ART. XLVI.—*Interference Phenomena in a new form of Refractometer*; by ALBERT A. MICHELSON.

IN an experiment undertaken with a view to detecting the relative motion of the earth and the luminiferous ether (*Am. Journal of Science*, No. 128, vol. xxii,) it was necessary to produce interference of two pencils of light, which had traversed paths at right angles with each other. This was accomplished as follows: The light from a lamp at *a*, fig. 1, was separated into two pencils at right angles, *bc*, *bd*, by the plane-parallel glass *b*, and these two pencils were returned to *b* by the mirrors *c* and *d*, whence they coincided along *be*, where they were viewed by the eye, or by a small telescope at *e*.

It is evident that, so far as the interference is concerned, the apparatus may be replaced by a film of air whose thickness is *bc-cd*, and whose angle is that formed by the image of *d* in *b*, with *c*.

The problem of interference in thin films has been studied by Feussner, but his equations do not appear to give the explanation of the phenomena observed. In particular, in the "*Annalen der Physik und Chemie*," No. 12, 1881, on page 558, Feussner draws the conclusion that the interference fringes are straight lines, whereas, in the above described apparatus they are in general curves: and there is but one case—that of the central fringe in white light—which is straight.

Fig. 1.

Fig. 2.

I have therefore thought it worth while to attempt the solution of the problem for a film of air, for small angles of incidence and neglecting successive reflections; and though the solution is not perhaps adapted to the general problem, it accounts for all the phenomena observed in the special case.

Let Om_1 , Om_2 , fig. 2, be two plane mirrors whose intersection is projected at *O*, and whose mutual inclination is ϕ . The illumination at any point, *P* (not necessarily in the plane of the figure), will depend on the mean difference of phase of all the pairs of rays starting from the source and reaching *P*, after

reflection from the mirrors; a pair of rays signifying two rays which have originated at the same point of the source.

If the area of the luminous surface is sufficiently large the illumination at P will be independent of the distance, form, or position of the surface. Suppose, therefore, that the luminous surface coincides with the surface Om_1 . Its image in Om_1 will also coincide with Om_1 , and its image in Om_2 will be a plane surface symmetrical with Om_1 with respect to the surface Om_2 , and for every point, p_1 of the first image there is a corresponding point, p_1' of the second, symmetrically placed and in the same phase of vibration. Suppressing, now, the source of light and the mirrors, and replacing them by the two images, the effect on any point, P, is unaltered.

Consider now a pair of points pp' . Let ϑ be the angle formed by the line joining P and p (or p') with the normal to the surface; ϑ and φ being both supposed small,

$$\Delta = Pp' - Pp = pp' \cos \vartheta.$$

The difference between this value of Δ and the true value is $2Pp \cdot \sin^2 \frac{\psi}{2}$, where ψ is the angle subtended by pp' at P. If ϑ is a small quantity, ψ is a small quantity of the second order, and $\sin^2 \frac{\psi}{2}$ is a small quantity of the fourth order; consequently $2Pp \sin^2 \frac{\psi}{2}$ may be neglected. We have therefore, to a very close approximation, $\Delta = pp' \cos \vartheta$; or, substituting $2t$ for pp' , $2t$ being the distance between the images, at the point where they are cut by the line Pp ,

$$\Delta = 2t \cos \vartheta.$$

Let $cdef$, $c'd'e'f'$, fig. 3, represent the two images, and let their intersection be parallel with cf' , and their inclination be

2φ . Let P be the point considered; P' , the projection of P on the surface $cdef$; and PB, the line forming with PP' the angle

3. Draw P'D parallel to cf, and P'C, at right angles, and complete the rectangle BDP'C. Let P'PC= i and DPP'= θ . Let PP'=P, and the distance between the surfaces at P'=2*t*₀. We have then

$$\begin{aligned} t &= t_0 + CP' \cdot \tan \varphi = t_0 + P \tan \varphi \cdot \tan i, \text{ and} \\ \Delta &= 2(t_0 + P \tan \varphi \tan i) \cos \vartheta, \text{ or} \\ \Delta &= \frac{2(t_0 + P \tan \varphi \tan i)}{\sqrt{1 + \tan^2 i + \tan^2 \theta}} \end{aligned} \quad (1)$$

We see that in general Δ has all possible values, and therefore all phenomena of interference would be obliterated. If, however, we observe the point P through a small aperture, *ab*, the pupil of the eye, for instance, the light which enters the eye from the surfaces will be limited to the small cone whose angle is bPa, and if the aperture be sufficiently small the differences in Δ may be reduced to any required degree.

It is proposed to find such a distance P, that with a given aperture these differences shall be as small as possible, which is equivalent to finding the distance from the mirrors at which the phenomena of interference are most distinct. The change of Δ for a change in θ , is

$$\frac{\delta \Delta}{\delta \theta} = - \frac{2(t_0 + P \tan \varphi \tan i) \frac{\tan \theta}{\cos^2 \theta}}{(1 + \tan^2 i + \tan^2 \theta)^{\frac{3}{2}}} \quad (2)$$

The change of Δ for a change in i , is

$$\frac{\delta \Delta}{\delta i} = 2 \frac{(1 + \tan^2 i + \tan^2 \theta) \frac{P \tan \varphi}{\cos^2 i} - (t_0 + P \tan \varphi \tan i) \frac{\tan i}{\cos^2 i}}{(1 + \tan^2 i + \tan^2 \theta)^{\frac{3}{2}}} \quad (3)$$

For $\frac{\delta \Delta}{\delta \theta} = 0$ we have $\theta = 0$ (or $\Delta = 0$).

For $\frac{\delta \Delta}{\delta i} = 0$ we have $(1 + \tan^2 i + \tan^2 \theta) P \tan \varphi$

$$= (t_0 + P \tan \varphi \tan i) \tan i, \text{ or}$$

$$(1 + \tan^2 \theta) P \tan \varphi = t_0 \tan i, \text{ whence } P = \frac{t_0}{\tan \varphi} \tan i \cos^2 \theta$$

Hence the fringes will be most distinct when $\theta = 0$ and when

$$P = \frac{t_0}{\tan \varphi} \tan i \quad (4)$$

This condition coincides nearly with that found by Feussner.

If the thickness of the film is zero, or if the angle of incidence is zero, the fringes are formed at the surface of the mirrors. If the film is of uniform thickness, the fringes appear at infinity. If at the same time $\varphi = 0$, and $t_0 = 0$, or $i = 0$ and $\varphi = 0$, the position of the fringes is indeterminate. If i has the

same sign as φ , the fringes appear in front of the mirrors; if i has the opposite sign, the fringes appear behind the mirrors.

To find the form of the curves as viewed by the eye at E, let $SE=D$; call T the distances between the surfaces at E' , the projection of E. From P draw PR parallel to DP' , and RS at right angles, and let $RS=c$. We have then $t_0=T+c \tan \varphi$, whence, substituting for c its value $D \tan i$,

$$\Delta = 2 \frac{T + (D+P) \tan \varphi \tan i}{\sqrt{1 + \tan^2 i + \tan^2 \theta}} \quad (5)$$

If on a plane perpendicular to EE' at distance D from E, we call x distances parallel to $P'C$, and y distances parallel to $P'D$, reckoned from the projection of E on this plane, then, putting $\tan \varphi = K$ and $D+P=S$, we have for the equation to the curves, as they would appear on this surface to an eye at E,

$$\Delta = 2 \frac{DT + SKx}{\sqrt{D^2 + x^2 + y^2}} \quad \text{or} \\ \Delta^2 y^2 = (4S^2 K^2 - \Delta^2) x^2 + 8TSKDx + (4T^2 - \Delta^2) D^2 \quad (6)$$

If, numerically,

$$\begin{aligned} \Delta < 2SK & \text{ the curve is a hyperbola,} \\ \Delta = 2SK & \text{ the curve is a parabola,} \\ \Delta > 2SK & \text{ the curve is an ellipse,} \\ K = 0 & \text{ the curve is a circle,} \\ \Delta = 0 & \text{ the curve is a straight line,} \end{aligned}$$

All the deductions from equations (4) and (6) have been approximately verified by experiment.

It is to be observed that in the most important case, and that most likely to occur in practice, namely, in the case of the central fringe in white light, we have $\Delta=0$, and therefore also $t_0=0$; and in this case the central fringe is a straight line formed on the surface of the mirrors. Practically, however, it is impossible to obtain a perfectly straight line, for the surface of the mirrors is never perfect.

It is to be noticed that the central fringe is black, for one of the pencils has experienced an external, the other an internal reflection from the surface b , fig. 1. This will not however be true unless the plate g (which is employed to compensate the effect of the plate b) is of exactly the same thickness as b , and placed parallel with b . When these conditions are not fulfilled, the true result is masked by the effect of "achromatism" investigated by Cornu (*Comptes Rendus*, vol. xciii, Nov. 21st, 1881). This remark leads naturally to the investigation of the effect of a plate of glass with plane parallel surfaces, interposed in the path of one of the pencils.

The effect is independent of the position of the glass plate, provided its surface is kept parallel with the corresponding mirror. Suppose, therefore, that it is in contact with the latter and let cd , fig. 4, represent the common surface. Let $t \doteq hi$ = thickness of the glass, i = angle of incidence, r = angle of refraction, n = index of refraction, λ = wave-length of light. Let ef represent the image of the other mirror, and put $n_0 = \frac{hk}{t}$.

Fig. 4.

It can be readily demonstrated that the path of the rays in the instrument is equivalent to that given in the figure, where one of the rays follows the path $qnmh$, and the other the path $rolh$. Suppose the mirrors cd and ef parallel. Then as has been previously shown, the curves of interference are concentric circles, formed at an infinite distance. Therefore the rays qn , ro , whose path is to be traced, are parallel, and from the point h they coincide. Their difference of path is $2nm - 2hl - op$, and their difference of phase is

$$\begin{aligned} \varphi &= \frac{2nm}{\lambda} - \frac{2hl}{\lambda} - \frac{op}{\lambda} = \frac{2n_0 \cdot t}{\lambda \cos i} - \frac{2t \cdot n}{\lambda \cos r} \\ &\quad - \frac{2t}{\lambda} (n_0 \tan i - \tan r) \sin i, \text{ whence} \\ \varphi &= \frac{2t}{\lambda} [n_0 \cos i - n \cos r] \end{aligned} \quad (7)$$

Let it be proposed to find the value of n_0 which renders any particular ring achromatic. The condition of achromatism, as given by Cornu, is $\frac{d\varphi}{d\lambda} = 0$, which gives

$$\varphi + 2t \left(\cos r \frac{dn}{d\lambda} - n \sin r \frac{dr}{dn} \cdot \frac{dn}{d\lambda} \right) = 0.$$

We have $n = \frac{\sin i}{\sin r}$ whence $\frac{dr}{dn} = -\frac{\sin^2 r}{\sin i \cos r}$, whence

$$\varphi + \frac{2t}{\cos r} \cdot \frac{dn}{d\lambda} = 0$$

By Cauchy's formula we have $n = a_1 + \frac{\alpha_2}{\lambda^2}$ whence $\frac{dn}{d\lambda} = -\frac{2\alpha_2}{\lambda^3}$.

Substituting, we have $\varphi - \frac{4\alpha_2 t}{\lambda^3 \cos r} = 0$ or $n_0 \cos i = n \cos r = \frac{2\alpha_2}{\lambda^2 \cos r}$

$$= \frac{\frac{2\alpha_2}{\lambda^2} + n \cos^2 r}{\cos i \cos r}, \text{ or finally,}$$

$$n_0 = \frac{2(n - \alpha_1) + n \cos^2 r}{\cos i \cos r}. \quad (8)$$

If the angle i is small, the value of n_0 will vary very little with i , consequently there will be a large number of circles all nearly achromatised. Under favorable circumstances as many as one hundred rings have been counted, using an ordinary lamp, as source of light. The difference of path of the two pencils which produce these rings in white light may exceed a thousand wave lengths.