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“The Field Practice of Laying Out Transition Curves in Connection with Circular Curves for Railways, with observations on compensating Gradients for Curvature and on Vertical Curves.”

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THE method of ranging and setting out circular curves by the transit theodolite and 100-foot chain or steel band, described by the late Professor Rankine in 1843,¹ depends upon the principle that the angle subtended by any arc of a circle at any point in its circumference is one half the angle subtended by the same arc at the centre of the circle. To make the change of curvature gradual it is the Author's practice to introduce the Froude curve of adjustment, approaching a cubic parabola, as a transition curve between a straight line and a circular curve of uniform radius. The following is a description of the process of ranging and setting out in the field a transition curve in connection with a circular curve, taking the data of a curve for illustration from the Field Book.

The curve of the example referred to had the tangents produced on the ground, and the exterior angle of the intersection was measured with a 5-inch transit theodolite, which had a movable plate for centering it over a station-point and adjusting it in line between two distant points. The angle DAB, *Fig. 1*, measured $77^{\circ} 40'$, being the same as the central angle included between the radii BO and CO, the half of which, viz., $38^{\circ} 50' =$ the angle between the tangent and chord of the curve. A radius of 1,000 feet for the circular curve was the greatest that would fit the locality, and, being convenient, was adopted. It was considered that 40 miles an hour would be the highest speed of the train, and that the superelevation of the outer rail would not exceed

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¹ “Description of a Method of Laying Down Railway Curves on the Ground,” *Minutes of Proceedings Inst. C.E.*, vol. ii. (1843), p. 108.

5 inches or 6 inches; a length therefore of 250 feet would be ample to allow for the change in the gradient due to the super-elevation of the outer rail being gradual and sufficient for easing the curvature, which is equally important. This length was consequently fixed upon for the transition curve. So great a length, however, is not frequently attainable in rough or mountainous countries, owing to the curves overlapping, but 180 feet may be obtained in most difficult situations. The change of curvature is accompanied by a change in the superelevation of the rail, both increasing regularly from the tangent point to the circular arc. The length of the transition curve having been

Fig. 1.

decided, the shift proper to the circular arc was computed by the equation—Shift = (length of curve of adjustment)² ÷ 24 radius = 250² ÷ 24 × 1,000 feet = 2.6 feet = BE or CF. When the method of setting out transition curves here described is adopted, the middle point of the transition curve bisects the shift. The transition curve therefore in this instance extended 125 feet on each side of the shift, and its middle point was 1.3 foot from the tangents at B and C, *Figs. 1 and 2*. Having the off-set of 1.3 foot (BN) from the tangent at a distance of 125 feet (GN) on the curve or tangent from G (the difference in length between them being inappreciable in field measurements) any other off-set is readily found, as all off-sets from the tangent to any point in the

transition curve are proportional to the cubes of the distances along the curves from G or H, each being measured square from the tangent. In order to range and set out the curves, the following calculations were made by simple formulas to obtain the necessary data for the purpose, Chambers's Mathematical Tables being used.

The tangent distance AG or AH = (radius + shift) $\times$ natural tangent of angle $38^{\circ} 50'$ + half the transition curve = $1,002.6 \times 0.80498 + 125 = 807$ (AB or AC) + 125 (BG or CH) = 932 feet.

The distance AK from the intersection of the tangents to the centre of the curve = the external secant AP + shift PK (the arcs being concentric, the distance between them equals the shift) = (radius + shift) $\times$ natural secant - 1 of angle $38^{\circ} 50'$ + shift = $1,002.6 \times 0.28374 + 2.6 = 284.5$ (AP) + 2.6 (PK) = 287.1 feet.

The distance AM may be found thus (although it is seldom required):—KM = radius $\times$ natural secant - 1 of angle $38^{\circ} 50'$ = $1,000 \text{ feet} \times 0.28374 = 283.7$; AM = AK - KM = $287.1 - 283.7 = 3.4$ feet.

The length of the circular curve EF = radius $\times$ length of arc to radius 1 of angle $77^{\circ} 40'$, the whole arc (Table 33, circular measure of angles) = $1,000 \times 1.355539 = 1,355\frac{1}{2}$ feet.

Having obtained sufficient data for setting out the curve, the line AK was ranged, bisecting the angle BAC $102^{\circ} 20'$, and the distance 287 feet was measured. A rod was then erected at the middle point of the curve K with a view to check the deflection angles for the pegs on arriving at half the angle of the whole deflection, viz., $19^{\circ} 25'$; also to check the chainage at the middle peg 108 + 78 feet. The distance AC, 807 feet, and CH, 125 feet, were then measured from A along the tangent and marked; H with three pegs coloured red. At C the off-set for the shift F, 2.6 feet, was marked for the point over which to set the instrument when ranging the circular arc hereafter referred to. On the other tangent, the same distances were measured from A to B and G and marked in a similar way, also the off-set from the shift at E. It was found on arriving at G that the point corresponded with peg 100 + 75 feet in the chainage of the centre line. The following off-sets from the tangent were then computed for the pegs in the transition curve, by the aid of a Table of cubes or a slide rule.

The last off-set, 10.4 feet, is at the point where the transition curve becomes converted to the circular, and is always equal to

four times the shift. It is also equal to the square of the chord divided by twice the radius + shift = $125^2 \div 2 \times 1,000 + 2.6 = 15,625 \div 2,000 + 2.6 = 10.4$ feet. These pegs were driven at the proper distances from the tangent in accordance with the Table, *Fig. 2*, and the peg at L was painted red.

Half Transition Curve.	Distance along Curve from G.	Off-set at Middle of Transition Curve = Half Shift.	Off-set from Tangent, Feet.	Chainage. Peg.
125 ³	25 ³	1.3	0.01	101
125 ³	75 ³	1.3	0.28	101 + 50 feet.
..	125	Half Transition Curve	1.30	102
125 ³	175 ³		3.56	102 + 50 „
125 ³	225 ³		7.58	103
125 ³	250 ³	1.3	10.40	103 + 25 „

With regard to the degree of curvature at L, the point where the transition curve unites with the circular, both will be the

Fig. 2.

same, having an equal radius, if the method given for calculating the shift and off-set is followed.

The pegs were driven to the surface of the ground, 50 feet distance apart, for the levelling staff to be held on, and, in addition, a numbered peg standing above the surface of the ground was inserted every 100 feet. This was done on all sharp curves, but both level pegs and numbered pegs were driven 100 feet apart along flat curves and straight lines. For the purpose of ranging the circular curve, the theodolite was planted at E, the off-set for the shift previously noticed, and ranging rods were placed 2.6 feet = shift, from the straight line and tangent GA, some distance away from the instrument, giving the line from which to lay off the deflection angles. For 100-foot chains these were

calculated by dividing 1,718·87, the number of minutes in one half the arc to radius 1 (circular measure of angles) by the radius of the circular curve in 100-foot chains = $1,718·87 \div 10·00$ chains = say 171·9 minutes = $2^{\circ} 51'·9''$. One half of this was used for 50-foot chords and a proportion for any shorter length. The ranging was then commenced by laying off the deflection angle $3^{\circ} 35'$ for L, 125 feet, and continued in the usual way for circular curves. It should be noticed that the point given by the angle $3^{\circ} 35'$ and chord 125 feet coincides with the extreme off-set 10·4 feet square from the tangent. The chainmen were instructed as to the chord off-set to be used for 50-foot lengths, to check the pegs as they were inserted, and to see that neither angle nor chain was slipped. By way of illustration, a few of the angles are given, copied from the field-book.

Instrument at E.	Angle.	Peg.
Angle for 125 feet (half transition curve, giving point coinciding with extreme off-set 10·4 feet)	$3^{\circ} 35'$	103 + 25 feet.
Angle for 150 feet	$4^{\circ} 18'$	103 + 50 "
" " 200 "	$5^{\circ} 44'$	104
" " 250 "	$7^{\circ} 10'$	104 + 50 "
" " 300 "	$8^{\circ} 36'$	105
" " 350 "	$10^{\circ} 1'$	105 + 50 "
" " 400 "	$11^{\circ} 27'$	106

The transition curve at the other tangent was staked out in the same way by working backwards. The chainage at the point H was found by adding together the lengths of the transition and circular curves and adding them to the chainage at point G = $(125 + 1,355·54 + 125) + 10,075 =$ peg 116 + (say) 81 feet. The following were the off-sets to the transition curve with a few of the angles in the circular curve.

TRANSITION CURVE.

Peg.	Distance along Curve from H.	Off-set from Tangent at Right Angles.
	Feet.	Feet.
116 + 81 feet.	0	0·00
116 + 50 "	31	0·02
116 "	81	0·35
115 + 56 "	125	1·30
115 + 50 "	131	1·34
114 "	181	3·95
114 + 50 "	231	8·20
114 + 31 "	250.	10·40

CIRCULAR CURVE.

Instrument at F.	Angle.	Peg.
Angle for 125 feet . .	3° 35'	114 + 31 ft.
" " 156 " . .	4° 28'	114
" " 206 " . .	5° 54'	113 + 50 "
" " 256 " . .	7° 20'	113
" " 306 " . .	8° 46'	112 + 50 "
" " 356 " . .	10° 12'	112

The theodolite used for ranging this curve had two rows of figures for the degrees, one row running round to 360° and the other to 90° for each quadrant, in opposite directions, with a Vernier reading both to the right hand and to the left. Verniers for engineers' transit theodolites are best placed in front to the left of the telescope, as they do not necessitate any movement in the position of the observer when curve ranging.

The difference between the length of the transition curve LN and the circular curve LE, *Fig. 2*, or between the transition curve NG and the tangent BG, is too small to cause any error in the field work of ordinary railway curves. For instance, a curve having a radius of 1,500 feet with a transition curve 400 feet in length, would only have a difference between GB and GN not exceeding $\frac{1}{4}$ inch, and between LN and LE not exceeding 1 inch. The excess length of a transition curve, however, would have to be allowed for in the case of a very sharp circular curve having at the ends unusually long transition curves; but a curve with a radius of 660 feet and a transition curve limited to 250 feet need not have the difference adjusted, because it would be too small to cause any serious error in the staking out. A sharper curve, with longer transition curves, rarely occurs in practice on main lines; indeed, about 800 feet is the smallest radius for a curve used by the Author on a main line through a very rough district with steep sidelong ground. He has, however, used curves of 330 feet radius for sidings in yards on which heavy tank-engines are run. He finds that a minimum transition curve 180 feet long and a maximum one 400 feet long cover most cases arising in practice; transition curves longer or shorter than these are exceptional. He also finds that where sharp curves have to be adopted on a line of railway long transition curves are unattainable.

Curves of more than 2,500 feet or perhaps 3,000 feet radius scarcely require transition curves, and where circular curves only are used the superelevation of the rail is made to begin some distance back from the tangent point of the curve where the full

elevation is attained : one half of the superelevation being given by elevating the outer rail above, and the other half by depressing the inner below the centre line. The elevation of the rail is kept uniform throughout the length of the circular curve, and is run out with an easy gradient on to the straight at each end, requiring nice adjustment when the main line itself is on a gradient.

CURVE NOTE.

Angle	77° 40'
Radius	1,000·0 feet
Transition curve	250·0 „
Shift	2·6 „
Tangent, 807 + 125 feet.	932·0 „
Intersection to centre, 284·5 + 2·6 feet	287·1 „
Off-set from tangent to junction of transition and circular curve	10·4 „
Transition curve . . . . . begins peg 100 + 75 feet; ends peg 103 + 25 „	
„ „ „ „ 114 + 31 „ ; „ „ 116 + 81 „	
Circular „ „ „ 103 + 25 „ ; „ „ 114 + 31 „	
Chord off-set for 50 feet.	2·5 „
Angle for 50 feet	1° 26'

Where sharp curves occur on steep ascents, the gradients should be flattened so that the combined resistance due to curvature and gravity does not exceed that due to the maximum gradient on the straight portions of the line. Compensation for curvature may be made by reducing the gradient at the rate of 0·04 foot per 100 feet per degree and at a somewhat greater rate at stopping places, for a gauge of 4 feet 8½ inches. The degree corresponding to the different radii is double the angle given in the column of the Table in the Appendix, headed "Deflection Angle for 100 feet." For uniting steep gradients the Author has used for many years parabolic vertical curves varying in length between 120 feet and 300 feet on each side of the intersection. The following is the office method of calculating and plotting these. Whatever length of curve is assumed, the difference in height between the two gradients at the point vertically over or under either end is ascertained by producing the opposite gradient from the apex. Having this height or vertical ordinate at one end of the curve the intermediate heights required are easily found, as all are proportional to the square of the horizontal distance from the other end of the curve. These are calculated and measured from the gradient line, their height being reduced to that above datum, and they are then plotted and figured on the working section.

The Paper is accompanied by two drawings, from which the *Figs.* in the text have been prepared.

APPENDIX.

TABLE OF CIRCULAR CURVES WITH SUITABLE LENGTHS OF TRANSITION CURVES AND THE SHIFT PROPER FOR EACH.

Radius of Circular Curve.	Deflection Angle for 100 Feet.	Length of Transition Curve, G L or H L.	Shift B E or C F.	Radius of Circular Curve.	Deflection Angle for 100 Feet.	Length of Transition Curve, G L or H L.	Shift B E or C F.
Feet.	Deg. Min.	Feet.	Feet.	Feet.	Deg. Min.	Feet.	Feet.
3,000·0	0 57·30	200	0·55	1,146·3	2 30·00	200	1·45
		300	1·25			240	2·09
		400	2·22			260	2·46
2,865·0	1 0·00	200	0·58			280	2·85
		300	1·31			300	3·27
		400	2·32			340	4·20
2,640·0	1 5·11	200	0·63	1,000·0	2 52·00	200	1·66
		260	1·06			220	2·02
		300	1·42			250	2·60
2,000·0	1 26·00	400	2·52			280	3·26
		200	0·83			300	3·75
		250	1·30			340	4·81
1,910·0	1 30·00	360	2·70	955·4	3 0·00	200	1·75
		400	3·33			240	2·51
		200	0·87			260	2·95
1,637·3	1 45·00	240	1·26			280	3·42
		280	1·71			300	3·92
1,432·7	2 0·00	300	1·96			340	5·03
		360	2·83	819·0	3 30·00	200	2·04
		400	3·49			240	2·93
1,320·0	2 10·00	200	1·02			260	3·44
		240	1·46			280	4·00
		280	2·00			300	4·57
1,320·0	2 10·00	300	2·28	800·0	3 35·00	320	5·20
		360	3·30			200	2·08
		400	4·07			240	3·00
1,432·7	2 0·00	200	1·16			280	4·08
		240	1·67			300	4·68
		280	2·28			180	1·88
1,320·0	2 10·00	300	2·62	716·8	4 0·00	200	2·32
		340	3·36			220	2·81
		400	4·65			240	3·35
1,320·0	2 10·00	200	1·26			260	3·93
		230	1·67			180	2·04
		360	2·13	660·0	4 20·50	200	2·52
1,320·0	2 10·00	300	2·84			220	3·05
		340	3·65			240	3·63
						250	3·04

Divide the shift by 2 for the middle off-set, and multiply the shift by 4 for the extreme off-set. Having these off-sets, any others may be found, all being proportional to the cube of the distance along the curve from the tangent points, and are to be measured perpendicular to the tangent.