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the mean free path of a molecule of air at the same pressure, and therefore only a very small fraction of the collisions can be effective in causing a negative particle to be expelled by the excited activity.

I am indebted to Professor Schuster both for placing the resources of his laboratory at my disposal and also for much valuable criticism during the course of these experiments.

LXVI. *Note on a Paper by W. Makower entitled "On the Method of Transmission of the Excited Activity of Radium to the Cathode."* By W. H. JACKSON, M.A., Assistant Lecturer in Mathematics in the University of Manchester*.

As a result of some experiments on the method of transmission of the excited activity of radium, Mr. Makower has arrived at the following hypothesis:—The molecules of excited activity, initially uncharged, are projected from radium emanation with velocities large compared with those of gas molecules, and at a certain percentage of encounters with the gas molecules a negative electron is shot off, leaving the activity positively charged.

It gives additional force to his argument to further show that the deviation of the experimental results obtained by him from those required by the above theory are actually less than the errors of observation.

Since the number of ions present in a gas ionized by a small quantity of radium emanation is exceedingly small compared with the number of molecules of gas, it is safe on this view, even though the positive ion influences a much larger field of force than an ordinary molecule, to assume that the importance of encounters with positive ions is negligible.

The amount of activity which, in the experiments described by Mr. Makower, is deposited on the cathode, is to be taken proportional to the number of excited activity molecules

* Communicated by Mr. W. Makower.

which have encounters with air molecules on their way to the sides of the cylindrical vessel.

Consider a number of particles travelling a distance x through air in which the average distance between molecules is λ , and the effective radii of molecules is σ . The distance may be divided into N layers each of thickness λ , in which each layer contains, on the average, $\frac{1}{\lambda^2}$ molecules per unit area, and the proportion, on the average, of encounters in each layer is $\frac{\pi\sigma^2}{\lambda^2}$. Hence the proportion of the particles which have no encounters is $\left(1 - \frac{\pi\sigma^2}{\lambda^2}\right)^N = \left(1 - \frac{\pi\sigma^2}{\lambda^2}\right)^{\frac{x}{\lambda}}$, and this for small values of λ is equal to $e^{-\frac{x}{L}}$, where $L = \frac{\lambda^3}{\pi\sigma^2}$ and is the "mean free path" of the particles in air.

Supposing now that the proportion of encounters effective in displacing a negative electron be R , the above result will be modified and the proportion of particles experiencing no "effective encounters" will be $e^{-\frac{R}{L}x}$.

In the experiments with which we are dealing, the central rod on which the activity was deposited was shielded at both ends, so that the calculation may be simplified by considering the problem as a two-dimensional one, that is by taking the cylinder to be of unlimited length. It is convenient to calculate the proportion of particles which reach the side of the cylinder without having any effective encounters, and then to subtract the ratio from unity.

Consider the number of particles which reach a small area of magnitude A , containing a point O . Use polar coordinates r, θ, ϕ , with O as origin; if P be any point of space let $OP=r$, and if Q be the projection of P on the right section of the cylinder through O , of which C is the centre, let $C\hat{O}Q=\theta$, and $Q\hat{O}P=\phi$.

Let M be the number of particles projected by the emanation per unit volume. The number of particles reaching the area A from an element of volume δV at P is proportional to the solid angle subtended by A at P , which is $\frac{A}{r^2} \cos C\hat{O}P$. In

particular, the number reaching A from P is

$$e^{-\frac{R}{L}r} \frac{MA}{4\pi r^2} \cos \theta \cos \phi \delta V.$$

This expression is to be integrated over the infinite cylinder, and then it is clear that the deposit on any portion of the sides is proportional to its area. The area of unit length of the cylinder is πd , where d is the diameter, and the value of δV expressed in terms of r, θ, ϕ is $r^2 \cos \phi \delta \theta \delta \phi \delta r$. Denote the number of particles reaching unit length of the sides by N , we have

$$N = \frac{1}{4} M d \iiint e^{-\frac{R}{L}r} \cos^2 \phi \cos \theta \, d\theta \, dr \, d\phi \quad . \quad . \quad . \quad (1)$$

The limits of integration being

$$\begin{array}{lll} \theta, & -\cos^{-1}\left(\frac{r}{d} \cos \phi\right) & \text{to } \cos^{-1}\left(\frac{r}{d} \cos \phi\right) \\ r, & 0 & \text{to } d \sec \phi, \\ \phi, & -\frac{\pi}{2} & \text{to } +\frac{\pi}{2}. \end{array}$$

Performing the first integration with respect to θ , we have

$$N = M d \int_0^{\frac{\pi}{2}} \int_0^{d \sec \phi} e^{-\frac{R}{L}r} \cos^2 \phi \left(1 - \frac{r^2}{d^2} \cos^2 \phi\right)^{\frac{1}{2}} dr \, d\phi \quad . \quad (2)$$

In this integral substitute $r = x d \sec \phi$, whence

$$N = M d^2 \int_0^{\frac{\pi}{2}} \int_0^1 e^{-\frac{R}{L}x d \sec \phi} \cos^2 \phi (1 - x^2)^{\frac{1}{2}} dx \, d\phi \quad . \quad (3)$$

The maximum value of this integral is obtained by omitting the exponential factor, and corresponds to a vacuum when all the activity would reach the sides. Denoting the maximum value of N by N_0 , we find, as is otherwise obvious, that

$$N_0 = \frac{1}{4} \pi M d^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Let the ratio $N : N_0$ be denoted by s .

An effective approximation to the integral expression for the

value of s may be found by expanding $(1-x^2)^{\frac{1}{2}}$ in powers of x . Introduce the notation

$$\gamma_n(z) = \int_0^1 e^{-zx} x^n dx, \quad . \quad . \quad . \quad (5)$$

and write

$$\mu = \frac{R}{L} d. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Equation (3) may now be written

$$s = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \cos \phi [\gamma_0(\mu \sec \phi) - \frac{1}{2} \gamma_2(\mu \sec \phi) - \frac{1}{8} \gamma_4(\mu \sec \phi) - \dots] d\phi \quad (7)$$

We also have

$$\left. \begin{aligned} \gamma_0(z) &= \frac{1}{z} (1 - e^{-z}), \\ \gamma_2(z) &= \frac{1}{3} - \frac{1}{4} z + \frac{1}{2!5} z^2 - \frac{1}{3!6} z^3 + \dots, \\ \gamma_4(z) &= \frac{1}{5} - \frac{1}{6} z + \frac{1}{2!7} z^2 - \frac{1}{3!8} z^3 + \dots \end{aligned} \right\} \quad . \quad . \quad (8)$$

Also $\gamma_n(z)$ decreases continuously as either z or n increases.

If $\mu = \frac{10}{3}$, we have, if $z = \mu \sec \phi$,

$$\frac{1}{z} e^{-z} < \frac{1}{\mu} e^{-\mu} < .012,$$

$$\frac{1}{2} \gamma_2(z) < \frac{1}{2} \gamma_2(\mu) < .015,$$

and with an error of order 3 per cent. we may write

$$s = \frac{4}{\mu\pi} \int_0^{\frac{\pi}{2}} \cos^2 \phi d\phi = \frac{1}{\mu} \quad . \quad . \quad . \quad . \quad (9)$$

This simplified result should therefore hold if the defect from the maximum deposit of activity on the cathode is less than 30 per cent. Hence, with this restriction, the deposit of activity y should be given by the formula

$$y = N_0(1-s) = N_0 \left(1 - \frac{L}{Rd} \right). \quad . \quad . \quad . \quad (10)$$

But the mean free path varies inversely as the pressure, and hence, if L_0 be the mean free path for a pressure of 1 cm. of mercury, and x cm. denote any other pressure, we obtain

$$y = N_0 \left(1 - \frac{L_0}{R\lambda} x \right). \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This equation affords a means of determining the values of N_0 and $\frac{L_0}{R}$ which best suit each of the Tables I., II., III. obtained by Mr. Makower. Owing to the restriction that the value of y must be greater than $\cdot 7N_0$, it is found that this is an inexact method of determining the value of $\frac{L_0}{R}$. In the accompanying figure, the maximum activities have all been reduced to 100, and the three curves are calculated by assuming the *mean* value of $\frac{L_0}{R}$ to be $\cdot 7$ cm.

The values of N_0 and $\frac{L_0}{R}$ were determined as follows:—
Assuming that the set of n points in question, *i. e.* $(x_1, y_1), \dots (x_n, y_n)$ belong to a curve of form

$$y = a - \frac{k}{x},$$

we make

$$\Sigma \left(y_r - a + \frac{k}{x_r} \right) = 0,$$

and

$$\Sigma \left(y_r - a + \frac{k}{x_r} \right)^2 \text{ a minimum.}$$

By differentiation this leads to the equations

$$\left. \begin{aligned} k &= \frac{\Sigma y \Sigma \frac{1}{x} - n \Sigma \frac{y}{x}}{n \Sigma \frac{1}{x^2} - \left(\Sigma \frac{1}{x} \right)^2} \\ na &= k \Sigma \frac{1}{x} + \Sigma y \end{aligned} \right\} \dots \dots \dots (12)$$

In the curves for vessels II. and III. all the points for which the deposit was greater than 70 per cent. of the maximum were used in the above formulæ.

In the curve for vessel I, the point slightly below this value was used because the next point was clearly inaccurate. The

result so obtained was practically identical with that obtained by leaving out both the points in question. On account of

the difference in the values of $\frac{L_0}{R}$ caused by omitting even single points, this value is reliable only to one figure. The results are given in the following table :—

	N_0 .	$\frac{L_0}{R}$ in cm.
Vessel I.	187	·71
Vessel II.	221	·73
Vessel III.	414	·64

It has only been possible to test the values of N_0 , as given by equation (4), very roughly, owing to the difficulty of finding the volume of vessel V and its connexions, with sufficient accuracy to determine the relative values of M in the three sets of experiments.