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XVIII. Permanent magnets.—I

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It still remains to be found which are the most convenient dimensions for the globe and lens, so that the minimum of alteration of position will be requisite to obtain a sharp image during all times of the year; and also the best method of fixing the paper so that it may be easily changed and the time-scale marked on it.

XVIII. *Permanent Magnets.*—I.

By R. H. M. BOSANQUET, *St. John's College, Oxford.*

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

IN some notes on this subject (*Phil. Mag.* xv. pp. 205, 257, 309) I showed that, when a permanent magnet is divided into short lengths, its total moment is considerably diminished. I also endeavoured to treat the theory of the question from what I take to be Faraday's point of view. The measures then executed were only of one element—the moment. The view of the phenomena was therefore insufficient. Having now in my hands processes for determining the magnetic induction in the magnet, I have made determinations of this element, as well as of the moment, in three compound magnets. Further, I have measured in one magnet the number of ampere-turns of a uniformly wound coil necessary to reduce the external action temporarily to zero. These measures form the first part of this communication. I find that the properties of magnets, between the state of thin disks and bars of about five times the length of their diameter, are governed by extremely simple laws. Yet these are of such a nature that I am unable to imagine how the vague generalities about induction, which some consider sufficient to account for the facts I have previously brought forward, can be applied to them with any useful result.

I then show how the theory I have based on Faraday's views offers an account of the phenomena, and especially of the cases comprised within the above limits.

The complete determination of the functions involved in the case of bars of greater length will require additional experiments.

N = total number of pieces into which a whole bar is divided.

Then, if m, n be two numbers such that $mn = N$,

m is the number of separate bars in any combination,

n the number of pieces in each separate bar of the combination,

x is the length of the separate bar,

l " " " " whole bar,

M , total moment of m bars of n pieces each,

$\frac{M}{m}$, moment of one bar of n pieces,

$\mathfrak{B}$, magnetic induction (lines of force per unit area, field-intensity) through equatorial section of bar of n pieces,

R radius of bar, $S = \pi R^2$,

$F = \frac{4\pi \text{ moment}}{S\mathfrak{B}} = \text{focal distance}^*$,

$f = \frac{F}{x} = \text{focal distance, length unity.}$

Bar A.

$N=18$. $l = 28.65$ centim. $R = .978$ centim.

	n		1.	2.	3.	6.	9.	18.
M {	Obs.		418	687	928	1773	2372	3494
	Calc.		418	686	954	1758	2562	4974
Obs.-Calc.			0	+1	-26	+15	-190	-1480
$\frac{M}{m}$			23.2	76.0	154.7	591.0	1186	3494
<hr/>								
$\mathfrak{B}$ {	Obs.		38.9	74.5	123.1	275.4	361	594
	Corr.		46	92	138	276	..	..
<hr/>								
f			1.33	1.09	.982	.934	.959	.859
F			2.11	3.45	4.69	8.92	13.7	24.6

$$\left. \begin{array}{l} M \text{ calc.} = 150 + n \ 268 \\ \mathfrak{B} \text{ corr.} = n \ 46 \end{array} \right\} \text{ up to } n=6.$$

The limiting value of F when $n=0$ is, according to these expressions, $F_0 = .758$ centim.

$$\frac{F_0}{R} = .775.$$

(As to the calculation of F_0 see *post.* (2).)

The total moment has a finite but small limit ($=150$), when the bar is supposed to be divided into an infinite number of thin disks. As to the persistence of the linear laws in the case of actual disks of small thickness, see bars B and C *post.* The moment increases with increments proportional to those

* The points thus defined would be the poles, if the conception of poles were true and applicable; in bars the distance of the foci is usually a little less than that of the poles as ordinarily defined. In the present paper the foci are made use of as indicating by their position the distribution of the magnetism. They may be also regarded, with reserve, as the points of application of the resultants of the lines of force.

of n till after the point where $n=6$, after which the moment falls below the linear value.

Bar A.

Curve of Total Moments. n pieces in each of the m bars into which the whole is divided.

Bar A.

Curve of Induction (intensity), in equatorial section of bar of n pieces.

The moments were measured in the bifilar and the torsion-balance. They were originally measured in the bifilar with all the pieces in one tray in each state of combination. The discrepancy then found between the limiting moment and the moment of the current-turns required for temporary demagnetization (see *post.*), gave rise to the suspicion that the induction between the pieces dispersed about the tray was not negligible. The measures were then repeated, the smaller combinations being taken three together. The numbers thus obtained are those above given. The accordance with the temporary demagnetizing force being still imperfect, though

much improved, a set was made with each piece separately. The moments had all fallen, but in proportion; so that there was no evidence of induction interfering in the values which are given.

The constant of the bifilar was determined by vibrating a leaden rectangle having the same weight as the tray and magnet, and a known moment of inertia. The same suspension-stirrup was used in both cases; its moment of inertia was determined separately. An approximate value of H was employed.

The constant of the torsion-balance was determined by vibrating a lead rectangle. The variation of the torsion with the weight was inconsiderable in amount with the wire now employed, but was somewhat irregular. It is very difficult to get wire at all suitable for this purpose. Besides the variation of the constant of torsion, most wires take sets in loading and unloading to such an extent as to make them nearly useless. The best wire I have had is excessively hard. By dint of great labour reliable readings were obtained.

The first numbers given for $\mathfrak{B}$, and the figure*, represent the results of experiment. Rowland's method was employed; ten induction-coils for $n=18, 9, 6, 3, 2$, and fifteen for the singles. It is clear, however, and was so from a preliminary set, that these results need correction. For, according to the course of the numbers, the induction would vanish, and then become negative for finite lengths, which is inconceivable.

The source of the error is doubtless in the fact that, with short lengths, the coils cannot be sufficiently concentrated about the equatorial sections. The coils, though bound up closely, extend over a considerable part of the length of the short bars. I have found by direct experiment that spreading the induction-coils leads to considerable diminution of the observed induction. In the present series it was impossible even to fit the 10-coil closely to the bars, as the packing had to pass within the coils. But the coil of 15 was packed so as to fit the singles very closely indeed: the error of the corresponding value was thus reduced to about two thirds of its former amount.

There can be no doubt that the straight line which marks the course of the lower inductions ought to go to the origin, instead of falling just below it (see figure above). Then up to $n=6$ we have $\mathfrak{B}=n46$. This assumes that the value for $n=6$ is correct, since the error due to the coil must be by this time small. The strengths of the different short bars varied a

* As originally drawn. The line in the woodcut passes through the origin instead of a very little below it.

good deal. They were all examined, and an average taken for each type.

In order to ascertain whether the law as to a finite limiting moment held good for further subdivision, two short bars were constructed of thin disks of sheet steel, hardened, ground, and fitted. Of these B had the same diameter as A, C a diameter about twice as large. As these bars were short, $\mathfrak{B}$ could not be accurately ascertained for their subdivisions. But as both were well within the linear range, it is assumed that a law holds similar to that in the case of A. The value of $\mathfrak{B}$ for the whole bar was ascertained by experiment in each case.

Bar B.

N=20. $l=3.992$ centim. $R=.978$ centim.

n	1.	2.	4.	5.	10.	20.
$M \left\{ \begin{array}{l} \text{Obs.} \dots \\ \text{Calc.} \dots \end{array} \right.$	22.2 22.6	27.6 27.2	35.5 36.4	41.6 41.0	66.2 64.0	109.4 110
Obs.—Calc.	—4	+4	—9	+6	+2.2	—6
$\frac{M}{m}$	1.11	2.76	7.1	10.4	33.1	109.4
$\mathfrak{B} \left\{ \begin{array}{l} \text{Obs.} \dots \\ \text{Calc.} \dots \end{array} \right.$	5.05	10.1	20.2	25.2	50.5	101
f	4.60	2.86	1.84	1.73	1.37	1.13
F	.919	1.14	1.47	1.73	2.74	4.53 centim.

$M \text{ calc.} = 18.0 + n \cdot 4.6.$

$\mathfrak{B} \text{ calc.} = n \cdot 5.05.$

The limiting value $F_0 = .745$ centim.

$$\frac{F_0}{R} = .762.$$

Bar C.

N=18. $l=3.647$ centim. $R=1.905$ centim.

n	1.	2.	3.	6.	9.	18.
$M \left\{ \begin{array}{l} \text{Obs.} \dots \\ \text{Calc.} \dots \end{array} \right.$	26.3 26.0	29.1 29.3	32.7 32.6	42.0 42.5	50.1 52.4	82.3 82.1
Obs.—Calc.	+3	—2	+1	—5	—2.3	+2
$\frac{M}{m}$	1.46	3.23	5.45	14.0	25.0	82.3
$\mathfrak{B} \left\{ \begin{array}{l} \text{Obs.} \dots \\ \text{Calc.} \dots \end{array} \right.$	1.11	2.22	3.34	6.67	10.0	20.0
f	7.15	3.96	2.96	1.91	1.51	1.24
F	1.45	1.60	1.80	2.32	2.76	4.54 centim.

$$M \text{ calc.} = 22.7 + n \cdot 3.3.$$

$$\mathfrak{B} \text{ calc.} = n \cdot 1.111.$$

The limiting value $F_0 = 1.25$ centim.,

$$\frac{F_0}{R} = .657.$$

The expressions above arrived at for bars within the range covered by the linear relations may be put in a more convenient form by introducing x , the length of the cylinder, instead of the number n , into the expressions. Remembering that

$$mn = N, \quad mx = l,$$

and putting

$$M = M_0 + nM',$$

where M_0, M' are constants, the moment of a bar of length x is

$$M_x = \frac{M}{m} = \frac{x}{l} M_0 + \frac{x^2}{l^2} NM'.$$

Again, putting $\mathfrak{B} = n\mathfrak{B}'$, where $\mathfrak{B}'$ is a constant,

$$\mathfrak{B} = \frac{x}{l} N\mathfrak{B}'; \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

also

$$\begin{aligned} f &= \frac{4\pi M_x}{x\mathfrak{B}S} \\ &= \frac{4\pi}{\mathfrak{B}'S} \left(\frac{M_0}{Nx} + \frac{M'}{l} \right), \\ F &= fx = \frac{4\pi}{\mathfrak{B}'S} \left(\frac{M_0}{N} + \frac{M'}{l} x \right). \end{aligned}$$

In this last put $x=0$, then

$$F_0 = \frac{4\pi M_0}{N\mathfrak{B}'S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Applying these formulæ, we find for bars of length x , magnetized in the same manner as A, B, C respectively, the following expressions, within the linear range:—

	A.	B.	C.
M_x . .	$5.24x + 5.88x^2,$	$4.51x + 5.77x^2,$	$6.22x + 4.46x^2.$
$\mathfrak{B}$. .	$28.9x,$	$25.30x,$	$5.483x.$
f . . .	$\frac{.758}{x} + .851,$	$\frac{.746}{x} + .955,$	$\frac{1.25}{x} + .898.$
F . . .	$.758 + .851x,$	$.746 + .955x,$	$1.25 + .898x.$
$\frac{F_0}{R}$. .	$.775,$	$.762,$	$.657.$

The moments of B and C were determined by suspending the combinations in a tray in the torsion-balance.

I have hitherto assumed that the resultant magnetism of a disk, or short cylinder, acts, as it is commonly supposed to do, at the terminal surfaces. It is now clear that the foci, as I define them, are at a distance greater than the length of the short cylinder. This distance tends to a finite limit for thin disks. The limiting focal distance appears to be about three fourths of the radius.

This idea is new to me, though it is an obvious deduction from the general appearance of the lines of force. It will probably also hold for electromagnets.

By the foregoing investigation we have reduced the moment of the magnet to that of a series of infinitely thin disks, which correspond to the magnetic shells of the amperian theory. In the case of bar A, for example, the total moment of the whole magnet is more than twenty times as great as the limiting disk-moment. It now appeared to be of interest to see what would be the number of current-turns which would be required to demagnetize the magnet temporarily. It was to be expected that the moment of the current-turns required would be equal to the limiting disk-moment.

The magnet was enclosed in a coil of 255 turns having exactly its own length, and was placed on the deflection-scale near the needle so that it produced a deflection of about 20° . A current was then sent through the coil, so as to reduce the deflection to zero. The first time this is done, it always reduces the permanent magnetism a little. The value employed must therefore be ascertained before all the other measurements are made.

Ampere-turns which demagnetize A = 430.

$$\frac{\text{Ampere-turns} \times \text{area of magnet}}{10} = 129.$$

Limiting disk-moment of A . . . = 150.

It has been mentioned that the distribution of the magnetism in A was far from uniform. It seems probable that the uniform coil acts with an advantage over the irregular distribution in the magnet. And if this be the case, we must suppose that the actual disk-moment of 150 is balanced by the effect of a uniform coil corresponding to 129. In fact it is known that a difference of potential localized at a point in a bar, as

by a small coil carrying a current, produces a magnetic short circuit through the surrounding medium, instead of contributing in full measure to strengthening the general distribution. However this may be, the broad features are that the total moment of A, amounting to about 3500, is neutralized by the action of a coil which would be measured by 129, or somewhat less than the disk-moment*. It is clear that the consideration of moments lends no convenient shape to the problem.

Consider instead the magnet as decomposed into a number of thin disks, each of which can be represented according to the fundamental convention of electromagnetism by a current round its edge. Then the number of current-turns is represented by the quotient of the moment by the area.

$$\begin{aligned}\text{In magnet A, } \frac{150}{\text{area}} &= 49.9 \text{ current-turns,} \\ &= 499 \text{ ampere-turns.}\end{aligned}$$

Ampere-turns in demagnetizing coil = 430.

Allowing, as above, for the discrepancy, we assume for the present that the demagnetizing current-turns are equivalent to those of the limiting disk-moment.

If, therefore, we conceive of a permanent magnet as having a magnetic potential imbedded in its substance of the same nature as that which would be due to amperian currents, and acting on the steel in the same manner as an external potential produced by a coil, we have a convenient statement of the facts. For the amperian currents, or the potential, have been measured in two ways, viz. by cutting up into disks and by neutralization with a coil; and the results are accordant, allowing for the imperfections and difficulty of the investigation.

I proceed to develop the mode of accounting for the foregoing facts by Faraday's analogy of the voltaic circuit; that is to say, by the application of an analogue of Ohm's law:—

$$\text{Magnetic resistance} = \frac{\text{Diff. of Potential}}{\text{Intensity}}.$$

I formerly called Difference of Potential, Magnetomotive

* The radius of the coil being about 1.27 centim., its total moment was about 218. But it is conceived that the space external to the magnet cannot have exercised any considerable action on the needle, nor any on the magnet.

Force, to emphasize the analogy with electromotive force. Intensity is the same as Magnetic Induction, or lines of force per unit area, and is represented by $\mathfrak{B}$.

We will first consider what I have called the linear range; that is to say, the case of cylinders whose length does not exceed ten times their radius, including thin disks.

We suppose the steel to have a difference of magnetic potential imbedded in it per unit of length, corresponding to the values observed.

$$\text{Potential per unit length} = \frac{4\pi M_0}{l \times \text{area}} = P',$$

and

$$\text{Potential of length } x = P'x.$$

Now within the linear range $\mathfrak{B} = B'x$ say, where B' is the coefficient given at (1),

$$\therefore \text{Magnetic resistance} = \frac{P'}{B'},$$

and is constant within the linear range.

Further, by (1) the value of B' is $\frac{N\mathfrak{B}'}{l}$;

$$\therefore \text{Magnetic resistance} = \frac{\frac{4\pi M_0}{lS}}{\frac{N\mathfrak{B}'}{l}} = \frac{4\pi M_0}{N\mathfrak{B}'S},$$

which is the same as the value of F_0 (2).

It is at once apparent that this must be so. For the resistance is the altitude of a cylinder which would have the same resistance as the actual distribution; and in the case of the disk this is the same as the distance between the points of application of the resultants of the lines of force.

Hence the magnetic resistance throughout the linear range is constant: it is nearly three fourths of the radius in A and B, and about two thirds of the radius in C. This is nearly equal to the resistance at one end of an open organ-pipe. If we could arrive at this independently by an extension of reasoning such as that employed in connexion with the open ends of organ-pipes, we could found the whole theory upon it. As it is, the admission of this law throws some light on the laws of the linear range.

For let P' be the potential per unit length, and ρ the

magnetic resistance, then

$$\mathfrak{B} = \frac{P'x}{\rho};$$

which accounts for $\mathfrak{B}$ being proportional to the length, or to the thickness of the disk, the point which it seems impossible to deal with by the method of induction.

As to the focal distance, or distance between the points of application of the resultants, it is easy to see that for thin disks it must be nearly equal to some constant, which we may call F_0 . We can thus establish for the moments of very thin disks the formula

$$\text{Moment} = \frac{\mathfrak{B}SF_0}{4\pi} \quad (S = \text{area});$$

and putting for $\mathfrak{B}$ its value,

$$\text{Moment} = \frac{P'SF_0}{4\pi\rho} x;$$

or if a bar of length l be divided into m pieces, so that

$$x = \frac{l}{m},$$

$$\text{then the moment of one piece} \quad . \quad . \quad . \quad = \frac{P'SF_0}{4\pi\rho} \frac{l}{m},$$

$$\text{and total moment of bar cut up into pieces} = \frac{P'SF_0}{4\pi\rho} l,$$

a finite limit when m is great.

We cannot deal accurately with the intervening distributions, as they require the knowledge of the value of F in order to obtain the moments from the inductions. But we may consider the case, which is not far from the truth, for bars whose length is about half the limit of the linear range, viz. where we suppose $F=x$, or $f=1$, or that the foci are at the ends of the bar. It will be seen that this is very nearly true for $n=3$ in bar A. In this case, putting x for F ,

$$\text{Moment} = \frac{P'S}{4\pi\rho} x^2;$$

and for a short range the moment can be expressed approximately in terms of the square of the length. The true forms of the expressions throughout the linear range have been given above.

Then if, as before, a bar of length l be divided into m pieces,
so that $x = \frac{l}{m}$,

$$\text{Moment of one piece.} \quad = \frac{P'S}{4\pi\rho} \frac{l^2}{m^2},$$

$$\text{Total moment of } m \text{ pieces} = \frac{P'S}{4\pi\rho} \frac{l^2}{m}.$$

Although it would probably not be easy to determine F by independent reasoning of a simple character, it is not difficult to see that it must be the sum of F_0 and some fraction of x such as $\cdot 8$ or $\cdot 9$; and, admitting this, we obtain the form of the general expression for the moment within the linear range, thus:—

$$\begin{aligned} \text{Moment} &= \frac{\mathfrak{B}SF}{4\pi} \\ &= \frac{P'S}{4\pi\rho} x(F_0 + \cdot 9x) \text{ say,} \end{aligned}$$

which fairly represents the actual expressions.

In the region beyond the linear range the magnetic resistance increases; the fall of both moments and inductions below the linear values (see figures to bar A) can be represented as due to this, as is clear from the formula last written down. Some values of the magnetic resistances are subjoined.

Bar A.

Magnetic Resistance of Bar of n pieces.

	n		1, 2, 3, 6.	9.	18.	
Calc. {	From disk-moment	.	.	$\cdot 758$	$\cdot 869$	$1\cdot 056$
	From demagnetization	.	.	.	.	$\cdot 909$
Induction {	pieces in right order	.	.	.	.	$\cdot 891$
	under H,					
100 coils {	reversed in pairs	.	.	.	.	$1\cdot 011$
By induction produced in demagnetization, 10 coils						$\cdot 928$

$$\text{Formula for resistance from disk-moment} \quad = \frac{4\pi 150}{mS\mathfrak{B}},$$

$$\text{,, ,, ,, demagnetization} \quad = \frac{4\pi \times 43}{m\mathfrak{B}}.$$

There is a fair accordance considering the difficulties introduced by the want of uniformity. An endeavour will be made to obtain a magnet in more uniform condition.

The increase of the magnetic resistance with the length of the bar is without doubt to be ascribed to the intrinsic magnetic resistance of the steel.

The discovery of the precise laws of magnetic resistance in such cases must form the subject of further experiment.

The bars B and C so slightly affect an external needle that the demagnetization-experiment cannot be made on them with any accuracy. It is not in any case an experiment of a very accurate character, and perhaps too much stress may be laid on the want of exact accordance between its results and others.

XIX. *Notices respecting New Books.*

The Scientific Papers of JAMES PRESCOTT JOULE. Published by the Physical Society. Vol. I. London: Taylor and Francis. 1884.

WE have the authority of the Prime Minister for the statement that the present age is not one abounding in minds of the first order. Perhaps not, from the popular (*i. e.* the superficial) point of view. But that it can hold its own, in this respect, with any previous age, will undoubtedly be the verdict of posterity. For the true and only test of a "mind of the first order" is to be sought in the creations of that mind, and their influence for good upon the intellectual and moral progress of our race. It may be that there is a dearth of popular idols—literary, artistic, political, philosophic, &c. What of that? Alike in the study of the mathematician and in the laboratories of the chemist, the physicist, and the physiologist, are now being developed ideas, results, and processes which will have an influence on the future, in comparison with which the utmost efforts of statecraft are but as a drop in the bucket. Watt and Faraday, Wöhler and Liebig, Lister and Pasteur, Helmholtz and Thomson:—what achievement of statesman or warrior in any age can be compared, either in intellectual grandeur or in true usefulness, with theirs?

"Regum timendorum in proprios greges,
Reges in ipsos imperium est Jovis."

The quiet, all but unknown, student is in these latter days the real master of monarchs and of civilization. But such a theme as this is unsuited to a scientific journal, and must be left to the next *Vates sacer*, if such a phenomenon be any longer possible.

The volume before us is the work of a mind undoubtedly "of the first order." As usually happens to a great discoverer, Joule's earlier efforts were received with coldness, even suspicion; then (as Whewell remarked of Forbes and his Glacier-Theory) they passed to the second stage, in which they were acknowledged to be true, *but not new*. This stage, also, was of short duration, and Joule's transcendent merits are now all but universally recognized.