XXX. On the determination of the distance of a given point on the earth's surface at, or very near, the level of the sea, by observations on its depression from a known height above it

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	per cent.
Of the <i>lodes</i> yielding both tin and copper ores . . .	37·2
... copper <i>lodes</i>	17·7
Of all the <i>lodes</i> the proportion <i>heaved</i> towards the <i>right-hand</i> is	51·1
Of the tin <i>lodes</i>	56·0
... <i>lodes</i> yielding both tin and copper ores . . .	44·0
... copper <i>lodes</i>	52·4
Of all the <i>lodes</i> the proportion <i>heaved</i> towards the <i>left-hand</i> is	26·2
Of the tin <i>lodes</i>	26·0
... <i>lodes</i> yielding both tin and copper ores . . .	18·6
... copper <i>lodes</i>	29·8
Of all the <i>lodes</i> the proportion <i>heaved</i> towards the <i>greater angle</i> is	63·5
Of the tin <i>lodes</i>	52·0
... <i>lodes</i> yielding both tin and copper ores . . .	56·0
... copper <i>lodes</i>	74·2
Of all the <i>lodes</i> the proportion <i>heaved</i> towards the <i>smaller angle</i> is	12·9
Of the tin <i>lodes</i>	30·0
... <i>lodes</i> yielding both tin and copper ores . . .	6·8
... copper <i>lodes</i>	8·8
The mean distance of the <i>heaves</i>	feet.
Of all the <i>lodes</i> is	16·4
... tin <i>lodes</i>	15·4
... <i>lodes</i> yielding both tin and copper ores . . .	14·6
... copper <i>lodes</i>	17·5
Towards the <i>right-hand</i>	18·7
... <i>left-hand</i>	12·0
... greater angle	16·3
... smaller angle	17·1

Gongo Soco Gold Mines, Brazil,
October 5, 1843.

W. J. HENWOOD.

XXX. *On the Determination of the Distance of a given point on the Earth's Surface at, or very near, the level of the sea, by observations on its depression from a known height above it.* By WILLIAM GALBRAITH, Esq., F.R.A.S.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

SHOULD the inclosed short paper meet with your approbation, you will oblige me by giving it a place in your Journal. It is intended as an improved solution of that problem in Horsburgh's edition of Mackenzie's Treatise on Marine

Surveying, in which both the effects of curvature and refraction are neglected, that in considerable distances are indispensable where even tolerable accuracy is required. I have occasionally found it useful, and others may probably also find it so.

I am, Gentlemen,

Your obedient Servant,

54 South Bridge, Edinburgh,
December 28, 1843.

WILLIAM GALBRAITH.

1. Let B H F be a section of the earth, A B the given height, the angle E A H, and B H the required distance.

In the triangle A B I right-angled at B, there are given A B, the height of the point A above B, the earth's surface at the mean level of the sea B H F, and the angle B A I, the complement of the angle of depression E A H.

Now let the height be denoted by h and the depression by D , then $R : \cot D :: h : B I = \cot D \times h$, a first approximation to the distance B H or K, the chord in this instance nearly when not great, or a small part of the distance of the visible horizon only.

But $I B H = M H B = H F B = \frac{1}{2} H C B$, of which an approximate value may be found from that of B I, when neither the height nor distance is great. For this purpose let ρ be the radius of the earth as usual, R'' an arc equal to the radius in seconds, and A'' the arc in seconds which measures the angle H F B, then

$$A'' = \frac{R''}{2\rho} \cot D \cdot h \text{ nearly} \quad (1.)$$

In the triangle A B H, the angle A H B = A I B = $D - A''$. Again, in the triangle A H B, $\sin A H B : \sin H A B :: A B : B H = K$,

or $K = \csc A H B \sin H A B \cdot A B = \csc (D - A'') \cos D \cdot h$ (2.)

The effect of refraction is expressed by $n K$ or $2 n A''$, n being the coefficient of refraction, a quantity by which D must be increased.

$$D - A'' + 2 n A'' = D - (1 - 2 n A'') = D - (0.5 - n) A''.$$

$$\text{Putting } a'' = (0.5 - n) A'' \dots (x),$$

$$\text{then } 2 n A'' = \frac{n}{0.5 - n} a'' = \alpha'' \dots (7.) \quad (3.)$$

Substituting these in formula (2.), and it becomes

$$K = \csc (D - \alpha'') \cos (D + \alpha'') h \quad (4.)$$

It is obvious also that

$$a'' = (0.5 - n) \frac{R''}{\rho} K = (0.5 - n) f K, \dots (5.)$$

in which f is the factor to convert feet on the earth's surface into seconds of arc in any given direction.

2. Formula (1.) fails to determine A'' with sufficient accuracy when the height h is considerable, and the distance K extends to, or near to, the visible horizon.

In this case we may proceed as follows:—

$$BC : AC :: \sin HAC : \sin AHC = \frac{AC}{BC} \sin HAC.$$

Whence $180^\circ - (AHC + HAC) = HCA$,

and $\frac{1}{2} HCB = HFB = IBH = A''$.

Wherefore $AIB - IBH = AHB$;

$$\rho : \rho + h :: \cos D : \sin AHC = \left(1 + \frac{h}{\rho}\right) \cos D. \quad (6.)$$

But h being very small in comparison with ρ , $1 + \frac{h}{\rho}$ exceeds unity by a very small fraction, therefore $\log \left(1 + \frac{h}{\rho}\right) = \frac{\mu h}{\rho}$, in which μ is the logarithmic modulus. If ρ be taken equal to the radius of the equator as a mean value, then $\log \left(1 + \frac{h}{\rho}\right) = .83171678$. Instead of the mean value of ρ being taken as above,

$$\log \rho = \log R'' - \log f \text{ exactly} \dots (7.)$$

$$\text{Hence} \quad \log \sin AHC = \log \cos D + \frac{\mu}{\rho} h \dots (8.)$$

always obtuse, of which in this problem its excess above 90° is taken.

In the preceding formula it is supposed that the coefficient of refraction, n , is determined by observation or calculation. The mean value, however, amounting to 0.08 of the intercepted arc, will frequently be sufficient. In this case the effect will be 0.08 K , or 0.16 A'' , a quantity to be added to D while A'' must be subtracted.

$$\text{Hence} \quad D - A'' + 0.16 A'' = D - 0.84 A'' = D - a'',$$

by making $a'' = 0.84 A''$.

Recurring to formula (1.), by substitution we have

$$\log a'' = \text{const.} \log 7.6170579 + \log \cot D + \log h \dots (9.)$$

$$\text{But } a'' = \frac{n}{0.5 - n} a'' = \frac{0.08}{0.42} a'' = 0.1904762 a'' = \frac{1}{5} a'' \text{ nearly.}$$

By combining this with the former, there results

$$\begin{aligned} \text{first const. log } 7.6170579 + \text{log of } 0.1904762 &= 6.8968986 \\ &= \text{the second constant logarithm.} \end{aligned}$$

Hence

$$\log a'' = 1^{\text{st}} \text{ const. log } 7.6170579 + \log K \quad . \quad (10.)$$

$$\log a'' = 2^{\text{nd}} \text{ const. log } 6.8968986 + \log K, \quad . \quad (11.)$$

which two results being substituted in formula (4.), and the operation repeated, will give K sufficiently correct, when the distance is only a small, or even a moderate part of that of the visible horizon.

In practice it will be found convenient and sufficiently accurate to use the mean quantities in the first and second approximations, and the true quantities under given circumstances for the last approximation only.

3. Again, let the angle A H C be denoted by H, and $\sin H = 2 \sin \frac{1}{2} H \cos \frac{1}{2} H = 2 \sin \frac{1}{2} H (1 - \sin^2 \frac{1}{2} H)^{\frac{1}{2}}$, therefore

$$2 \sin \frac{1}{2} H (1 - \sin^2 \frac{1}{2} H)^{\frac{1}{2}} = \frac{g+h}{g} \cos D.$$

Squaring both sides, and there results

$$4 \sin^2 \frac{1}{2} H - 4 \sin^4 \frac{1}{2} H = \left(\frac{g+h}{g} \right)^2 \cos^2 D,$$

or, by arranging,

$$4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H = - \left(\frac{g+h}{g} \right)^2 \cos^2 D.$$

Add 1 to each side of the equation, and

$$4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H + 1 = 1 - \left(\frac{g+h}{g} \right)^2 \cos^2 D,$$

$$\begin{aligned} \text{or } 4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H + 1 &= \left(1 + \frac{g+h}{g} \right) \cos D \\ &\times \left(1 - \frac{g+h}{g} \right) \cos D. \end{aligned}$$

Multiplying and dividing by $\left(\frac{g+h}{g} \right)^2$, and

$$\begin{aligned} 4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H + 1 &= \left(\frac{g+h}{g} \right)^2 \left(\frac{g}{g+h} + \cos D \right) \left(\frac{g}{g+h} - \cos D \right). \end{aligned}$$

But $\frac{g+h}{g} = \sec D_p$, D_p being the depression of the horizon,

$$\begin{aligned} 4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H + 1 &= \sec^2 D_p (\cos D_p + \cos D) (\cos D_p - \cos D). \\ \text{Since } (\cos D_p + \cos D) (\cos D_p - \cos D) &= \sin (D + D_p) \sin (D - D_p) \\ 4 \sin^4 \frac{1}{2} H - 4 \sin^2 \frac{1}{2} H + 1 &= \sec^2 D_p \{ \sin (D + D_p) \sin (D - D_p) \}. \end{aligned}$$

Extracting the square root, and there results

$$2 \sin^2 \frac{1}{2} H - 1 = \pm \sec D_i \{ \sin (D + D_i) \sin (D - D_i) \}^{\frac{1}{2}}. \quad (12.)$$

$$2 \sin^2 \frac{1}{2} H = 1 \pm \sec D_i \{ \sin (D + D_i) \sin (D - D_i) \}^{\frac{1}{2}}, \text{ and}$$

$$\sin \frac{1}{2} H = \{ \frac{1}{2} (1 \pm \sec D_i [\sin (D + D_i) \sin (D - D_i)]^{\frac{1}{2}} \}^{\frac{1}{2}}. \quad (13.)$$

From formula (16.) it appears that $H - 90^\circ = 2 \sin^2 \frac{1}{2} H - 1$, the radius, or $\sin 90^\circ$ being unity, therefore calling $H - 90^\circ = d$, we have

$$\sin d = \sec D_i \{ \sin (D + D_i) \sin (D - D_i) \}^{\frac{1}{2}}. \quad (14.)$$

the most useful form, since d in this case is the arc required.

Remarks.—1. If the *negative* sign in formula (13.) be employed, the resulting value of H will be the supplement of that by the positive.

2. Since D is the depression of the given point, and D_i that of the horizon, $D - D_i$ will then be the difference between the depression of the given point D and that of the horizon D_i , which difference may be measured with a sextant or reflecting circle, while the depression of the horizon may be computed by the usual formulæ; and then D and $D - D_i$ become known without an altitude circle on shore, as required in the first method.

3. From the same figure the method of determining the height above the level of the sea may be very readily derived.

Let H' be the visible horizon, the angle $E A H' = D$ the depression, $A H'$ a tangent to the horizon at H' passing through the point of observation A , and $A B = h$, the required height.

From the property of the circle $E A H' = H' C A = 2 H' F D$, and $H' F B = \frac{1}{2} D$. The triangles $A B H'$, $H' B F$ give

$$\sin \frac{1}{2} D : \cos D :: A B : B H' = A B \times \frac{\cos D}{\sin \frac{1}{2} D},$$

and

$$\begin{aligned} \sin D : \cos \frac{1}{2} D :: B H' : H' C &= A B \times \frac{\cos D \cos \frac{1}{2} D}{\sin D \sin \frac{1}{2} D} \\ &= A B \cot D \cot \frac{1}{2} D. \end{aligned}$$

Whence $h = H' C \tan D \tan \frac{1}{2} D = \frac{1}{2} \rho \tan^2 D$ nearly. (15.)

Introducing the effects of refraction, n ,

$$h = \frac{1}{2} \rho (1 + n)^2 \tan^2 D \quad (16.)$$

$$\text{or } h = \frac{1}{2} \rho (1 + n) \sin^2 1'' D'' = 0.5832 \rho \sin^2 1'' D'' \quad (17.)$$

when the dip is expressed in seconds and $n = 0.08$,

$$\text{const. log or log of } 0.5832 \rho \sin^2 1'' = 6.457582.$$