440. To Find the Condition That the Four Straight Lines Represented by $ax^4+4bx^3y+6cx^2y^2+4dxy^3+ey^4=0$ Shall Form a Harmonic Pencil

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operation. Let us suppose, for example, that O , operating on any complex number $a+ib$ or (a, b) , produces

$$O(a, b) = (\lambda a + \mu b, \lambda' a + \mu' b),$$

where $\lambda, \mu, \lambda', \mu'$ are real.

Then

$$O\{O(a, b)\} = (A, B),$$

where

$$A = \lambda(\lambda a + \mu b) + \mu(\lambda' a + \mu' b),$$

$$B = \lambda'(\lambda a + \mu b) + \mu'(\lambda' a + \mu' b).$$

It will easily be verified that we shall have $A = -a, B = -b$, if the two conditions

$$\lambda + \mu' = 0, \quad \lambda^2 + \lambda'\mu + \mu'^2 = -1$$

are satisfied. We can satisfy these equations by taking

$$\lambda = \sinh \theta, \quad \mu = -\rho \cosh \theta, \quad \lambda' = \frac{\cosh \theta}{\rho}, \quad \mu' = -\sinh \theta,$$

θ and ρ being any real numbers; so that

$$O(a, b) = \left(a \sinh \theta - b \rho \cosh \theta, \frac{a \cosh \theta}{\rho} - b \sinh \theta \right).$$

In particular the operation O , when performed on a real number a , produces the complex number

$$a \sinh \theta + \frac{ia \cosh \theta}{\rho}.$$

The special case required is of course that in which $\theta = 0, \rho = 1$; when

$$O(a, b) = (-b, a), \quad O(a) = ia. \quad \text{G. H. HARDY.}$$

440. [K¹. 7. a.] To find the condition that the four straight lines represented by

$$ax^4 + 4bx^3y + 6cx^2y^2 + 4dxy^3 + ey^4 = 0$$

shall form a harmonic pencil.

The lines joining the origin to the intersections of the straight lines $l_1x + m_1y - 1 = 0$ and $l_2x + m_2y - 1 = 0$ with the parabola $y^2 = x$ have for their equation

$$\{y^2 - x(l_1x + m_1y)\} \cdot \{y^2 - x(l_2x + m_2y)\} = 0.$$

These lines form a harmonic pencil if $m_1m_2 + 2(l_1 + l_2) = 0$; that is to say, if the coefficient of y^2 in $(l_1x + m_1y - 1)(l_2x + m_2y - 1) = 0$ is twice the coefficient of x .

The given equation may be written

$$ax^2 + 4bxy + 2\lambda y^2 + 2(3c - \lambda)x(y^2/x) + 4dy(y^2/x) + e(y^4/x^2) = 0,$$

and therefore represents the straight lines joining the origin to the points of intersection of the parabola $y^2 = x$ with the conic

$$ax^2 + 4bxy + 2\lambda y^2 + 2(3c - \lambda)x + 4dy + e = 0.$$

This conic will represent two straight lines if

$$\begin{vmatrix} a & 2b & 3c - \lambda \\ 2b & 2\lambda & 2d \\ 3c - \lambda & 2d & e \end{vmatrix} = 0,$$

and moreover, the joins of the origin to its intersections with $y^2 = x$ will form a harmonic pencil if (as above) $2\lambda = 12c - 4\lambda$ or $\lambda = 2c$.

Thus

$$\begin{vmatrix} a & 2b & c \\ 2b & 4c & 2d \\ c & 2d & e \end{vmatrix} = 4 \begin{vmatrix} a & b & c \\ b & c & d \\ c & d & e \end{vmatrix} = 0,$$

which is the required condition.

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441. [A.] (Vol. vii. p. 48, No. 391.) It may be worth noting that the symbol $\simeq$, which is suggested by C. F. Boon (*Gazette*, No. 103, January 1913, p. 48) as an abbreviation for "approximately equal to," has been already so used. It occurs in Ernst Gottfried Fischer's *Lehrbuch der Elementar-Mathematik, vierter Theil:—Anfangsgründe der Algebra*. Berlin and Leipzig, 1829, p. 147 ff.

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