Weather Prediction by Numerical Process by Lewis F. Richardson
Review by: Nelson K. Johnson

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if such comparison were always theoretically possible, every probability would be greater than, equal to, or less than $\frac{1}{2}$. If, then, p and q were two independent propositions each of which was less likely than its contradictory, the probability of pq would be less than $\frac{1}{4}$. In this way, an indefinite number of numerical approximations would become possible. I do not know whether it can be maintained that the probability of a proposition is always comparable with that of its contradictory, but it seems evident that, where this is not the case, it is not rational to allow the probability of the proposition to influence conduct. In the chapter on "The Application of Probability to Conduct," Mr. Keynes seems to suggest, though he does not definitely assert, that non-numerical probabilities may rationally influence conduct; and elsewhere he suggests that it is rational to give weight to an uncertain induction when no better induction can be ascertained. I cannot see how this can be justified except when the probability of the proposition in question is known (or believed likely) to be greater than that of its contradictory. It seems to me, therefore, by no means certain that probabilities are incapable of numerical measurement (at least approximately), except in the very special cases allowed by Mr. Keynes's use of the principle of indifference. These are, however, only doubts; I have nothing positive to urge as against Mr. Keynes's scepticism.

Much of the criticism in the book is extremely valuable, and the mathematical calculus is astonishingly powerful considering the very restricted premisses which form its foundation. The book as a whole is one which it is impossible to praise too highly, and it is to be hoped that it will stimulate further work on a most important subject which philosophers and logicians have unduly neglected. BERTRAND RUSSELL.

Weather Prediction by Numerical Process. By LEWIS F. RICHARDSON. xi + 231 pp. 4to. Diagrams. 30s. net. 1922. (Cambridge University Press.)

The enormous advances which were made during the last half century in most of the physical sciences were not shared by meteorology. Certain very material accessions to our knowledge of meteorological processes have been gained, it is true; but in view of their importance and the fact that all the various meteorological processes are always at work before our eyes, it must be admitted that the advance was small when compared with the results achieved in some branches of science. And yet a very definite step forward has been made, and it is to be found in the manner in which the meteorologist regards his science. The vast accumulations of systematic observations,* which were regarded as the essence of meteorology not very long ago, to-day occupy but a secondary place. The present day meteorologist seeks the physical explanation of observed phenomena by different means. He realises that statistical analysis alone will not lead far, on account of the complexity of the phenomena caused by the close inter-dependence of all the factors involved. He strives, therefore, to place the science on a mathematical and dynamical basis, and measures his success by the extent to which he is able to achieve this end.

The first step in this direction was taken by Ferrel, who applied to the atmosphere the equations of motion of a fluid on a revolving sphere. The value of this method of attacking meteorological problems was not realised for some time, and it is only comparatively recently that it has been followed up by Shaw, Jeffreys and the late Lord Rayleigh. The dynamics of revolving fluid as represented by a cyclone has attracted the attention of all these investigators, but as Shaw remarks, "the application of equations of revolving fluid to the phenomena of cyclones" is a subject that "is as yet almost unexplored." The first two investigators have shown the necessity of taking account of the small terms in the differential equations, and Shaw has shown

* ["Whether the effect of (printing the observations of many observatories) will be that millions of useless observations will be added to the millions that already exist, or whether something may be expected to result which will lead to a meteorological theory, I cannot hazard a conjecture. This only I believe, that it will be useless at present to attempt a process of mechanical theory; and that all that can be done must be to connect phenomena by laws of induction. But the induction must be carried out by numerous and troublesome trials in different directions, the greater part of which would probably be failures."
SIR GEORGE AIRY, 1867.]

how the distribution of velocity in ordinary cyclones is consistent with their being regarded as simple vortices. But even now we know practically nothing about the mechanism by which cyclones are developed and maintained. These and other problems still await elucidation, and there can be little doubt that their solution will come from a more detailed study of the dynamics of revolving fluid.

But perhaps the most important and the most brilliant of recent contributions to dynamical meteorology is the work of G. I. Taylor on atmospheric turbulence. The phenomena of eddy motion in the atmosphere appeared to be almost hopelessly insoluble, until Taylor showed that the problem could be tackled by dealing with eddies *en masse*, and by examining the effects produced by large numbers of eddies acting over relative long intervals of time. His original conception, according to which momentum, heat and water-vapour are all propagated in the atmosphere by eddies in a similar manner to the conduction of heat in a solid, was developed in order to explain the vertical distribution of temperature over the sea. But its most fertile application has been to the case in which momentum is the entity propagated by eddy diffusion. Taylor himself first used it thus to provide a partial dynamical explanation of the observed vertical distribution of wind velocity and direction.

He also showed that the observed diurnal variation in the wind near the ground follows from a corresponding variation in the amount of atmospheric turbulence. Richardson and others have objected to Taylor's work inasmuch as it contains the assumption that turbulence does not vary with height above the ground. This criticism is now generally conceded, and the determination of the laws of the variation of turbulence with height forms one of the problems on which meteorologists are at present engaged. Here again the mathematical method promises to be at least as effective as the experimental; for the laws of the variation with height of wind velocity and direction—which have since been obtained—can only be compatible with definite corresponding laws for the vertical distribution of atmospheric turbulence.

Yet another meteorological problem which is being attacked mathematically is that of evaporation from water surfaces. The subject is closely connected with atmospheric turbulence and is, moreover, one of extraordinary complexity. Nevertheless, the work of Jeffreys shows that it is, at least partially, amenable to mathematical treatment.

In the above sketch a brief attempt has been made to indicate the important part which mathematics plays in modern meteorology.

The consummation of the mathematical method is to be found in this latest work of Mr. Richardson. The author compares his book with the Nautical Almanac, but the task which he has set himself is in reality a far more complex one. The factors which he endeavours to cope with include the horizontal and vertical movements of the air, the conveyance of water and heat, the effects of latent heat, precipitation, radiation, atmospheric turbulence and evaporation from the sea, from land and from foliage.

Differential equations are developed which represent the contributions of all these factors towards the future weather, and we are then shown how to solve them by the method of finite differences on a set of twenty-three different computing forms. The originality of the whole conception is such as one has learnt to associate with Mr. Richardson's name. This process of forecasting is necessarily extremely laborious, and for this reason alone has little chance of competing with the present empirical method, for the present at least. But the main value of the book lies in the fact that it presents a co-ordinated dynamical treatment of meteorological processes which has not hitherto been attempted. Many investigators have devoted their attention to the dynamics of wind, to the problems arising from atmospheric turbulence, or to the thermodynamics of meteorological processes, but in the present work we see for the first time the true relationship between all these various factors, and how each contributes to that very complicated phenomenon which we call "the weather."

Regarded as a method of forecasting the weather, Mr. Richardson's process suffers from the fact that a local deviation of the wind in the lower layers of the atmosphere is liable to produce grievous errors in the forecast. In Chapter IX.

the anthor applies his process to an actually observed set of conditions. His success is only partial, and the failure appears to be due to the above cause.

Again, the co-ordinate intervals employed are—for time 6 hours, for latitude 3 degrees, and for longitude 200 km. Small scale phenomena, such as thunder-storms and " local showers " are consequently smoothed out, and the method will " not help us for example to say whether it will hail or not on Mr. X's field."

Throughout the book Mr. Richardson employs consistently a system of notation which is explained in Chapter XII. Like the rest of the book, the notation possesses certain original features. In addition to the English and Greek alphabets, both small and capital letters, we find weird Coptic letters and a number of symbols of the author's own invention. The meaning of each letter and symbol is described both in English and also in the international language Ido. An excellent system is employed for facilitating references.

The book is published by the Cambridge University Press and is a good example of their best work. N. K. J.

The Fourth Dimension. By E. H. NEVILLE. Pp. 55. 5s. net. 1921. (Cambridge University Press).

Professor Neville has fastened upon a fact which should have been obvious but has been seriously overlooked. While many have been talking at length upon Non-Euclidean geometry in four-dimensions, and particularly upon the very complex notions of the curvature of three-dimensional manifolds in four-dimensional space, or of four-dimensional manifolds in five-dimensional space, very few have had any preliminary preparation in the shape of a detailed consideration of the most elementary propositions in four-dimensional Euclidean geometry. The ordinary procedure in everyday geometry is to begin with the study of straight lines and planes, then to proceed to spheres, and so gradually to the differential geometry of surfaces in general. This may or may not turn out to be the proper order in which the systematic study of hyperspace should be undertaken; but at any rate it is worth trying. The first chapters of the text-book are here presented to us.

What are the meanings to be attached to the words " line," " plane," " space," when we come into the world of four-dimensions ? This is the question which is answered in detail in the first half of this useful little book. It is shewn how the geometry of four-dimensions is but algebra with the addition of some terminology adopted by analogy with three-dimensional geometry. Unfortunately, the dictionary which Prof. Neville tells us he is writing, is faulty at one point. A line is 1-dimensional, a plane is 2-dimensional a space is 3-dimensional, but there is no distinctive name for the 4-dimensional domain within which these others may be described; and the adoption of the word " space " for a 3-dimensional variety satisfying one linear equation, brings us into confusion with the term " hyperplane," which has often been used in this sense.

These are small matters, however, and the student who will take the trouble to read these pages attentively will be well rewarded by finding his power of thinking in four-dimensions strengthened and clarified.

The later part of the book dwells particularly on the generalization of the idea of " rotation." In ordinary space the displacement of a rigid body with one point fixed from any one position to any other leaves a certain axis in the body fixed in position. Prof. Neville proves in detail that in four-dimensions the displacement of any system of points, the intervals between each pair being unaltered, and one point being fixed, leaves all the points of a certain plane (2-dimensional manifold) undisturbed. He gives us, in fact, a rational and complete description of the idea which, noticed by Minkowski, was the germ-thought of all the analytic development of the theory of relativity. In 1908 Minkowski pointed out that the Lorentz-Einstein transformation was, formally, precisely a rotation in four-dimensions about a fixed plane. This led him to the unification of space and time into what is now called a space-time. It is greatly to Prof. Neville's credit that he has perceived the need for an elementary exposition of the matter, and that he has had the courage and humility to set his hand to it. We are very grateful to him for doing so.

E. CUNNINGHAM.