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XXVIII. *On a new Method of reducing Lunar Observations for the Determination of the Longitude.* By CHARLES RUMKER, Esq., F.R.A.S.*

To Lieut.-General Sir Thomas Macdougall Brisbane, K.C.B., F.R.S. L. & E., F. Ast. S., &c.

SIR,

THE lively interest which you took in the determination of the longitude by lunars, and the success which attended your observations, prompt me to propose to you the following method of their reduction, which admits of greater accuracy when either altitude is low. Already when I had the honour of accompanying you on your passage to New South Wales, I had an opportunity of remarking the necessity of a more correct allowance for the refraction of the lower object. This subject has since been resumed by a celebrated astronomer, whose method appears to me, however, not likely to be generally adopted by seafaring men, requiring, moreover, a particular ephemeris, which becomes inapplicable under ordinary circumstances, where the usual methods are sufficiently correct. I trust, therefore, that the following method, which requires no other ephemeris than the Nautical Almanac, and may, as circumstances require, be computed with more or less precision, will be found of more practical utility.

The object of the present lines is the correction of the error committed in the usual methods of clearing the apparent distance between sun and moon by taking out the refractions

* Communicated by Sir T. M. Brisbane, K.C.B. &c.

for their central altitudes, whereas it is the refraction of those points of their limbs whereof the distance is actually measured that should be used in the calculation. As introduction, it may not be useless to remind, that the usual methods of clearing the distance, which suppose the altitudes known by observation, can be classed in direct and approximatory ones. For as all the former methods are derived from the equation existing between the sides of the two triangles formed by the apparent zenith distance and apparent distance, and the true zenith distance and true distance, and only differ by slight variations in the manner of finding thence the true distance, or side of the latter triangle opposite to the angle at the zenith which is common to both triangles, thus all the latter methods agree in the former or apparent triangle, the angles at the sun and moon, and the sides adjacent thereto in two right-angled triangles; having for hypotenuses the corrections of the sun's and moon's altitudes, these sides are the corresponding corrections of the distance by a first approximation.

Let D designate the apparent distance of centres, S and M the above angles at the sun and moon, $\varphi - \pi$ the differences between parallax and refraction or corrections of altitudes, then is the true distance of centres $= D + \cosine S. (\varphi - \pi) + \cosine M. (\varphi' - \pi') + \alpha (\varphi - \pi)^2 + \beta (\varphi' - \pi')^2 + \dots$

The moon's parallax being greater than her refraction, the second correction becomes negative; and when one of the angles is obtuse, its cosine takes the opposite sign. The fundamental formula of all approximatory methods is, therefore,

$$\begin{aligned} \text{true distance} = D + \left(\frac{\sin h - \sin H \cos D}{\cosin H \sin D} \right) (\varphi - \pi) \\ + \left(\frac{\sin H - \sin h \cos D}{\cosin h \sin D} \right) (\varphi - \pi) + \dots \end{aligned}$$

where φ and π refer to that altitude h or H which stands first and by itself in the parenthesis. Lions obtained by executing the division,

$$D + (\sin h \secant H \cscant D - \tang H \cotang D) (\varphi - \pi) + (\sin H \secant h \cscant D - \tang h \cotang D) (\varphi - \pi).$$

$$\text{By calling } \left(\frac{\sin h - \sin H \cosin D}{\cosin H \sin D} \right) (\varphi - \pi)$$

$$\left(1 - 1 + \frac{\sin h - \sin H \cosin D}{\cosin H \sin D} \right) (\varphi - \pi)$$

$$= \left(\frac{1 - 2 \cosin \frac{1}{2}(D + H + h) \sin \frac{1}{2}(D + H - h)}{\cosin H \sin D} \right) (\varphi - \pi),$$

we obtain the one part of Mendoza y Rio's approximate formula, to which the other is analogous.

By separating in Lions's method the moon's parallax from the refraction, we obtain the thence proceeding correction of

the distance = hor. par. $\left[\frac{\sin \odot\text{'s alt.}}{\tan D} - \frac{\sin \ominus\text{'s alt.}}{\sin D} \right]$, which

is identical in Elford's, Thomson's, and Lynn's tables, and is found graphically by Kelly and Norie in his linear tables, and by Thomson with his lunar scale; and it is only the *manner* of allowing for the refraction and sun's parallax wherein the above authors differ, all of whom, however, with the exception of Lynn, have erred in using in the calculation the apparent altitude in lieu of the altitude corrected for refraction. On the same account the usual tables giving the moon's correction by inspection are erroneously computed. By approximate methods are not to be understood methods that admit of less accuracy, but such ones that approach the truth by a series of which the last terms vanish, or, which answers the same purpose, by successively substituting in the calculation terms found by a former approximation. The approximative methods have, particularly to seamen, that advantage above the direct ones, that the trigonometrical calculation need only to be executed to the nearest minute; and as the corrections of the distance never can exceed those of the altitudes, and as their amount, as well as sign, may nearly be estimated from the places which the observed bodies occupy in the heavens, essential errors may easily be avoided; alterations may be made in the refraction and parallax without rendering it necessary to resume the calculation; and, moreover, they afford an easy mode of reducing this refraction to the points of the limbs brought into contact when observing their distance, as shall be shown by the following lines: we will designate by

D , the apparent distance of centres;

d , the distance of the points of contact or observed distance of limbs;

H , the greater, h the less, apparent altitudes of centres;

H' , the greater, h' the less, apparent altitudes of the points of contact;

r , the semidiameter parallel to the horizon (augmented if the moon's) of the upper body;

$g - \pi$, the correction of the upper altitude;

$g - \pi'$, the correction of the lesser altitude;

θ , the true distance of the limbs, to which the equatorial horizontal semidiameters must be added to obtain the true distance of the centres.

Let A be the distance of the middle of the apparent distance of the centres, and A' the distance of the middle of the

apparent distance of the limbs from the highest point in the distance—or its prolongation, that is, from that point in a great circle drawn through sun and moon, where the effects of parallax and refraction in distance is 0,—then is

$$\text{tang } A = \frac{\text{tang } \frac{1}{2} (H-h)}{\text{tang } \frac{1}{2} (H+h) \cdot \text{tang } \frac{1}{2} D} \text{ and } A' = \frac{1}{2} d + r + A - \frac{1}{2} D,$$

whence we obtain the apparent altitudes of the points of contact,

$$\sin H' = \frac{\cos (\frac{1}{2} d - A') \sin H}{\cos (\frac{1}{2} D - A)} \text{ and } \sin h' = \frac{\cos (\frac{1}{2} d + A') \cos h}{\cos (\frac{1}{2} D + A)},$$

which can also be found by

$H' - H = r \cdot \text{tang } H \text{ tang } (\frac{1}{2} D - A)$ and $h' - h = r' \text{ tang } h \cdot \text{tang } (\frac{1}{2} D + A)$ and adding $H' - H$ and $h' - h$ to the apparent altitudes of the centre above the sensible horizon and subtracting from the sun the change of refraction from H' to H and h' to h . But if $A > \frac{1}{2} D$, $H' - H$ is to be subtracted from the greater altitude, and the change of declination to be added thereto. The same is to be observed when the distance of the moon's remote limb from a star is observed. It being, however, not so much an error in the altitude as an error in the refraction which materially affects the calculation, and this refraction not being sensibly altered by a few seconds of change of altitude, the change produced by a change of refraction in the altitude may safely be neglected. For the apparent altitudes of the points of contact of the sun and moon above the horizon, compute strictly the refraction ρ and ρ' with regard to barometer and thermometer, and add the sums of these refractions in altitude to the apparent distance of their limbs. From the same apparent altitudes of the points of contact above the sensible horizon, find, by applying thereto the above-stated reduction, the altitudes H' and h' with respect to the true zenith, and deduct from each the corresponding refraction ρ found before, and compute for the rest the parallax in altitude π and π' , and subtract their sums $\pi + \pi'$ from the observed distance augmented by the refractions, and call the rest $d + \rho + \rho' - \pi - \pi' = \delta$.

Find also for each altitude the corrections $\rho - \pi$ and $\rho' - \pi'$; then is

$$\begin{aligned} \theta = \delta &- \frac{\cos (H' + \frac{1}{2} d - A') \cdot (\rho - \pi)}{\cos H' \cos (\frac{1}{2} d - A')} - \frac{\cos (h' + \frac{1}{2} d + A') \cdot (\rho' - \pi')}{\cos h' \cos (\frac{1}{2} d + A')} \\ &+ \frac{\cos (H' + \frac{1}{2} d - A') \cos [H' - (\frac{1}{2} d - A')] (\rho - \pi)^2 \cdot \sin 1''}{2 \text{ tang } d \cos^2 H' \cos^2 (\frac{1}{2} d - A')} \\ &+ \frac{\cos (h' + \frac{1}{2} d + A') \cos [h' - (\frac{1}{2} d + A')] (\rho' - \pi')^2 \cdot \sin 1''}{2 \text{ tang } d \cos^2 h' \cos^2 (\frac{1}{2} d + A')} - - + \end{aligned}$$

[To be continued.]

C. RUMKER.