REPLY TO "PROBLEM OF THE LOADED TABLE."

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In the discussion of the "Problem of the Loaded Table" in the March issue of SCHOOL SCIENCE AND MATHEMATICS the author of the article seems to have some doubt that the problem has a definite solution. I will endeavor to show in the following that the problem has one definite solution and, furthermore, this solution is based on certain well known principles of physics and mechanics.

A brief restatement of the problem may be necessary. "A table 3 ft. by 4 ft., has a leg at each of its four corners, A, B, C, and D. The table supports a load of 120 pounds at a point 1 ft. from the long side, AB, and 1 ft. from the short side, AD. What is the pressure due to this load at the foot of each leg?"

I agree with the author of the article referred to, that the solution in the text-book of which he speaks is absurd.

Letting A, B, C, D, denote the respective stresses in the legs and using in succession the line joining the points of application of each pair of forces, DC, AD, BA, CB, as an axis of moments, the following equations are readily obtained:

$$(1) \quad A+B=80 \text{ lbs.}$$

$$(2) \quad B+C=30 \text{ lbs.}$$

$$(3) \quad C+D=40 \text{ lbs.}$$

$$(4) \quad D+A=90 \text{ lbs.}$$

These equations are not independent of each other. Another and independent equation must be obtained from other considerations.

To solve the problem with the data given we must make the following assumption, that the floor is rigid, or at least so rigid that the yielding of the floor is a negligible quantity compared to the yielding of the legs. This is readily conceivable. A concrete floor resting on the ground would fulfill these conditions. A very thin floor resting on supports, say twenty feet apart, probably would not.

Under this assumption the legs, being elastic, would yield due to the load on the table. Quoting the March issue, "We might assume the yielding proportional to the square root or cube root of the pressure." It is true we might, had not a certain gentleman, named Hooke, made the remarkable discovery some two hundred years ago, that "Within the elastic limit the deforma-

tion of a body is proportional to the stress producing it." See Henry Crew's *General Physics*, page 132; Millikan and Gale's *First Course in Physics*, page 110; Kimball's *College Physics*, page 155. The above principle is generally known as Hooke's Law and the mathematical statement is as follows:

Let p be the unit stress per unit area of cross section of the body. Let s be the unit deformation or ratio of change in length to the original length. Then $\frac{p}{s} = E$, where E is a constant depending on the material and is known as the Modulus of Elasticity or Young's Modulus. See above references.

One other principle will be used to obtain that elusive fourth equation. It is, I believe, due to Clerk Maxwell, and is as follows: "For stable equilibrium the potential energy of any system is a minimum."¹ The results of the following process of reasoning make the application of this principle permissible. If we compress a spring we store up an amount of potential energy equal to the product of one-half the force and the distance the spring is compressed, or equal to the amount of work done in compressing the spring. The legs of the table are elastic and their conduct under compression, by Hooke's Law, is strictly analogous to the case of the spring. If then, we apply a force to one of the table legs, we store up in it potential energy, the amount of which is equal to the average force times the distance the leg is compressed, or equal to the "work of deformation." We may then for our case restate the above principle as follows: "For stable equilibrium the work of deformation is a minimum."

The principles involved in the solution will not be affected in the least if we consider the legs of the table as all having the same cross-sectional area.

Consider the leg carrying the stress A .

Let F be the area of the cross-section of the legs in sq. in.

Let l be the length in inches.

From Hooke's Law the unit deformation is $s = \frac{p}{E}$, and the total deformation is $ls = \frac{Al}{FE}$.

The work of deformation is, therefore, $= \frac{1}{2}A \times \frac{Al}{FE} = \frac{A^2 l}{2FE}$.

If we solve the above system of equations for B , C , and D in

¹ *Ency. Brit.*, 9th Edition, "Mechanics," Vol. XV, page 722, Art. 198.

terms of A , we may write the expression for the work of deformation in each of the legs, as follows:

$$W_b = \frac{(80-A)^2 l}{2FE}$$

$$W_c = \frac{(A-50)^2 l}{2FE}$$

$$W_d = \frac{(90-A)^2 l}{2FE}$$

The total work of deformation is then:

$$W = \frac{l}{2FE} [A^2 + (80-A)^2 + (A-50)^2 + (90-A)^2]$$

and the one and only condition necessary that W be a minimum is, that

$$8A - 440 = 0 \text{ or, that } A = 55.^2$$

B , C , and D are readily found to be 25 lbs., 5 lbs., and 35 lbs., respectively. These results are the same as were published in the March issue of *SCHOOL SCIENCE AND MATHEMATICS*. It is evident that this is also the only solution, because the expression for W is a quadratic and the curve represented by it is a parabola. It can, therefore, have only one minimum value. Furthermore the solution is entirely independent of the material or the sizes of the members of which the table is constructed, provided it is homogeneous and symmetrical throughout. Even if we did have the load "resting on an infinitely rigid table supported by an infinitely rigid floor" the load would still be distributed in the same manner.

The principle "For stable equilibrium the work of deformation is a minimum" is known as the "Principle of Least Work." It is an important one in mechanics and is met with frequently in structural work. I will illustrate it further by another problem.

Suppose we have two short posts of the same length and of different material glued together and carrying a load P . Let it be required to find how much of the load is carried by each post.

Let F_1 and F_2 , E_1 and E_2 be the respective cross sections and Moduli of Elasticity. Let l be their length and R be the load carried by post F_1 . Then post F_2 will carry a load $(P-R)$. The total work of deformation will be,

$$W = \frac{R^2 l}{2F_1 E_1} + \frac{(P-R)^2 l}{2F_2 E_2}$$

The condition for a minimum is that,

² See Wentworth's *Complete Algebra*, page 211. Rietz and Crathorne's *College Algebra*, page 67.

$$\frac{R}{E_1 F_1} - \frac{P-R}{F_2 E_2} = 0$$

$$\text{Whence } R = P \frac{E_1 F_1}{F_2 E_2 + F_1 E_1}$$

$$\text{and } (P-R) = P \frac{E_2 F_2}{F_2 E_2 + F_1 E_1}.$$

VARIATION OF FOCUS IN LENSES.

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The problem of variation in the focus of lenses presented itself in a discussion of the moving picture camera. Objects of wide variation in distance are seen in good focus. Even though objects have different distances their respective images must fall upon the film in good focus and a clear picture results. It was thought curious that this was true and the lens formula was suggested as the means of clarifying the difficulty. The formula, however, requires that for a certain value of object distance, a certain value of image distance results. According to the problem at hand, it must necessarily be true that for different values of object distance, the values of the corresponding image distance do not vary much. However one might expect that this is not true at all values of object distance. Further it may be inquired as to the importance of the focal length of the lens in the accomplishing of the result.

Consider the formula as usually applied.

$$1/y + 1/x = 1/f.$$

Where y = image distance.

x = object distance.

f = focal length.

We want to know the variation of x with the variation of y at different values of y . This is the fundamental concept of calculus and has a striking application in this case.

We want $\frac{Dy}{Dx}$ for different values of x .

$$\frac{Dy}{Dx} = - \frac{\dot{y}^2}{x^2} = \text{magnification}^2.$$

In terms of the independent variable alone, namely x .

$$\frac{Dy}{Dx} = - \frac{f^2}{(x-f)^2}$$