LIV. On certain small corrections in a newton's-rings system

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Type.	Principle employed.	Reference.	Smallest detectable pressure.
Rayleigh	Tilting mercury U-tube.	Phil. Trans. A 196, p. 205 (1901).	0.68 dyne per cm. ²
Morley & Brush .	„ „ „	Amer. Journ. Sci. xiii. June 1902.	
Threlfall	Micrometer screw adjustments to surfaces of water U-tube.	Inst. Mech. Eng. Proc. Feb. 1904, p. 273.	0.5 dyne per cm. ²
Hering	Tilting mercury U-tube with electric contacts.	Ann. d. Phys. xxi. 1906.	0.3 „ „
Scheel & Heuse...	Metal diaphragm with interference fringes.	Deutsch. Phys. Gesell. Jan. 1909.	0.1 „ „
Chattock & Fry...	Tilting water U-tube with benzene bubble indicator.	Stanton, Proc. Inst. C. E. clvi.	} 0.01 „ „
		Morrow, P. R. S. A 76, 1905.	
		Chattock & Tyndall, Phil. Mag. xix. p. 450 (1910).	
		Fry & Tyndall, Phil. Mag. xxi. p. 348 (1911).	
Fry.....	Diaphragm.		0.001 „ „

Experiments on a manometer of this type for use at low absolute pressures are now in progress.

University of Bristol.
Jan. 1913.

LIV. *On certain Small Corrections in a Newton's-rings System.* By ALLAN FERGUSON, B.Sc. (Lond.), Assistant Lecturer in Physics in the University College of North Wales, Bangor*.

IN the progress of some experimental work which demanded the production and examination of a system of Newton's rings formed under oblique incidence, it seemed that measurements of the diameters of the rings—which, of course, viewed under oblique incidence appear elliptical—would furnish an accurate and easy method for the determination of the angle of incidence.

For if the rings be viewed through a microscope provided with independent vertical and horizontal scales, and the

* Communicated by Professor E. Taylor Jones.

minor axes of the ellipse be measured *on the vertical scale*, then, as the figure shows,

Fig. 1.

$$\tan i = \frac{ab}{cd},$$

if we assume that the rings are truly circular, and concentric with the point of contact of the surfaces between which they are formed, so that ab may be assumed to be given by the measurement of the major axis of the ellipse.

Such a method is probably more satisfactory than those which depend on the measurement of the inclination either of microscope or collimating system, but its use obviously depends on the correctness of the assumption italicised above.

This assumption again depends on the manner in which the equations of the ring-curves are formed. It is a remarkable fact that, even in the definitive treatises on the subject, it is tacitly—or, as for example in the case of Airy's "Undulatory Theory," overtly—assumed that the parallel-plate formulæ can be used without modification, and this, despite the fact that there is a somewhat extensive literature dealing with the subject*. The papers cited below deal fairly exhaustively with a variety of points of importance—*e. g.* the exact shape of the interference surfaces, and the plane in which the fringes are formed—but the expressions arrived at are somewhat cumbrous in form, and not well adapted for practical use.

The variety and importance of the measurements of physical quantities which depend on the production of interference systems, are such as render it desirable to put

* See *e. g.*, Wangerin, Pogg. *Ann.* cxxxi. p. 497 (1867); Feussner, Wied. *Ann.* xiv. p. 564 (1881); Sohneke and Wangerin, Wied. *Ann.* xii. pp. 1 & 207 (1881), and Wied. *Ann.* xx. pp. 177 & 391 (1883); Mascart, *Traité d'Optique*, i. pp. 439 *seqq.*

parallel to the plane of xy . We shall consider the magnitude of the correction to be applied for light which, after refraction at the upper surface of the lens, is incident at the symmetrical points P and P'.

Let R be the radius of the lens-surface and x the radius of the n th dark ring, and let us assume in what follows that $\frac{x^2}{R^2}$ and $\tan^2 \psi$ ($\tan \psi \equiv \frac{dy}{dx}$) are negligible compared with unity. Taking first the case in which the refraction occurs at P , we have from fig. 3

Fig. 3.

$$\phi = i' - r' + r, \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{ii.})$$

and $\psi = i' - \phi$.

Also, if the origin be at O,

$$x^2 + y^2 - 2Ry = 0,$$

and therefore $\tan \psi = \frac{dy}{dx} = \frac{x}{R-y},$

giving $\sin \psi = \frac{x}{R} = p$, say.

Again,

$$i' - \phi = \psi = \tan \psi = \sin \psi = p, \quad . \quad . \quad . \quad (iii.)$$

to the order considered, which, combined with (ii.), gives

$$r' = r + p.$$

Hence

$$\sin i' = N \sin r' = N \sin (p + r) = N(p \cos r + \sin r).$$

Similarly from (iii.),

$$\sin i' = \sin (p + \phi) = p \cos \phi + \sin \phi,$$

and therefore $p \cos \phi + \sin \phi = N(p \cos r + \sin r)$,

which, solving for $\cos \phi$ and reducing, gives

$$\cos \phi = \cos i + \frac{x}{R} \sin i \left\{ 1 - \frac{\sqrt{N^2 - \sin^2 i}}{\cos i} \right\}.$$

Hence $\cos \phi$ is *not* equal to $\cos i$, but is subject to a small first-order correction. It is important to notice that the correction vanishes to the first order for normal incidence.

Turning now to the case (fig. 2) where the ray is incident at P', a point in the YZ plane, the direction-cosines of the ray after refraction at the plane surface of the lens are

$$l_2 = \sin r, \quad m_2 = \cos r, \quad n_2 = 0.$$

If the tangent at P' makes an angle ψ with OZ, the direction-cosines of the normal at P' are

$$p' = 0, \quad q' = \cos \psi, \quad r' = \cos (90^\circ + \psi) = -\sin \psi.$$

Then if ϕ_1 and ϕ_2 be the angles between the ray and the normal before and after refraction at P', l_3 , m_3 , n_3 the direction-cosines of the ray after refraction at P',

$$\cos \phi_1 = m_2 q' = \cos r \cos \psi,$$

$$\cos \phi_2 = m_3 \cos \psi - n_3 \sin \psi.$$

From the general formulæ for refraction, where μ and μ' are the "absolute" refractive indices for glass and air*

(so that $\frac{\mu}{\mu'} = N$),

$$\left. \begin{aligned} \mu l_2 - \mu' l_3 &= (\mu \cos \phi_1 - \mu' \cos \phi_2) p', \\ \mu m_2 - \mu' m_3 &= (\mu \cos \phi_1 - \mu' \cos \phi_2) q', \\ \mu n_2 - \mu' n_3 &= (\mu \cos \phi_1 - \mu' \cos \phi_2) r'. \end{aligned} \right\} . \quad . \quad . \quad (iv.)$$

* Heath, 'Geom. Optics,' p. 21.

From the first of these we have

$$\mu l_2 - \mu' l_3 = 0,$$

$$\text{and therefore} \quad l_3 = N \sin r,$$

Hence

$$m_3^2 + n_3^2 = 1 - l_3^2 = 1 - N^2 \sin^2 r. \quad . \quad . \quad . \quad (\text{v.})$$

From the second or third of (iv.) we obtain

$$m_3 \sin \psi + n_3 \cos \psi = N \cos r \sin \psi.$$

Substituting this value of n_3 in (v.), we obtain, if $\tan^2 \psi$ be negligible compared with unity,

$$m_3^2 = 1 - N^2 \sin^2 r = \cos^2 i.$$

If, then, ϕ be the angle between the ray in the film and a line parallel to OY whose direction-cosines are therefore 0, 1, 0, we have from the usual formula

$$\cos \phi = m_3 = \cos i.$$

It seems, then, that the correction, which is of the first order in the plane of XY, and only vanishes there at normal incidence, is of the second order in the YZ plane, vanishing there for all angles of incidence. These results are in substantial agreement with those of Wangerin*.

If it be desired, therefore, to eliminate first-order corrections, the analysis points to the following experimental precautions. (α) Either the light must be used at normal incidence, or (β) if obliquely incident light be used, measurements must be restricted to those rings for which $\frac{x}{R}$ is small compared with unity. But, in this case, it is only to the measurement of diameters in the plane of incidence to which this restriction applies. Even with oblique incidence the diameters of the rings measured perpendicularly to the plane of incidence are only subject to a second-order correction.

* *Loc. cit.*