ART. VI.—Coronas with Mercury Light; by C. BARUS.

1. *Preliminary Survey.*—The inferences of the preceding papers* gave promise that on judiciously using monochromatic light as the source of illumination, the optical nature of the coronas might be fully brought out. Such light must be strictly homogenous and at the same time very intense. Hence the usual methods of obtaining it are unsatisfactory. The strong green line of a mercury lamp, however, fulfills the requirements admirably, and this was therefore used. The results show that the green disc and the first green ring alternately vanish as the result of the interference phenomenon superimposed on the diffraction phenomenon. If therefore the nucleation of a highly charged medium is systematically reduced, a series of angular diameters may be obtained both for the green disc and the inner or outer edge of the first green ring. From the loci of these values the position of the *first diffraction minimum* for green light may be inferred, and the size of droplets computed from the usual equation for small opaque particles.

If the reduction of the nucleation is accomplished by successive partial exhaustions, all of them identical, while filtered air is allowed to enter the receiver systematically between the exhaustions, the nucleations of any two consecutive exhaustions should show a constant ratio. Allowances must, however, be made for the subsidence during the later fogs and for time losses, if any. This is the method used hitherto in my work and the results seem to have been trustworthy.

In the case of mercury light, however, it is now possible to compare the latter with the former (diffraction) method of obtaining the diameters of particles, with a view to throwing definite light on the optical phenomenon. Subsidence methods are out of the question for large coronas, as these are invariably fleeting in character and pass at once into smaller coarse coronas.

The results of the two methods may be regarded as coincident as long as not more than 300,000 nuclei per cubic centimeter, or diameters of particles not smaller than $\cdot 0003^{\text{cm}}$ are in question. For larger numbers and smaller diameters the divergence rapidly increases. Indeed, for particles larger than the size given, the optical loss per exhaustion exceeds the exhaustion ratio, a result which is satisfactorily explained by the contemporaneous subsidence of these relatively large particles. For particles smaller than the limit in question, the loss

* This Jour., xxv, p. 224, 1908; xxvi, p. 87, 1908; xxvi, p. 324, 1908.

computed by the optic method is larger than the exhaustion loss, as if fresh nuclei were produced, or rather made available at each exhaustion. It is this result which the present paper purposes to bring out in detail and to consider in its bearings on the optical phenomenon.

2. *Apparatus.*—The fog chamber was of the usual pattern, cylindrical in form, with its axis horizontal. The clear walls being of blown glass showed some refracting disturbances, not however of serious character. The fog chamber was connected with a large vacuum chamber by a short, wide passage way, though width is of little consequence here. The cylinder was lined with wet cloth, closely adhering, except at the narrow horizontal windows for observation.

For exhaustion this stopcock was suddenly opened at the beginning of the first second, closed after five seconds and the corona quickly measured. Filtered air was then at once introduced and the next exhaustion made at the beginning of the sixtieth second. This rhythm is essential. The isothermal value of a drop of pressure $[\delta p_s]$ was carefully predetermined. It fixes the ratio, y , of the geometric progression of nucleation, since

$$y = (p - \pi - [\delta p_s]) / (p - \pi)$$

where p is the barometric pressure and π the vapor pressure at the given temperature. If the cock were left open for a longer time than five seconds, $[\delta p_s]$ would increase to the limit δp_s .

The goniometer was of the usual type, the eye being at the center or apex, while two needles on radii 30^{cm} long registered the angular diameter of the coronal discs or annuli. Formerly the whole instrument was placed on the *near* side of a fog chamber, the eye being about 30^{cm} from the nearest wall. It conduces to much greater sharpness of vision, however, and admits of a measurement of larger coronas, *caet. par.*, if the eye is placed all but in contact with the nearer wall and the needles (or in this case preferably the inner edges of round rods) beyond the further wall. In such a case the refraction errors are also diminished. In addition to these advantages I may mention the decidedly increased (about 25 per cent) value of the aperture obtained. These excessive apertures show, however, that the ordinary diffraction equation for coronas is not fully applicable; for aperture varies with the position of the eye along the line of sight. It is often surprising how large a corona can be measured by the second method in a small fog chamber, scarcely six inches long. The distance between lamp and chamber is kept about $D = 250^{\text{cm}}$.

3. *Equations.*—The equations needed in the present work are derived in my last Report,* and need merely be summarized

* Carnegie Publications, No. 96, 1908, Chapters I, III (equat. 1 to 12).

here. If y is the exhaustion ratio, the nucleation n_z of the z th exhaustion in terms of the original nucleation n_0 will be

$$n_z = n_0 y^z (1 - S/s_0^2) (1 - S/s_1^2) \dots (1 - S/s_{z-1}^2)$$

where S is a subsidence constant and s the chord of the angular diameter of the coronal disc on a radius of 30^{cm} .

To find S , two consecutive values of s suffice, or

$$S = s_z^2 (1 - s_{z+1}^2 / y s_z^2)$$

approximately. The diameter of particles, d , is in terms of n ,

$$d^3 = 6m / \pi n$$

If $\lambda = 54.6 \times 10^{-6}$ cm. is the wave length of green mercury light and θ the angular radius of the first green minimum of the coronas

$$\sin \theta = .61 \lambda / (d' / 2). \quad \text{Since } \sin \theta = s / 2R,$$

R being the radius of the goniometer of which s is the chord, d' the diameter of fog particle (the primes referring to optic measurements)

$$d' = .004 / s$$

for mercury light. Hence, optically, the nucleation

$$10^{-6} n' = 29.8 m s^3,$$

where m grams of water are precipitated per cubic centimeter on exhaustion, and found in the exhausted fog chamber.

4. *Data with green mercury light*, $10^6 \lambda = 54.6^{\text{cm}}$.—Omitting the correlative data with white light and removing all tables for lack of space, the charts figure 1 contain results obtained with a mercury arc lamp as the source of light. Different annuli are measured and the chords, s , on a radius of 30^{cm} are distinguished by accents, as follows:

s' is the chord of the green disc,

s'' is the chord of the inner edge of the first green ring,

s''' is the chord of the outer edge of the first green ring.

The edges are fairly sharp. Hence

$$s = (s' + s'') / 2$$

may be taken as the chord of the first green minimum. Optically the diameter of the particle is then $d' = .004 / s$.

The water precipitated per cubic centimeter, m , differs with the drop of pressure, $\delta p_s / p$, and the temperature. In the successive parts of the chart the data are

Parts I, II,	$10^6 m = 4.1$ grams,	$n' = 122 s^3$ (omitted)
Parts III, IV,	$= 4.12$ grams,	$n' = 123 s^3$
Parts V–VIII,	$= 4.02$ grams,	$n' = 120 s^3$

where n' is the optical value of the number of nuclei per cubic centimeter.

In parts I–IV the angular diameter, s' , of the green discs

only were measured. The diameter s of the corresponding first minims may, however, be obtained by using the method of reduction found in parts V and VI where $s = .44 + .85 s'$.

In parts V and VI the data for s' and s'' , the angular diam-

eter of the inner edge of the first ring, are both observed, while in parts VII and VIII data for s' and s''' , the outer diameter of the first ring, appear.

In parts I and II (omitted) the goniometer was in front of the fog chamber, and in series I a two-minute interval between exhaustions was (exceptionally) introduced. The result is not good; for a sudden break of the curve appeared after the seventh exhaustion, probably due to the time losses in the extra minutes. The reason, however, is by no means obvious. In series II, for one minute intervals there was no break and the locus passing through the points for green discs (the others, not marked g , are here and elsewhere to be disregarded) is persistently straight throughout. The curves show

For series I, $ds/dz = 1.00$, and

For series II, $ds/dz = .80$,

suggesting a time loss in the first case.

Compared with the preceding table, the values of ds/dz should be in the ratio of red and green minims, or

$$ds_r/dz : ds_g/dz = .95/.80 \text{ correspond to } \lambda_r/\lambda_g = 63.0/54.6;$$

the last ratio, 1.15, is somewhat short of the former, 1.19.

In series III and IV the pins of the goniometer are behind the fog chamber, the eye being at the front wall. In series III the relation,

$$s' = 2.30 + 1.15 (19 - z)$$

is remarkably well sustained throughout; and in series IV

$$s' = 2.50 + 1.13 (18 - z)$$

gives a good account of the green coronas, if the dull cases are ignored.

The most interesting results are given in parts V and VI of figure 1, and in the subsequent paragraphs the computations have been fully carried out. In these cases the chords on a radius of 30^{cm} of the edge of the green disc, s' , and the inner edge of the first green ring, s'' , were successively observed. Figure 1 contains both pairs of curves and their linear character is again astonishing. We may write

$$\begin{aligned} \text{Part V, } s' &= 2.0 + 1.10 (19 - z), & \text{Part VI, } s' &= 2.0 + 1.13 (18 - z), \\ s'' &= 3.4 + 1.21 (19 - z), & s'' &= 3.3 + 1.30 (18 - z), \\ s &= .59 + 1.06 s' = -.56 + .96 s'', & s &= .50 + 1.07 s' = -.44 + .93 s'', \end{aligned}$$

where the minimum is located midway between s' and s'' , both of which are fairly sharp.

From both series the mean value

$$s = .55 + 1.06 s' = -.50 + .93 s''$$

may be derived, for the general reductions in this and other cases where s' is observed. The individual data are omitted.

With the given value for s the optical data for the diameters of the particles, $d'=.004/s$ and for the nucleation, $n'=120 s^3$, were computed.

The subsidence constant

$$S = s_z^2 (1 - s_{z+1}^3 / y s_z^3)$$

was obtained from the observations between $z=13$ and $z=19$, and a mean datum, $S=12$, accepted as most satisfactory. It is then possible to compute n_0 , the original nucleation, as

$$n_0 = n_z / \{y^z (1 - S/s_0^3) \dots (1 - S/s_{z-1}^3)\}$$

from each of the groups of values specified. The mean results are

$$\text{Part V, } n_0 = 4\ 540\ 000$$

$$\text{Part VI, } n_0 = 3\ 430\ 000.$$

Knowing n_0 , the series of data for the nucleation $n \times 10^{-3}$ follow; from these the diameter $10'd=199 n^{-\frac{1}{3}}$ of fog particles and the apertures $s=.004/d$ are finally computed. In other words n and d are the data obtained in view of the occurrence of geometric series of nucleations with allowance for subsidence.

All these results have been constructed graphically in figures 2 and 3, for parts V and VI of the above charts respectively. The observed and computed apertures s are first given; then the observed are optically computed d' and the computed d from sequences appear, and finally the observed or optic n' and the computed nucleation n from sequences. The corresponding data coincide very closely after 9 to 11 exhaustions, i. e. for moderately small coronas: but in the case of large coronas and nucleations, there is marked systematic divergence. A discussion of these results will be given presently.

Finally, parts VII and VIII of fig. 1 contain measurements of the chord of the angular diameter (radius of goniometer of 30^{cm}) of the outer edge of the green disc s' and the outer edge of the first ring s'' . The minimum may be deduced from s' , since $s=.54+1.06s'$ has been accepted. From s the optic value of diameter of fog particle d' and the nucleation n' have been computed. The apertures are very fully given in figure 1, in relation to the number z of the exhaustion. The curves are again linear in character, but in case of the outer edge s'' , the insufficient size of my fog chambers and the greater vagueness of definition interferes with close measurement. The results are

$$\begin{array}{ll} \text{Part VIII } s' = 1.00 + 1.07 (18-z), & \text{part VII } s' = .50 + 1.13 (17-z), \\ s'' = 3.00 + 1.93 (18-z), & s'' = 1.50 + 2.13 (17-z), \\ s = 1.61 + 1.14 (18-z), & s = 1.08 + 1.20 (17-z). \end{array}$$

As a rule s''' is less than twice s' . The data for s' and s are obtained alternately as the figure shows, the green color passing from the disc of the first ring and returning again, in succession.

Inferences.

5. *Interference and diffraction.*—The tables and chart show in the first place, that the disc and first ring of the coronas are alternately of a vivid green, other colors being dull because the remaining lines of the mercury spectrum are faint. At inter-

vals neither disc nor ring are quite green. Hence there is a periodic term impressed on the diffractions, which may be identified as an interference similar to the case of the lamellar grating referred to in another paper.*

When monochromatic light is used it is necessary, therefore, to observe both the edge of the disc and the inner edge of the first ring; for neither appear vividly at the same time. The chord s on a radius of 30^{cm} for the minimum in terms of the corresponding chords s' and s'' , of the first and second edges specified, may then be written.

$$\begin{aligned}s &= .55 + 1.06 s', \\ s &= -.50 + .93 s''\end{aligned}$$

Probably the ratio s/s' and s/s'' should be constant and the absolute term in these equations is an error of observation; but as it is small, little depends upon it, millimeters only being significant.

If we summarize all the observations for ds/dz , the agreement as a whole is in keeping with the nature of the observations and reasonably satisfactory. Thus, in

$$\begin{array}{lcl}\text{Series I, II, } d's/dz = .90, \} & s = .44 + .85 s' & (\text{goniometer in front}), \\ \text{III,} & 1.15, & \\ \text{IV,} & 1.13, & \\ \text{V,} & 1.10, & \\ \text{VI,} & 1.13, & \\ \text{VII,} & 1.13, & \\ \text{VIII,} & 1.07, & \end{array} \left\{ \begin{array}{l} s = .54 + 1.06 s' \text{ (goniometer behind),} \\ \text{mean } ds'/dz = 1.12. \end{array} \right.$$

The feature of these data is the occurrence of linear loci for s and z nearly throughout the extent of the curves. It is as difficult to even conjecture a reason for this, as it is easy to find reasons against it. The presumptive equation for s is

$$s = .004 (\pi/6m)^{\frac{1}{2}} n^{\frac{1}{2}}, \propto 3\sqrt{n}$$

and for n'_z if Π denotes the product,

$$n_z = n_0 y^z \Pi.$$

Hence,

$$s = .004 (\pi n_0/6m)^{\frac{1}{2}} (y^z \Pi)^{\frac{1}{2}}$$

if we disregard the subsidence correction for large coronas

$$ds/dz \propto y^{2/3}$$

in which there is no suggestion of a sustained constancy of the coefficient ds/dz such as the experiments show.

To come to some conclusion as to the cause of the discrepancy between the optic value of the nucleation n' and the presumable value n (geometric progression), we may compare the

* This Journal, xxv, p. 224, 1908.

successive value $n'_{z+1}/n'_z = s^3_{z+1}/s^3_z$ in their relation to $y = .771$ the exhaustion applied. The table shows that for very large coronas $n'_{z+1}/n'_z > y$, whereas for very small coronas $n'_{z+1}/n'_z < y$. For the intermediate coronas (g_3), i. e., from the seventh to the tenth exhaustion among twenty, the ratio is nearly equal to y . Ratios smaller than y may be reasonably interpreted as due to subsidence and the constant S is actually of the order of values to be computed from the viscosity of the medium and the size of the vessels and fog particles. Within this range (coronas smaller than g_3) the optic and the presumptive nucleation may in fact be brought into agreement.

In case of the large coronas, however, subsidence is virtually absent and the occurrence of $n'_{z+1}/n'_z > y$ calls for some apparent production of nuclei at each exhaustion, which is altogether improbable. Therefore, fig. 2 and 3, compare n' and n , whence $n-n'$ shows the number of nuclei not registered by condensation.

For, no matter whether condensation on a given group of nuclei occurs or not, no matter how many nuclei have failed of catching a charge of water, the identical removal of nuclei by partial exhaustion must take place. Such removal is independent of condensation, and would occur in a dry atmosphere under similar treatment. Consequently y cannot be too large. It may be too small not only from subsidence, but from time losses (decay), or as the result of the purification of air due to turbulent motion across a solid or liquid surface. Consequently n may be regarded as an inferior limit of the nucleation with a probably close approximation to the true value. A comparison of n and $n-n'$ would, in such a case, show the percentage of nuclei of irregular size which have failed of capture, the number being $n-n'$.

At the same time it must always be recalled, that no adequate theory of coronas exists and that therefore the meaning of n' is obscure. We must in any case place a part if not all the discrepancy between n and n' within the province of such a theory as is evidenced by the dependence of aperture on the position of the eye. The need is particularly manifest for the large coronas, in which there is accentuated superposition of interference and diffraction. Small coronas may be tested by coincident results obtained from subsidence and the agreement is then well within the errors of observation.

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