IV. Remarks on the trisection of a circular arc

Mr. Paul Newton

To cite this article: Mr. Paul Newton (1823) IV. Remarks on the trisection of a circular arc , Philosophical Magazine Series 1, 62:303, 10-14, DOI: [10.1080/14786442308644368](https://doi.org/10.1080/14786442308644368)

To link to this article: <http://dx.doi.org/10.1080/14786442308644368>

Published online: 23 Jul 2009.

Submit your article to this journal [↗](#)

Article views: 2

View related articles [↗](#)

part of oil of turpentine, with seven of alcohol, be used, the flame becomes very luminous.

In order to equalize and regulate the efflux, I have contrived a boiler, like a gazometer. It consists of two concentric cylinders, opening upwards, leaving an interstice of about one quarter of an inch between them; and a third cylinder, opening downwards, which slides up and down in the interstice. The interstice being filled with boiling water, and alcohol introduced into the innermost cylinder, it soon boils and escapes by the pipes. These pass through stuffing boxes in the bottom of the cylinder. Hence their orifices, and of course the flame, may be made to approach nearer to, or recede further from, the boiler.

The construction of this instrument, which I call the compound blowpipe by alcohol, may be understood from the engraving (Plate I. Fig. 4.).

The idea of making the flame of hydrogen gas, or alcoholic vapour, more luminous by an admixture of oil of turpentine, occurred to me in 1819; and I put the idea into practice in the summer or succeeding winter of that year, when my pupils witnessed the result.

It seems, that Mr. Morey, by another catenation of ideas, was led to a similar inference, employing, in an alcohol blowpipe, whiskey and turpentine. He endeavours so to regulate the efflux of a single jet of the vapour of these fluids, as that it may continue to burn, when once lighted.

This process is too troublesome and precarious for ordinary use. A mixture of alcohol and turpentine are burned with a wick in a lamp, in the same way as oil, according to my plan. It is of course perfectly practicable, and I shall be surprised if it be not adopted in the western country, where alcohol may be had very cheap, and oil must be comparatively dear.

IV. *Remarks on the Trisection of a Circular Arc.* By Mr. PAUL NEWTON.

To the Editors of the Philosophical Magazine and Journal.

Newark, May 4, 1823.

SHOULD the following remarks, the result of much patience, and of many attempts, on the trisection of an angle or a circular arc, obtain a place in your *Philosophical Magazine and Journal*, they may perhaps prove interesting to some of your mathematical readers. That geniuses so exalted as Newton, Barrow, Halley, in conjunction with an endless list of other illustrious names of our own countrymen, and of celebrated

celebrated foreigners,—that nearly all these authors, ancient and modern, should concur in observing a profound silence on trisection; or that none of them should speak or write thereon, but confidently to pronounce its impossibility, except so far as chiefly relates to a right angle; is discouraging in the extreme, and is sufficient to damp the ardour of the most resolute, zealous, aspiring mind. Hopeless, however, as the performance of this task which I have assigned myself, may still appear to the reader, yet what I have observed permit me to communicate. I am, gentlemen,

Your very obedient servant,

PAUL NEWTON.

With any radius AF, or FB, describe the given circle AEHRB, &c. With half the radius of AF, or of FB, describe the circle *cnmto* D, &c. Let the arc AH, or the arc HB, be that of a quadrant, and let the arc AE be equal to the arc RB, the co-arc EH will in consequence be equal to the co-arc HR. It is required to find a third part of the quadrantal arc AH, or HB, a third part of the arc AE, or RB, a third part of its co-arc EH, or HR, and a third part of the arc AR, or of its equal BE.

To find a third part of the quadrant AH, or HB.—Draw the diameter HG perpendicularly to the diameter AB. Draw the chord HcL through the point c, in which the circumference of the circle *cnmto* D bisects the radius AF, to meet the given circle

circle in L. Draw the sine Lx ; the chord LA ; and, through s , the extremity of the semi-radius Fs , and in a direction parallel to the diameter AB draw the chord YZ . But the sine Lx , and the semi-radius Fs , being perpendiculars to the parallels AB , YZ , are themselves parallels, and from the nature of parallel lines the sine Lx is longer than Fs . And because the third part of a quadrant = 30 degrees, and the sine of 30 degrees = half the radius, the sine Lx , being longer than the half radius Fs , exceeds its proper magnitude. The sine Lx is here considered as = the sine of $\frac{1}{3}$ d part of the quadrantal arc AH or HB augmented by an arc commencing at L , and described from the centre c , with the radius Lc , part of the chord LcH . With cL , therefore, as radius, and centre c , describe the arc $LkMNb$, forming a lune with the given circle, and meeting or intersecting the given circle in the points L , and b , at equal distances from N , the extension of the diameter AB . With the chord AL as a distance, and L as a centre, describe an arc at M . With the same distance AL , and centre N , describe an arc at k . The arc NM = the arc kL . From which I infer that if we apply the chord AL , from N to k , on the lunar arc $LkMNb$, the point k fixes the situation of the sine kiv , drawn of course perpendicularly from the point k , or parallel to the sine Lx ; and the point i , in which the sine kiv intersects the given circle, is one extremity of the true sine; the other extremity v is consequently on the diameter; and $iv = Fs$ is = the sine of $\frac{1}{3}$ d part of the quadrant AH , or HB .

For the sake of illustration, let us suppose the radius cL , the sine Lx , and the chord LA , to be composed of, or represented by, three inflexible rods. Let the extremity c of the radius Lc be a fixed point, and let the chord LA , the sine Lx , and the radius Lc , all meeting in the point L , be so joined or attached to one another in this point, that the end A of the chord or rod LA shall, in quitting the arc LA , fall upon the lunar arc $LkMNb$ at M , and that while the extremity L of the lunar radius cL , bearing with it the sine Lx , shall move along the arc $LkMNb$, from L to k ; the extremity M of the chord or rod LM shall be driven along the same arc $LkMNb$, from M to N , and shall lie in the direction kN , during which time and motion the sine Lx , preserving its perpendicular direction, is carried into the situation of the sine kiv ; for the arc MN is = the arc kL . It hence appears that the chord LA , applied to the arc $LkMNb$, leaves a remainder of this arc MN , or kL , = the intervening arc which separates the sine Lx from the sine kiv ; the part iv of the latter of which

is

is the true sine. For, produce now the sine iv , to meet the given arc in a on the opposite side of the diameter AB. Then because the chord of an arc = double sine of half that arc, the chord ia = double the sine iv , or the sine av = sine iv , and the arc Ai = the arc Aa . Draw aH , joining a , the upper extremity of the sine av , with H , the termination of the quadrant AH ; and parallel to aH draw vm , connecting the lower extremity v , of the sine av , with m , the termination of the quadrant cnm described with half radius. Join aF = radius of the given circle. Bisect the sine av in p , and parallel to the diameter AB draw pno , passing through the point n of intersection of the radius aF with the line vm .

Because aF = radius of given circle, and nF = half radius; therefore an = half radius or = nF . And because the sine av is bisected in p , and pn is perpendicular to av , the sides ap , pn , of the triangle apn are respectively equal to the sides vp , pn , of the triangle vpn , and they include equal angles apn , vpn . Consequently the side vn = the side an , or = nF . The line pno being parallel to the diameter AB, the arc nc = the arc oD , and the co-arc nm = the co-arc mo . Consequently the $\angle nFc$ = the $\angle oFD$. And because the triangle vFn is isosceles, and the side vn = the side nF , the $\angle nvF$ = the $\angle nFv$ = the $\angle oFD$, or by sim. $\Delta s.$ = the $\angle mno$. But the $\angle mno$ at the circumference = only half the $\angle mFo$ at the centre. Therefore the arc mo (= the arc nm) is double the arc oD , or double the arc nc . See Leslie's Geometrical Analysis, book i. prop. 31st, prefixed to his Geometry of Curve Lines. Or, since the line po , which bisects the sine av in p , bisects its equal mF in w , the $\angle mnw$ or $\angle mno$ = $\angle nFc$; because the sine mw of the former $\angle$ is = the sine wF of the latter $\angle$. But because the $\angle mno$ is an angle at the circumference, the arc mo , or its equal the arc mn , is double the arc nc , the $\angle nFc$ being an $\angle$ at the centre. Again, because the circumferences of circles have the same ratio to each other as their radii, and the radius aF , or aF , is double the radius nF or cF , the arcs Aa and aH are respectively double of the arcs cn and nm , and the arc aH is double the arc Aa : or this latter arc Aa is = $\frac{1}{3}d$ of the arc AH , or = $\frac{1}{3}d$ of HB .

In a similar manner we may find the third of any fractional part of the quadrant, as well as a third of the whole, because for every variable position of the chord HL we describe a new circle with the varying radius cI . That is, when HB ceases to be the arc of a right angle, or, which is the same thing, when HIC moves into the position of Rc, or when, instead

stead of a right angle, the given arc becomes only $= RB$, then cL varies to cq , which now becomes a substituting radius for cL .

To find one third part of the arc AE , or of RB .—Draw the line $RfFr$ through the centre F of both the concentric circles. Through the point c , which bisects the radius AF , draw the chord Rcq , meeting the given circle in q . With the radius qc , and centre c , describe the lunar arc qPd , intersecting or meeting the given circle in the points q and d points equidistant from P , the extension of the diameter AB . To prevent a confusion of sines, we will pass to the other side of the diameter AB . Through the point of intersection d draw the augmented sine dh . Apply the chord of the arc Ad to the arc Pd (as for the quadrant) from P to c , and through this latter point c draw the sine cg , intersecting the given circle in the point u . The sine ug is the true sine, or is the sine of $\frac{1}{3}d$ of the arc AE , or of $\frac{1}{3}d$ of the arc RB , and consequently the arc $Au = \frac{1}{3}d$ of the arc AE , or $\frac{1}{3}d$ RB . Make the arc $Ae =$ the arc Au , then will the arc $Ae = \frac{1}{3}d$ of the arc AE , or $= \frac{1}{3}d$ of RB . If the upper extremity u of the sine ug were connected with R , the termination of the arc RB , by means of a straight line; and if a straight line were drawn from g , the lower extremity of the sine ug , to t , the termination of the arc Dt ; and if, moreover, the sine ug were bisected in a manner similar to that employed for the right angle, a similar proof would follow; viz. that the arc $Au = \frac{1}{3}d$ of the arc AE , or $= \frac{1}{3}d$ of the arc RB . The straight line drawn from g would fall on the arc cn between c and n , and would be $= vn$, or $= nF$, and on being produced would pass through the point of intersection made by the other two lines; drawn from u , and from the bisection of the sine ug .

Since the arc $Aa = \frac{1}{3}d$ of the arc AH , or $= \frac{1}{3}d$ of HB , and the arc $Au = \frac{1}{3}d$ of the arc AE , or $= \frac{1}{3}d$ of RB ; the difference between these thirds, viz. the arc ua , or the arc ei , $= \frac{1}{3}d$ of the co-arc EH , or of HR . Make, next, the arc $iQ =$ the arc Ai , then will the arc $eQ = \frac{1}{3}d$ of the arc EB , or $= \frac{1}{3}d$ of the arc AR .

Scholium.—When the proposed arc is less than half a quadrant, as the arc EII or HR ; the complement AE , or RB , may be trisected, and the difference between $\frac{1}{3}d$ of this complement, and $\frac{1}{3}d$ of a quadrant as the arc ua , will be $= \frac{1}{3}d$ of the proposed arc EII or HR .

P. N.

V. On