved parallel and D perpendicular to the axes without destroying the parallelism.

The coils A and C were connected with the battery L and with the key K to form the primary circuit; the coils B and D were connected with the galvanometer G, or sometimes with a telephone, to form the secondary circuit. Now it is evident that with this arrangement, in order that there may be no current in the secondary circuit on making and breaking the primary one, the coils C and D must be so placed that their mutual induction is equal to that between A and B; and, with reference to the secondary circuit, that the electromotive forces induced in the coils B and D must oppose each other. Hence, when A and B were clamped at a fixed distance apart, every pair of positions in which the coils C and D could be placed, so as not to cause a deflection of the galvanometer on making and breaking the primary circuit, were positions in which their mutual induction was constant, and determined one point of a curve of constant induction. Beginning with the coil D at P, the middle point of the guide MN, the coil C was moved until the induction between C and D balanced that between A and B. C was then moved nearer and nearer to P, being shifted a centimetre or so at a time; and the balance was restored in each case by moving D towards M or N. Now, as the arrangement is symmetrical with respect to the axis of C, it is evident that when a balance was obtained by displacing the coil D on one side of P, it could also be obtained by moving it to the corresponding position on the other side. In all cases these two balancing positions were found; and the half-difference of the readings on the scale MN, which was numbered from one end, was taken as the result of the experiment. In this way the measurements obtained were independent of any uncertainty in the determination of the point P. In using the numbers thus obtained for plotting a curve of constant induction, the distances read off along OP were taken as abscissæ, and the half-differences of the pairs of readings along MN were taken as ordinates. Thus in all the figures which accompany this paper, the axis of x is taken as coinciding with the axis of the primary coil with its centre fixed at the origin of coordinates, and the curves of constant induction are to be taken as representing

the paths traced out by the centre of the secondary coil in a plane containing the axes of x and y . The induced electromotive force changes sign when the centre of the secondary coil passes from one side to the other of the curve of no induction. In what follows, the induction is reckoned positive when the axes of the primary and secondary coils coincide. Hence, in the figures, all the curves lying between the axis of abscissæ and the zero-curve are to be taken as curves of positive induction; while those which lie further from the axis of abscissæ than the zero-curve are to be taken as curves of negative induction. The negative curves were traced experimentally in the same way as the positive curves, except that the two leading wires of one of the four coils were interchanged. Thus the inversion of electromotive force, due to the relative positions of the coils C and D, was counteracted by the inversion of the connections; and the inductive action between A and B could be balanced by that between C and D, just as in the measurements which gave the curves of positive induction.

The positive and negative divisions of the set of curves already alluded to, which, in what follows, are spoken of as the first set of curves, were each traced out with the apparatus arranged as in the diagram. But in order to trace out another set of curves, which are afterwards spoken of as the second set of curves, the axis of the coil D was set at right angles to that of C, but otherwise every thing remained unaltered. The zero-curve of the second set coincides with the axes of x and y ; and therefore it did not require to be traced; each of the other curves is complete in one quadrant. The induced electromotive force changes sign when the centre of the coil D passes from one side to the other of the axes of x or of y : hence in the figures the second set of curves are to be taken as positive in one quadrant and negative in another alternately. The measurements for the second set of curves were made experimentally in the same way as for the first set; but as the axes of the coils C and D were at right angles, there was a considerable part of the field near the origin in which no measurements could be obtained, on account of the coils coming into contact with each other.

The positive or negative induction corresponding to any one

of the plotted curves being equal to that between A and B when placed at some definite distance from each other, it became necessary, in order to place the curves so that the numerical value of the induction-coefficient might have a known relative for each, to determine the law according to which the induction between the two coaxal coils A and B varied with the distance between them. For this purpose the coil A was connected with a contact-key and with a battery of 20 Grove's cells to form the primary circuit; the secondary circuit was formed by connecting the coil B with a contact-key and with the coils C and D, which were placed close together and used as a galvanometer. The needle and mirror employed were rather heavy: this had the advantage of allowing the induced currents to exert their full effect on the needle before it had moved perceptibly from its position of equilibrium. The key was a double-contact key, so arranged that on depressing the key the primary circuit was completed; and immediately afterwards, when the current was established, the secondary circuit, which was already complete, was opened, and was kept open by a spring acting on the lower key, while the primary circuit was then broken. The time occupied in depressing and releasing the key was in general about one third of a second; hence the time during which the primary current circulated every time that contact was made was approximately that period. With this arrangement only the induced current on making was allowed to act on the galvanometer, while that on breaking was prevented from circulating in the secondary circuit.

The temperature of the primary coil was indicated by a delicate thermometer divided to tenths of a degree Centigrade, which had its bulb placed in contact with the silk covering of the coil. The variation of the temperature of the coil, due to the heating effect of the current during the experiments (as indicated by this thermometer) was approximately $1^{\circ}0$ C. The scale-readings obtained at the highest and lowest temperatures were found to agree so closely that it was unnecessary to make any correction on account of the variation of the temperature of the primary coil: its mean temperature during the experiments was $11^{\circ}5$ C.

As the state of the battery was liable to change and the

strength of the current to vary, it was important that errors arising from these causes should as far as possible be eliminated. In order to effect this, a double set of observations was taken, first with increasing distances between the coils, and next with decreasing distances. In every position deflections were taken towards both ends of the scale; so that the recorded results represent the means of never less than four readings; and in most cases they are the means of twelve or sixteen. The observations were begun with the coils A and B as close together as possible without touching, and were continued till the distance between them was 70 centim.; but beyond about 50 centim. the galvanometer above mentioned, formed by the juxtaposition of the coils C and D, was not sufficiently sensitive. It was therefore replaced for these distances by a delicate reflecting galvanometer; and the necessary reduction was made after the ratio of the indications of the two instruments had been found. The deflections of the needle were indicated by the movements of a spot of light on a circular scale of one metre radius. The readings were obtained in centimetres and millimetres, and, after having been corrected for the effect of damping, were reduced to degrees. The value of the induced electromotive force, or, in other words, the strength of the induced current, was in each case taken as being proportional to the sine of half the angle through which the needle was deflected—that is, proportional to the sine of one fourth of the corrected scale-reading reduced to degrees. In using the numbers thus obtained for plotting a curve which graphically represents the values of the coefficient of mutual induction M for different distances of the coils A and B, the distances read off along the scale EF for the positions of the coil B were taken as abscissæ, and the values obtained for the sines were taken as ordinates. The results of these experiments are given in Table I., where the columns headed x give distances between the coils, and the columns z give the corresponding values of the coefficient of mutual induction. The curve plotted by means of these numbers is given in fig. 1, Plate XV.; the centres of the small circles in that and the following curves represent the points found in the experiments.

TABLE I.

Distance between centres of coils = x .	Relative value of induction-coefficient = z .	Distance between centres of coils = x .	Relative value of induction-coefficient = z .
centim.		centim.	
1.6	39.567	15.0	0.9
1.7	37.731	16.0	0.758
1.8	35.936	17.0	0.631
1.9	34.016	18.0	0.557
2.0	32.251	19.0	0.48
2.2	29.551	20.0	0.423
2.5	25.593	21.0	0.364
3.0	21.194	22.0	0.321
3.5	17.341	23.0	0.284
4.0	14.468	24.0	0.25
5.0	10.227	25.0	0.224
6.0	7.449	30.0	0.131
7.0	5.504	35.0	0.083
8.0	4.163	40.0	0.056
9.0	3.158	45.0	0.040
10.0	2.528	50.0	0.029
11.0	2.021	55.0	0.022
12.0	1.641	60.0	0.017
13.0	1.338	65.0	0.013
14.0	1.093	70.0	0.011

This curve is related to the curves of constant induction which form the main subject of this paper as the vertical section of a surface is related to the contour-lines of that surface. Imagine, then, a surface such that the three rectangular coordinates x , y , and z of any point upon it represent respectively the distance of that point from the centre of the primary coil, x being measured parallel to the axis of the coil, y perpendicular to it, and z being taken equal to M , the coefficient of mutual induction between the primary coil and the secondary coil placed with its centre at the point x, y, z . Then, the curve just described may be viewed as a section of this surface in a plane containing the axes of x and z , the curves of constant induction may be looked upon as contour-lines of the surface, or as sections of it in planes parallel to the plane of x and y , and the curve whose coefficient of mutual induction is equal in value to z will pass through the point in question.

The values of M , whether positive or negative, are synonymous with those of z : hence in the figures the curves of variable induction, which are situated in vertical planes, are

to be taken as positive if they are above the plane of x and y , and negative if they are below it. Where the values of the coordinates of any curve are represented in the Tables by x and y , the curve is situated in a horizontal plane; where they are represented by x and z , the curve is situated in a vertical plane. On examining the numbers in this Table, it appears that the values of z or M are approximately inversely proportional to the cube of the distance from the centre of one coil to the mean circumference of the other—that is, that the product Mc^3 is approximately constant, c being put for $\sqrt{a^2 + x^2}$, where a is the mean radius of one of the coils. On closer examination it is seen that this product decreases somewhat from $x=0$ to about $x=a$, and then slowly increases for greater values of x .

By means of the curve (fig. 1, Plate XV.) it was easy to place the coils A and B so that their coefficient of mutual induction might have any desired value within the available range, and thus to assign determinate relative values to the coefficients of mutual induction corresponding to the curves to be traced out by the coils C and D. In Table II. the *first* and *fourth* columns give reference numbers referring to the several curves plotted in fig. 2, Pl. XVI., whose coordinates are given in Table III.; the *third* and *sixth* give the corresponding values of M , the coefficient of mutual induction; and the *second* and *fifth* the distance along the scale E F at which the coils A and B had to be clamped in order to obtain these values.

TABLE II.

No. of curve.	x .	M .	No. of curve.	x .	M .
1	0.5	Uncertain.			
2	1.0	Uncertain.			
3	2.025	32.0	13	31.6	— 0.125
4	3.72	16.0	14	24.0	— 0.25
5	5.78	8.0	15	18.7	— 0.5
6	8.15	4.0	16	14.45	— 1.0
7	11.05	2.0	17	11.05	— 2.0
8	14.45	1.0	18	8.15	— 4.0
9	18.7	0.5	19	5.78	— 8.0
10	24.0	0.25	20	3.72	— 16.0
11	31.6	0.125	21	2.025	— 32.0
12		Zero.			

The position on the scale *E F*, in which the coil *B* had to be placed in order to give the required value to the induction-coefficient of any curve, was found by inspecting Table II.; the coil was clamped in the position indicated; and a number of balancing pairs of positions of the coils *C* and *D* were then found.

In determining the values of the coordinates of the curve of zero induction the coils *C* and *D* only were employed. The primary circuit was the same as that in the diagram, omitting the coil *A*; the secondary circuit was also the same as that in the diagram, omitting the coil *B*. The coil *C* was brought as near as possible to the point *P*, and clamped in that position, sufficient room being left for the coil *D* to pass without touching it. *D* was then moved to a balancing position on one side of *P*, and then to the corresponding position on the other side of it, and the half-difference of the readings on the scale *M N* was taken as the value of the ordinate, the reading on the scale *O P* for the position of the coil *C* during the experiment being taken as the value of the corresponding abscissa.

The least distance between the centres of the coils *C* and *D* at which readings could be obtained when their axes were coincident was 1·6 centimetre, as their thickness prevented a nearer approach; the least distance between their axes at which readings could be obtained, when the mean planes of the coils were coincident or at a less distance than 1·6 centim. from each other, was about 9·2 centim. There was, therefore, in each quadrant a rectangular area of which these dimensions are the sides within which no results could be obtained.

In order to extend the experimental curves within this area, two coils, whose centres could be brought within 2 millim. of one another, were constructed from the remainder of the piece of wire from which the other four were made. Their diameters were approximately the same as those of the other coils; but, owing to their thinness and the consequent small number of convolutions of wire, their coefficient of mutual induction was much less; but this defect was compensated to some extent by their proximity. The lines of force due to them were slightly different in form from those due to the thick coils; but as in

most positions of the thin secondary coil the lines of force passed through it nearly at right angles, there was less objection to the use of the thin coils than if they had been at a greater distance apart. They were fitted one to each of the coils C and D, so that the same mountings served for both, and the readings were obtained from the same scales. That which was fitted to the former we shall call C_0 , that which was fitted to the latter we shall call D_0 .

In continuing the measurements within the area already mentioned, the coils C_0 and D_0 were placed in the same balancing pair of positions which C and D had occupied when the last point of a given positive curve was determined. The coil B was then moved towards F along the guide EF to a position in which the mutual induction between A and B balanced that between C_0 and D_0 ; this gave the desired value to the induction-coefficient of the curve; and the remaining points were found as in previous cases. Without again shifting the coil B, a negative curve, the induction-coefficient of which had the same relative value as that of the positive curve, could either be continued or completely traced out by interchanging the leading wires of the coil D_0 and passing it to the negative region. Thus the curves, whether positive or negative, could either be continued or completely traced out with the thin coils just as with the thick ones; and a constant value could be given to the mutual induction between the coils C_0 and D_0 in each pair of balancing positions, proportional to, although not the same as the value assigned to M for a given curve. Hence the path traced out by the centre of the coil D_0 was practically the same as that which would have been followed by the centre of the coil D, had it been possible to use it in tracing the same curve in that part of the field. The value of the induction-coefficient of each of the two curves nearest to the origin is uncertain, as the value of z which corresponds to that of x for either of these curves is beyond the range of the curve, fig. 1. A convenient, although uncertain value was given to the induction-coefficient of either of these curves by placing D_0 at the point P and moving C_0 towards O until the distance between the centres of C_0 and D_0 was equal to that between the origin and the point where the

curve was required to cross the axis of x , then balancing by the coil B and finding the remaining points of the curve. In completing several of these curves for which the value of M was comparatively small, it was found that the mutual induction between A and B was greater than that between C_0 and D_0 , even when B was placed at the extreme end of the scale E F. Hence, in order to reduce it in such cases, B was laid down flat on the guide and displaced towards E or F until a balance was obtained; and although the value of the induction-coefficient was uncertain, the mutual induction between A and B was equal to that between C_0 and D_0 when they occupied a balancing pair of positions previously occupied by C and D, and remained constant while A and B retained the same relative positions.

The coordinates of the first set of curves of constant induction are given in Table III., and are numbered in accordance with Table II.

The results obtained by means of the thin coils C_0 and D_0 are distinguished in the Table by being enclosed in square brackets [].

TABLE III.

Curve 1, M uncertain.		Curve 2, M uncertain.	
<i>x.</i>	<i>y.</i>	<i>x.</i>	<i>y.</i>
[0.2]	[0.97]	[0.2]	[1.77]
[0.3]	[0.81]	[0.5]	[1.43]
[0.4]	[0.58]	[0.7]	[1.14]
[0.5]	[0.0]	[0.9]	[0.67]
		[1.0]	[0.0]
Curve 3, M = 32.		Curve 4, M = 16.	
[0.2]	[3.06]	[0.2]	[4.35]
[1.0]	[2.4]	[1.0]	[4.07]
1.6	1.59	1.6	3.74
1.9	0.88	2.0	3.46
2.025	0.0	3.0	2.37
		3.5	1.35
		3.72	0.0
Curve 5, M = 8.		Curve 6, M = 4.	
[0.2]	[5.17]	[0.2]	[5.65]
[1.0]	[5.07]	[1.0]	[5.69]
1.6	4.985	1.6	5.695
2.0	4.885	2.0	5.705
3.0	4.505	3.0	5.68
4.0	3.855	4.0	5.5
5.0	2.705	5.0	5.15
5.5	1.66	6.0	4.53
5.78	0.0	7.0	3.5
		7.5	2.69
		8.0	1.345
		8.15	0.0
Curve 7, M = 2.		Curve 8, M = 1.	
[0.2]	[5.89]	[0.2]	[6.0]
[1.0]	[6.0]	[1.0]	[6.15]
1.6	6.105	1.6	6.325
2.0	6.195	2.0	6.475
3.0	6.43	3.0	6.89
4.0	6.585	4.0	7.28
6.0	6.495	6.0	7.84
8.0	5.69	8.0	7.84
9.0	4.935	10.0	7.25
10.0	3.685	12.0	5.925
10.5	2.73	13.0	4.685
10.8	1.8	14.0	2.885
11.05	0.0	14.3	1.555
		14.45	0.0

Table III. (*continued*).

Curve 9, $M=0.5$.		Curve 10, $M=0.25$.	
x .	y .	x .	y .
[0.2]	[6.07]	[0.2]	[6.12]
[1.0]	[6.24]	[1.0]	[6.29]
1.6	6.445	1.6	6.515
2.0	6.63	2.0	6.715
3.0	7.18	3.0	7.325
4.0	7.745	4.0	8.04
6.0	8.79	6.0	9.475
8.0	9.435	8.0	10.66
10.0	9.61	10.0	11.465
12.0	9.33	12.0	11.845
14.0	8.455	14.0	11.825
16.0	6.845	16.0	11.45
17.0	5.55	18.0	10.56
18.0	3.49	19.0	9.9
18.5	1.8	20.0	9.05
18.7	0.0	21.0	8.05
		22.0	6.775
		23.0	4.85
		23.5	3.2
		23.8	1.74
		24.0	0.0
Curve 11, $M=0.125$.			
[0.2]	[6.14]	20.0	15.0
[1.0]	[6.32]	22.0	14.45
1.6	6.545	24.0	13.55
2.0	6.76	25.5	12.525
3.0	7.42	27.0	11.175
4.0	8.215	28.0	10.05
6.0	9.965	29.0	8.725
8.0	11.635	30.0	6.83
10.0	13.025	30.5	5.5
12.0	14.075	31.0	3.95
14.0	14.82	31.4	2.05
16.0	15.2	31.6	0.0
18.0	15.2		
Curve 12, $M=zero$.			
[0.2]	[6.15]	12.0	18.055
[1.0]	[6.35]	13.0	19.45
1.6	6.575	14.0	20.775
2.0	6.8	15.0	22.15
2.5	7.135	16.0	23.54
3.0	7.5	17.0	24.9
4.0	8.38	18.0	26.275
5.0	9.395	19.0	27.65
6.0	10.505	20.0	29.0
7.0	11.675	21.0	30.376
8.0	12.80	22.0	31.775
9.0	14.115	23.0	33.13
10.0	15.4	24.0	34.5
11.0	16.745		

Table III. (*continued*).

Curve 13, $M = -0.125$.			Curve 14, $M = -0.25$.		
x .	y .	y' .	x .	y .	y' .
0		26.0	0		20.22
[0.2]	[6.16]		[0.2]	[6.18]	
[1.0]	[6.38]	25.97	[1.0]	[6.42]	20.105
1.6	6.615	25.86	1.6	6.64	19.97
2.0	6.845	25.81	2.0	6.89	19.83
3.0	7.6	25.5	3.0	7.72	19.35
4.0	8.58	25.0	4.0	8.855	18.56
5.0	9.795	24.35	5.0	10.475	17.365
6.0	11.275	23.4	5.7	12.4	15.75
7.0	13.33	21.95	5.9	14.05	14.05
7.7	15.325	19.95			
8.0	18.05	18.05			

Curve 15, $M = -0.5$.			Curve 16, $M = -1.0$.		
0		16.27	0		13.305
[0.2]	[6.2]		[0.2]	[6.24]	
[1.0]	[6.48]	16.21	[1.0]	[6.56]	13.125
1.6	6.725	15.975	1.6	6.865	12.85
2.0	7.0	15.79	2.0	7.195	12.6
3.0	7.96	15.09	3.0	8.73	11.27
4.0	9.62	13.74	3.2	9.49	10.605
4.4	11.9	12.33			

Curve 17, $M = -2.0$.			Curve 18, $M = -4.0$.		
0		11.085	0		9.445
[0.2]	[6.35]		[0.2]	[6.64]	
[1.0]	[6.7]	10.835	[0.5]	[6.81]	9.33
1.6	7.215	10.4	[0.7]	[7.02]	9.225
2.0	7.825	9.845	[1.0]	[7.445]	[8.85]
2.2	8.48	9.18	[1.2]	[8.15]	[8.15]

Curve 19, $M = -8.0$.			Curve 20, $M = -16.0$.		
[0.2]	[7.165]	[8.42]	0	7.45	8.0
[0.3]	[7.33]	[8.29]	0.1	7.52	7.95
[0.4]	[7.6]	[8.02]	0.15	7.7	7.83

Point of maximum negative
intensity,
No. 21, $M = -32.0$.

x .	y .
0	7.7

The first set of curves of constant induction are given in fig. 2, Plate XVI. Curve 20 and the point of maximum negative induction could not be found experimentally; they are therefore only given as near approximations. The rectangular space near the origin shows the region within which the auxiliary coils C_0 and D_0 (see p. 369) had to be employed. These curves represent the contour-lines of one quadrant of the surface above described, and show at what distance from the origin any curve cuts the axis of y . Knowing this, and knowing also the relative value of M or of z for each curve, we obtain a set of values for the coordinates of a curve which may be viewed as a section of the surface in question in a plane containing the axes of y and z . These coordinates are given in Table IV.; the curve plotted by means of them is given in fig. 3, Pl. XVII.; but the dotted part of it is only approximate.

TABLE IV.

y .	z .	y .	y' .	z .
2.6	39.55	6.15	26.0	— 0.125
3.07	32.0	6.17	20.22	— 0.25
4.38	16.0	6.19	16.27	— 0.5
5.18	8.0	6.23	13.305	— 1.0
5.64	4.0	6.33	11.085	— 2.0
5.88	2.0	6.58	9.44	— 4.0
5.99	1.0	7.02	8.48	— 8.0
6.06	0.5	7.45	8.0	— 16.0
6.11	0.25	7.7	7.7	— 32.0
6.13	0.125			
6.14	0.0			

In order to give a better idea of the symmetry of the curves of constant induction than can be got from fig. 2, the curves in that figure have been repeated in the other quadrants, and are given in fig. 4, Pl. XVII., for the whole of the magnetic field of the primary coil. Now every thing is symmetrical relatively to the axis of x , and each curve represents a section of a surface of revolution about that axis. Hence, if the curves are supposed to revolve round the axis of x , a number of surfaces of revolution will be generated, each of which will be a surface of constant induction, the surfaces of positive induction being separated from those of negative induction by the surface of no induction. The positive surfaces may be described as shells which enclose one another, and each of which turns inwards, closing up round the axis of x on each side of the origin. The zero-surface, which divides the positive from the

negative surfaces, instead of closing up, may be supposed to extend in space to an infinite distance. The negative surfaces turn outwards and close up in a plane containing the axes of x and z ; they are hollow circular rings which enclose one another, and which have their common centre at the origin of coordinates.

In the series of measurements for the determination of the second set of curves, the axis of the coil D was perpendicular to that of C, but otherwise every thing was arranged as in the diagram. The same values were given to M for these curves as for the others, by setting the coil B in the same positions on the scale EF which it had occupied during the previous experiments. These curves are numbered in accordance with Table V.

TABLE V.

Number of curve.	x .	M.
1	31.6	0.125
2	24.0	0.25
3	18.7	0.5
4	14.45	1.0
5	11.05	2.0
6	8.15	4.0

The zero-curve of this set coincides with the axes of coordinates; and, in general, when two similar coils with their axes perpendicular are employed, one as primary and the other as secondary, their mutual induction is zero when the axis of the one lies in the mean plane of the other. This shows that any curve of the second set for which the value of M is small must make a near approach to the axes of coordinates in the neighbourhood of the origin, and that a part of each curve in that region cannot be traced experimentally with the coils employed. The coordinates of these curves, as far as it was found possible to determine them experimentally, are given in Table VI.; but there is with this set a larger space near the origin within which the dimensions of the coils made it impossible to get measurements than was the case with the first set of curves. In the figures, the curves are continued conjecturally within this space by dotted lines. The second set of curves are given in fig. 5, Pl. XVI.

TABLE VI.

Curve 1, $M=0.125$.					
x .	y .	y' .	x .	y .	y' .
0.06		5.02	5.5	0.05	21.88
0.1		7.1	6.0	0.05	22.22
0.2		9.26	8.0	0.12	23.3
0.3		10.45	10.0	0.23	23.85
0.4		11.34	12.0	0.47	24.15
0.5		12.1	14.0	0.84	23.97
0.7		13.3	16.0	1.35	23.49
1.0		14.54	18.0	2.3	22.57
1.5		16.2	20.0	3.67	21.1
2.0		17.37	22.0	5.72	18.9
3.0		19.18	23.0	7.29	17.05
4.0		20.48	24.0	10.25	13.48
5.0		21.42			
Curve 2, $M=0.25$.					
x .	y .	y' .	x .	y .	y' .
0.1		5.09	5.0		17.26
0.2		7.18	5.5	0.1	17.54
0.3		8.28	6.0	0.11	17.77
0.4		9.16	8.0	0.24	18.36
0.5		9.77	10.0	0.53	18.45
0.7		10.74	12.0	1.03	18.02
1.0		11.82	14.0	1.9	17.1
1.5		13.13	16.0	3.44	15.45
2.0		14.13	17.0	4.55	14.22
3.0		15.55	18.0	6.51	12.27
4.0		16.54	18.4	8.0	10.4
Curve 3, $M=0.5$.			Curve 4, $M=1.0$.		
x .	y .	y' .	x .	y .	y' .
0.21		5.4	0.4		5.42
0.3		6.4	0.5		6.1
0.4		7.24	0.7		7.02
0.5		7.84	1.0		7.92
0.7		8.72	1.5		8.89
1.0		9.72	2.0		9.57
1.5		10.88	3.0		10.52
2.0		11.65	4.0		11.06
3.0		12.82	5.5	0.39	
4.0		13.59	6.0	0.46	11.46
5.0	0.17	14.1	8.0	1.01	11.04
5.5	0.19	14.28	10.0	2.33	9.64
6.0	0.23	14.38	11.0	3.85	8.11
8.0	0.5	14.54	11.4	5.0	6.9
10.0	1.07	14.06			
12.0	2.19	12.87			
13.0	3.17	11.82			
14.0	4.69	10.17			
14.5	6.34	8.35			

Table VI. (*continued*).

Curve 5, $M=2$.			Curve 6, $M=4$.		
x .	y .	y' .	x .	y .	y' .
0.8		5.54	1.6		5.5
1.0		6.17	1.8		5.76
1.6		7.27	2.0		6.0
2.0		7.76	2.5		6.45
3.0		8.54	3.0		6.72
4.0		8.91	4.0		6.96
5.0		9.0	5.0		6.82
5.2	0.69		5.5	1.55	6.66
6.0	0.92	8.85	6.0	1.9	6.31
7.0	1.37	8.41	6.5	2.45	5.78
8.0	2.23	7.58	7.0	3.62	4.61
9.0	4.13	5.56			

The positive and negative curves of the second set have the same form, and are positive in one quadrant and negative in another alternately. They have been treated as an independent set of curves; but, as will afterwards appear, they are not so, but are merely a special case in which the positive curves of the first set have become separated by the intervening zero-curve which coincides with the axes of x and y . In this case any corresponding pair of positive and negative curves, besides having the same form, have also the same linear dimensions. In every other case the form and linear dimensions of any corresponding pair of positive and negative curves are different, the linear dimension of the negative curve being always less than that of the positive. Hence this is the case in which the linear dimension of any negative curve is greatest, and in which that of the corresponding positive curve is least. The second set of curves, for the whole of the magnetic field, are given in fig. 6, Pl. XV., which shows the relative positions occupied by them in the various quadrants.

When a system of curves of constant induction have been obtained for a given pair of coils, they may be used to give the total inductive effect produced on one of the coils by a given change of relative position while the other coil is traversed by a current of known strength. To simplify the statement, suppose the primary coil in which there is a current of uniform strength C to remain at rest while the secondary coil is moved from a position such that the coefficient of mutual

induction is M_1 , to a position in which this coefficient becomes M_2 . Then, if t is the time occupied by the movement, the average electromotive force e which acts in the secondary circuit during the movement is

$$e = \frac{M_2 - M_1}{t} C;$$

and if r be the resistance of the secondary circuit, the average strength of the secondary current is

$$c = \frac{e}{r} = \frac{M_2 - M_1}{r t} C.$$

The total quantity, q , of electricity conveyed by the secondary current is consequently

$$q = c t = \frac{M_2 - M_1}{r} C,$$

and is therefore independent of the time occupied in the displacement of the secondary coil.

The values of M_1 and M_2 , the coefficients of induction corresponding to the initial and final positions of the secondary coil, can be obtained by inspection of diagrams of curves of constant induction such as those which accompany this paper. It is evident that the total secondary current is nothing in every case in which the secondary coil is moved so as to make $M_2 = M_1$ (that is, whenever the secondary coil in its final position is on the same curve as in its first position), and that the secondary current is positive when the absolute value of M_2 is greater than that of M_1 , and negative when M_2 is less than M_1 .

The spaces between the curves in the figures may be filled up as follows by other curves, for which the values of M lie within the available range of the curve fig. 1. Assume a value for M which will lie between its values for two curves already drawn; take this as the value of z , and find the point of the curve fig. 1 to which it corresponds. A point on the axis of x will thus be found, which will divide the distance between the two points on that axis, whose values are those of x for the given curves, into two parts whose ratio may easily be found. Then, if a series of points between the two given curves are found for which this ratio is constant, a curve joining these points will be a curve of constant induction, for which the value of M will be that which was assumed.

The curves of constant induction may also be employed in a graphic method for determining, at a number of points in the magnetic field, the direction of the resultant inductive effect on the secondary coil in the positions in which it was used in the measurements for determining the two sets of curves. If the curves figs. 2 and 5 are superposed as in fig. 7, Pl. XVI., we may resolve by the parallelogram of forces at a number of points where the curves intersect. For this purpose components are taken whose values are proportional to the values of M for the curves, and, from the point where any two curves intersect, lines are drawn proportional to these values for the two intersecting curves. The component which is proportional to the value of M for the curve of the first set is drawn parallel to the axis of x ; and that for the curve of the second set is drawn at right angles to it. Compounding, we then obtain as resultant a straight line proportional to the magnitude, and in the direction, of the resultant inductive effect on the secondary coil in two positions at right angles to each other. The arrows at the points of intersection of the curves in fig. 7 show the directions of these resultants. If the curves had been obtained by means of an indefinitely small secondary coil, the resultants would have been tangents to the lines of force, and the straight lines drawn through the same points at right angles to the resultants would have been tangents to the equipotential curves; but with the coils employed this is not strictly the case.

The two sets of curves in the figures are so related to one another that the one set may be viewed as a modified form of the other. If we begin with the axes of the primary and secondary coils parallel, and gradually increase the angle between them, the curves of maximum induction, as it were, carry the curves of constant induction round with them, while the curves of zero induction, in moving round, always form lines of demarcation which separate the positive from the negative regions. The positive curves of constant induction at the same time become distorted and gradually contract, the linear dimensions of each curve becoming less until the angle between the axes is about 90° . When this angle has become exactly 90° , the positive curves of constant induction have become separated into two distinct divisions by the intervening zero-curve (which coincides with the axes

of x and y), so that they then occupy two opposite quadrants. Now, both the positive and the negative curves move round together; but as the negative curves are carried round they become distorted and gradually expand, the linear dimensions of each curve becoming greater until the angle between the axes of the coils is 90° . When this, which is the extreme or limiting case, has been reached, the form and linear dimensions of any negative curve are the same as those of either of the two corresponding positive curves for which the value of M is the same, and the two sets of negative curves, which originally were each bisected by the axis of y , have then come to occupy the two remaining opposite quadrants. Thus the one set of curves of constant induction merges into the other, and the forms of both sets, which may be described as ovals, depend upon the angle between the axes of the primary and secondary coils.

We see, then, that the second set of curves, although found experimentally, are not entirely independent of the first set, and that being positive and negative in alternate quadrants, they have no corresponding surfaces of revolution, that the only curves of constant induction which have corresponding surfaces of revolution are those which are obtained when the axes of the primary and secondary coils are parallel, and that all curves of constant induction which are due to the coils when their axes make any other angle with each other are merely modified forms of them.

The arrangement of apparatus employed in this investigation, namely two similar coils in the primary and two other similar coils in the secondary circuit, was employed by Dove in his researches, and was called by him the "Differential Inductor" (*Annales de Chimie et de Physique*, tome iv. 1842).

The same arrangement was employed by Felici, a short account of whose researches is given in Maxwell's 'Electricity and Magnetism,' vol. ii. pp. 169-172.

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