Two Unsolved Problems of Astronomy*

Planetary Perturbations and the Moon's Motion

By Arthur B. Turner

THE recent appearance of a valuable memoir¹ by Prof. Eric Doolittle brings again forcibly to mind the fact that the Newtonian theory of gravitation is not in complete harmony with all astronomical observations. From the vague ideas of Ptolemy of a universal pervading force, through the observations of the skilful Tycho Brahe, through the empirical laws laboriously evolved by Kepler from these, we come to the transcendent genius of Newton, who showed that a force of attraction varying with the product of the masses of two bodies and inversely as the square of the distance between their centers was sufficient to explain the motion of the planets, their satellites and even the motion of comets. Since Newton, many other minds—like those of Lagrange and Laplace—have woven a wonderful fabric based upon this law of universal attraction. Several times when theory and observation did not agree, these mathematicians were tempted to abandon the law, but in each case a little more labor, a few more neglected terms of a series, and theory was found to agree with observation. Since the time of the Herschels, double star astronomy and still more recently the spectroscopic binaries have testified in favor of the same law in distant stellar systems, as far as the accuracy of our observations is able to show. But there are two cases where the refinements of mathematical analysis and the increased accuracy of observations have not succeeded in verifying the Newtonian law. Let us suppose that the solar system consists of the sun and Mercury. The planet Mercury would then describe an ellipse about the sun as a focus, due to their mutual attraction—an ellipse which would remain for all time fixed in size and position. As soon as we introduce another planet, as Venus, this equilibrium—if we may call it by such a term—is disturbed, and Mercury will now travel in an ellipse which varies continuously, but slowly, both in size and position. Each planet path of any planet is, as a consequence, a very complicated curve about the sun, no point of which, however, is very distant from the fixed ellipse of the undisturbed planet.

If upon the quiet surface of water in a vessel, water is allowed slowly to drop, the surface will be disturbed, the particles rising and falling as the ripples spread over it. At the same time the surface will be slowly raised, due to the increase of volume of the water in the vessel. The result is that every particle of the surface has two distinct motions (a) periodic (motion in ripple) and (b) progressive (rising of the level). When the powerful mathematical analysis is applied to planetary perturbations a similar two-fold nature is found to exist. That is, we have periodic perturbations and secular variations. The periodic perturbations depend on the relative positions of the two planets in their respective orbits, that is, upon their heliocentric longitudes. They are completed in most cases in a few revolutions of the two planets. The secular variations, on the other hand, depend upon the position of the two orbits to one another, and are entirely independent of the positions of the planets in their respective orbits. A famous example of the first class is the "long period" of Jupiter and Saturn, taking about 918 years to run its course—the time of a complete revolution of a point of conjunction. This perturbation displaces Jupiter a total of 21' and Saturn 49' in longitude. It was Laplace who brought, after much difficulty, this observed displacement into harmony with the Newtonian law. Examples of the second class are: (1) the line of apsides of Neptune's orbit makes a complete revolution in 540,000 years; and (2) the line of nodes of Mercury, a revolution in 166,000 years.

Some interesting facts can be stated regarding the second class of perturbations.

(a) Each planet's mean distance and consequently its year remains entirely free from any secular variation.

(b) The line of nodes and apsides revolve continuously.

(c) The inclination and eccentricity confine their variations within very narrow limits as shown by Laplace. [(a) and (c) insure the stability of the solar system.]

(d) The eccentricities and inclinations being small, also the masses of the planets being small, in comparison with the sun; the secular variation produced in any element by the remaining planets can be computed separately, and added algebraically to get the total secular variation of that element. Modern investiga-

tions, which include higher powers in the development of the perturbing function than those of Laplace, indicate that these facts in regard to the stability of the solar system are not strictly true.

In 1818 the astronomer Gauss proved that if the mass of each planet were spread over its orbit to form an elliptic ring of infinitesimal thickness, and with a density at each point inversely proportional to the velocity of the body at that point, the mutual attractions of such rings would change their shapes and positions, and these changes would be exactly the same as the secular variations of the elements. The American astronomer G. W. Hill has given a very accurate method,² based on this theorem of Gauss, for the practical calculation of these secular variations. Since 1896 there have been appearing from time to time in the *Astronomical Journal* applications of Hill's Method by Professor Eric Doolittle, of the University of Pennsylvania. Final results for the four inner planets have recently been published.³ Professor Doolittle finds that observation⁴ gives an excess of 45".4 per century over the calculated motion of Mercury's perihelion; 10".6 in case of the node of Venus; and 8".7 in the case of the perihelion of Mars. The smallest of these residuals is over three times its probable error. The remaining variations agree with observation. The explanation of the above residuals is one of the old unsolved problems of astronomy. The first is the most important and was first stated by Leverrier in 1845 as 35"; Newcomb found it to be 43". Since Leverrier's announcement astronomers have been looking for other masses—intra-Mercurial planets—whose added attraction might harmonize the Newtonian law with observation. They used first the telescope, then later the camera during the total eclipses of the sun, but no images other than of known objects have ever been definitely seen. Director Campbell,⁵ of the Lick Observatory, feels sure from the results of the Croker Eclipse Expedition to the South Seas in 1903, that there exists no planet as bright as the 8th magnitude nearer the sun than Mercury. Dr. Perrine was able on this expedition to photograph stars to the 9th magnitude. He estimated that it would take about a million planets of the 8th magnitude, 30 miles in diameter, to produce the observed motion of Mercury's perihelion. In 1895 Newcomb tried the hypothesis of placing a ring of finely divided material between the sun and Mercury, and a second hypothesis of a ring between Mercury and Venus having a mass of $\frac{1}{37000000}$ of the sun. Both rings were inclined to the ecliptic. Each of these suggestions he found gave rise to certain conditions which made them untenable. The zodiacal light which extends for more than 90 degrees along the ecliptic, on the two sides of the sun, is supposed by astronomers to be sunlight reflected from finely divided material in the neighborhood of the sun. In 1906 H. H. Seeliger supposed this material to be arranged around the sun in the form of ellipsoids of revolution. He then computed the secular variation produced by the attraction of the ellipsoids: equated these to the residuals as determined by Newcomb in his "Astronomical Constants," excepting the four eccentricities which he considered as not perturbed by the ellipsoids. The solution of these equations gave:

Density of inner ellipsoid = $2.52 + 10^{-11}$ times sun's density.

Density of the outer ellipsoid = $0.0026 + 10^{-11}$ times sun's density.

Total mass = $35000 + 10^{-11}$ times sun's density.

Inclination of equator of inner ellipse = $6^{\circ}.95$

Longitude of node of inner ellipse = $40^{\circ}.03$

The length of the axes of the ellipsoids and the position of the equator of the outer one were arbitrarily fixed. The total mass stated above has been computed as being comparable to $\frac{1}{27}$ of a cubic foot of water per cubic kilometer of space. It remains to be seen whether further observations upon the zodiacal light will strengthen this plausible hypothesis of Seeliger.

A. Hall⁶ has tried the effect of changing the Newtonian law without adding any new masses to the solar system. His discussion is based on a theorem stated by Bertrand in 1873. That is, if the central force varies with the

n th power of the radius vector, the angle θ between the maximum and the minimum radii vectors will be $\theta =$

$\frac{\pi}{\sqrt{n+3}}$. When $n = -2$, which is the Newtonian

law, $\theta = \pi$ and the perihelion is fixed. He also found that a value of $n = -2.00000016$ is sufficient to move Mercury's perihelion 43" per century as found by Newcomb. There are no secular variations of the other elements due to this change in the value of n . But such a change Newcomb calculated would also make the moon's perigee move 1".5 faster per year. E. W. Brown, of Yale, however, finds that the motion of the moon's perigee is in agreement with the Newtonian law.⁷

The other old unsolved problem of gravitational astronomy was first stated by Halley, the friend of Newton, who found that the mean motion of the moon around the earth was slowly increasing; that is, the month was getting shorter. He arrived at this conclusion from a comparison of ancient with modern solar eclipses. It was left for Laplace, however, to apparently explain this phenomenon. It can easily be shown⁸ that the average disturbing effect of the sun upon the moon—the earth, moon and sun being in a line—varies, approximately, inversely as $(1 - e^2)^{\frac{3}{2}}$ where e is the eccentricity of the earth's orbit. Therefore if the earth's orbit becomes more circular (i. e. its eccentricity diminished), the average disturbing effect of the sun upon the moon during a revolution of the earth will be weakened, and as a consequence the earth's attraction will tend to accelerate the moon's mean motions and bring it closer to the earth. The computed secular variation of the earth's orbit shows that its eccentricity is slowly decreasing, sufficient to cause a shortening of the mean distance of the moon from the earth 14 feet in 200 years, and if continued indefinitely would bring the moon down upon the earth. This decrease will continue, however, for about 24,000 years, the orbit of the earth becoming almost a true circle, after which interval the eccentricity will increase for several thousands of years. Laplace computed the acceleration of the moon's mean motion and found it to be 10" per century. This was increased to 12".5 by later astronomers, and agreed with the observed discrepancy obtained from the comparison of eclipses. In 1853 Adams reduced this to about half, finding that the former computations were rather rough approximations. The present value as found by Brown, and considered as accurate, is 5".91. In 1877 Newcomb, from the comparison of the records of lunar eclipses, kept by Ptolemy and the Arabians, found the moon's observed acceleration to be 8".3 per century. This still leaves a difference of 2".4 per century between theory and observation. There are also some unexplained periodic perturbations of the moon, thought to be due to the action of planets, which complicates the solution of the problem of the moon's motion.

As a result, the tables which predict the longitudes of the moon for given dates, do not agree with observation. For instance, in 1910.5 the moon's longitude was 4".5 in excess of its predicted position.⁹ Also the computed times of contact in the case of solar eclipses are found to be in error a few seconds, and the plotted path of totality on the earth slightly shifted.

The early suggestion in 1856 by Ferrel that the discrepancy might be due to the lengthening of the unit of time, the day, by tidal friction or other causes seems to meet with the approval of most of the present day astronomers. Some, however, are suggesting a possible explanation through the finite propagation of gravitation based on the Principle of Relativity.¹⁰ Still others suppose that during a lunar eclipse the earth acts as a partial absorber of the gravitational force between the sun and the moon.¹¹

In conclusion it is well to remember that the discovery of aberration, nutation and the planet Neptune were due to the presence of small residuals; and that the whole structure of modern investigations depends upon the explanation of small quantitative changes. Some day this excess motion of Mercury and the moon may lead to the discovery of some phenomena as simple as those given above. The changing of the law seems further from our minds than the presence of some subtle force like light pressure or magnetism.

* Moulton's *Celestial Mechanics*, p. 242.

² The Observatory, No. 454, p. 391.

³ Poincaré *Comptes Rendus*, Vol. CXL., p. 1504; C. O. James, *Astronomical Journal*, 633-634.

⁴ W. De Sitter, *The Observatory*, No. 454.

* Reproduced from the *Journal of the Royal Astronomical Society of Canada*.

¹ "The Secular Variations of the Elements of the Orbits of the Four Inner Planets." *Transactions of the American Philosophical Society*, Volume XXII, Part 2.

² On Gauss's Method of Computing Secular Perturbations *Astronomical Papers of the American Ephemeris*, Vol. I.

³ *Transactions of the American Philosophical Society*, Vol. XXII, Part 2.

⁴ Newcomb—"Astronomical Constants."

⁵ *Popular Science Monthly*, Vol. LXXIV.

⁶ Doolittle—"Secular Variations of the Elements of the Four Inner Planets."

⁷ *Astronomical Journal*, Vol. XIV.