En admettant que les masses des compagnons se trouvaient dans le plan de l'orbite de A , on peut calculer leurs positions par rapport à A à l'aide des formules employées pour les queues des comètes, et on obtient ainsi pour E , par exemple :

$$\begin{array}{lll} \text{août } 5.25 & A = 0.0076 & \varphi = 319^{\circ}5 \\ \text{oct. } 23.42 & 0.0170 & 207.8 \end{array}$$

D'après les éléments respectifs on a pour ces dates :

	A	E
août 5.25	$v = -24^{\circ}56'47''$	$-27^{\circ}51'29''$
	$\log r = 0.3024104$	0.3038484
oct. 23.42	$v = +9^{\circ}20'58''$	$+6^{\circ}29'54''$
	$\log r = 0.2911104$	0.2882955

Avec ces données on calcule pour E :

$$\begin{array}{lll} \text{août } 5.25 & A = 0.0077 & \varphi = 331^{\circ}2 \\ \text{oct. } 23.42 & 0.0139 & 204.1 \end{array}$$

Ces valeurs s'accordent avec les valeurs tirées de l'observation dans les limites des erreurs des estimations.

1889 décembre 17.

Il est très intéressant que les éléments de C et de E indiquent presque le même point pour la séparation des masses de la comète génératrice.

Probablement cette séparation est produite par l'attraction excessivement forte de Jupiter (Chandler, Astron. Journal, No. 205).

Si l'on voulait l'attribuer à l'action éruptive de la comète, on trouverait pour la masse C le choc $j = +0.0014$ et dirigé vers le Soleil, suivant le rayon vecteur, et pour la masse E $j = -0.0147$, c'est à dire le choc dirigé à l'opposite du Soleil, suivant le rayon vecteur prolongé. —

Note.

La petite queue de la comète 1889 I observée par M. Barnard le 3 juin 1889, a paru anormale par l'effet de la perspective.

En calculant sa position réelle par rapport au noyau nous avons trouvé $A = 0.0701$ et $\varphi = 57^{\circ}8$ en arrière du rayon vecteur prolongé. Donc c'est une queue normale et du III type, car il est facile de trouver pour lui la valeur de $1-\mu = 0.04$. Il ne faut pas perdre de vue la petitesse de cette queue et l'insuffisance d'une seule estimation pour le calcul tant soit peu exact de la force répulsive.

Th. Bredichin.

Some formulae for the correction of meridian transit observations

which have been reduced with erroneous values of the instrumental constants.

By *Milton Updegraff*, Assistant Astronomer.

It sometimes happens, after the complete reduction of a series of transit observations, that the necessity appears of adopting different values of one or more of the instrumental constants, a , b , c or n . This is especially liable to occur in case of the collimation constant c through change in the wire intervals of a spider-line reticule or in case of the constants n and a through the adoption of new places for the circumpolar stars.

The labor of re-reduction may be avoided by the use

of formulae for computing the necessary corrections to the erroneous Right Ascensions which were derived from the erroneous values of the constants first adopted. I have derived two sets of formulae for this purpose, the first to be used when the observations have been reduced by Bessel's formula and the second when the observations have been reduced by Mayer's formula for the reduction of observations in Right Ascension.

I. For Bessel's Formula.

When a night's work in Right Ascension has been reduced with a wrong value of the collimation constant, let c' be this erroneous collimation and c the correct one.

Then we have, from Bessel's formula (see Chauvenet's Spherical and Practical Astronomy Vol. II Page 143), for any star observed at declination δ

$$\alpha = T + (\Delta T + m) + n \tan \delta + c \sec \delta \quad (1)$$

$$\alpha' = T + (\Delta T' + m') + n' \tan \delta + c' \sec \delta, \quad (2)$$

α , $\Delta T + m$ and n corresponding to the true value of the collimation, c , and α' , $\Delta T' + m'$ and n' to the erroneous value c' . Subtracting Eq. (2) from Eq. (1)

$$\alpha - \alpha' = (\Delta T + m) - (\Delta T' + m') + (n - n') \tan \delta + (c - c') \sec \delta \quad (3)$$

For a time star at declination δ_1 , $\Delta T + m = \alpha_1 - T_1 - c \sec \delta_1 - n \tan \delta_1$

$$\Delta T' + m' = \alpha_1 - T_1 - c' \sec \delta_1 - n' \tan \delta_1$$

$$(\Delta T + m) - (\Delta T' + m') = -(c - c') \sec \delta_1 - (n - n') \tan \delta_1 \quad (4)$$

From equations (3) and (4)

$$\alpha - \alpha' = (c - c') (\sec \delta - \sec \delta_1) + (n - n') (\tan \delta - \tan \delta_1) \quad (5)$$

For a pair of circumpolar stars U. C. and L. C. whose declinations are δ_2 and $180^\circ - \delta_3$ (L. C.) and which have been observed for determining the constant n we have for collimation c ,

$$\begin{aligned} n \tan \delta_2 &= \alpha_2 - T_2 - (\Delta T + m) - c \sec \delta_2 \\ n \tan (180^\circ - \delta_3) &= \alpha_3 - T_3 - (\Delta T + m) - c \sec (180^\circ - \delta_3) \\ n (\tan \delta_2 + \tan \delta_3) &= \alpha_2 - \alpha_3 - T_2 + T_3 - c (\sec \delta_2 + \sec \delta_3) \\ n &= \frac{\alpha_2 - \alpha_3 - T_2 + T_3}{\tan \delta_2 + \tan \delta_3} - c \frac{\sec \delta_2 + \sec \delta_3}{\tan \delta_2 + \tan \delta_3} \end{aligned}$$

For collimation c'

$$\begin{aligned} n' &= \frac{\alpha_2 - \alpha_3 - T_2 + T_3}{\tan \delta_2 + \tan \delta_3} - c' \frac{\sec \delta_2 + \sec \delta_3}{\tan \delta_2 + \tan \delta_3} \quad \text{subtracting,} \\ n - n' &= -(c - c') \frac{\sec \delta_2 + \sec \delta_3}{\tan \delta_2 + \tan \delta_3} \quad (6) \end{aligned}$$

From equations (5) and (6)

$$\alpha - \alpha' = (c - c') (\sec \delta - \sec \delta_1) - (c - c') \frac{\sec \delta_2 + \sec \delta_3}{\tan \delta_2 + \tan \delta_3} (\tan \delta - \tan \delta_1) \quad (7)$$

Equation (7) gives the correction $\alpha - \alpha'$ for a star at declination δ referred to a time star at declination δ_1 , the constant n having been determined from a pair of circum-

polar stars at declinations δ_2 (U. C.) and $180^\circ - \delta_3$ (L. C.). When there are p time stars and q pairs of circumpolars, equation (7) assumes this form

$$\alpha - \alpha' = (c - c') \left(\sec \delta - \frac{\tan \delta}{q} \sum \frac{\sec \delta_u + \sec \delta_l}{\tan \delta_u + \tan \delta_l} \right) - \frac{c - c'}{p} \left(\sum \sec \delta_l - \frac{\sum \tan \delta_l}{q} \sum \frac{\sec \delta_u + \sec \delta_l}{\tan \delta_u + \tan \delta_l} \right)$$

in which $\sum \sec \delta_l$ and $\sum \tan \delta_l$ are the sums of the secants and tangents of the declinations of the p time stars and $\sum \frac{\sec \delta_u + \sec \delta_l}{\tan \delta_u + \tan \delta_l}$ is the sum of the secants of the circumpolars U. C. and L. C. divided by the sum of their tangents. When the declinations of the circumpolar stars are all plus,

$\frac{1}{q} \sum \frac{\sec \delta_u + \sec \delta_l}{\tan \delta_u + \tan \delta_l}$ may be taken equal to $+1$ and when the declinations δ_u and δ_l are all minus $\frac{1}{q} \sum \frac{\sec \delta_u + \sec \delta_l}{\tan \delta_u + \tan \delta_l}$ may be taken equal to -1 , with sufficient accuracy, and we have

$$\alpha - \alpha' = (c - c') (\sec \delta \mp \tan \delta) - \frac{c - c'}{p} \sum (\sec \delta_l \mp \tan \delta_l) \quad (I)$$

the ambiguous signs to be taken minus when the circumpolars are north stars and plus when the circumpolars are south stars.

In case the error of the collimation $c - c'$ is large enough or the distribution of the time stars such, that the rate of variation of $\Delta T + m$ is sensibly affected, the time stars must be divided into groups and a variable value of $\frac{c - c'}{p} \sum (\sec \delta_l \mp \tan \delta_l)$ used.

When a wrong value of n has been employed let n' be the erroneous value and n the correct value. Then for any star

$$\begin{aligned} \alpha &= T + (\Delta T + m) + n \tan \delta + c \sec \delta \\ \alpha' &= T + (\Delta T' + m') + n' \tan \delta + c \sec \delta \\ \alpha - \alpha' &= (\Delta T + m) - (\Delta T' + m') + (n - n') \tan \delta \end{aligned}$$

For a time star at declination δ_1

$$\begin{aligned} \Delta T + m &= \alpha_1 - T_1 - c \sec \delta_1 - n \tan \delta_1 \\ \Delta T' + m' &= \alpha_1 - T_1 - c \sec \delta_1 - n' \tan \delta_1 \\ (\Delta T + m) - (\Delta T' + m') &= -(n - n') \tan \delta_1 \\ \alpha - \alpha' &= -(n - n') \tan \delta_1 + (n - n') \tan \delta \\ \alpha - \alpha' &= (n - n') (\tan \delta - \tan \delta_1) \end{aligned}$$

In case of p time stars, using the same notation as before, we have

$$\alpha - \alpha' = (n - n') \left(\tan \delta - \frac{1}{p} \sum \tan \delta_l \right) \quad (II)$$

II. For Mayer's Formula.

A , B and C being the transit factors whose values depend on the latitude of the place of observation and the declination of the star, we have from Mayer's formula (Chauvenet Vol. II Page 145) for the two values of the collimation c and c'

$$\alpha = T + \Delta T + aA + bB + cC \quad (1)$$

$$\alpha' = T + \Delta T' + a'A + bB + c'C \quad (2)$$

$$\alpha - \alpha' = \Delta T - \Delta T' + (a - a')A + (c - c')C \quad (3)$$

$$\Delta T = \alpha_1 - T_1 - aA_1 - bB_1 - cC_1$$

$$\Delta T' = \alpha_1 - T_1 - a'A_1 - bB_1 - c'C_1$$

$$\Delta T - \Delta T' = -(a - a')A_1 - (c - c')C_1 \quad (4)$$

For a time star at declination δ_1

From equations (3) and (4)

$$\alpha - \alpha' = (a - a')(A - A_1) + (c - c')(C - C_1) \quad (5)$$

For any two stars at declinations δ_2 and δ_3 used for determining the azimuth constant a we have for collimation c

$$-A_2 a = T_2 + \Delta T - \alpha_2 + bB_2 + cC_2$$

$$-A_3 a = T_3 + \Delta T - \alpha_3 + bB_3 + cC_3$$

$$-a(A_2 - A_3) = T_2 - T_3 - \alpha_2 + \alpha_3 + b(B_2 - B_3) + c(C_2 - C_3).$$

And for collimation c'

$$-a'(A_2 - A_3) = T_2 - T_3 - \alpha_2 + \alpha_3 + b(B_2 - B_3) + c'(C_2 - C_3)$$

$$-(a - a')(A_2 - A_3) = (c - c')(C_2 - C_3).$$

$$a - a' = -(c - c') \frac{C_2 - C_3}{A_2 - A_3}. \quad (6)$$

From equations (5) and (6)

$$\alpha - \alpha' = (c - c')(C - C_1) - (c - c')(A - A_1) \frac{C_2 - C_3}{A_2 - A_3} \quad (7)$$

When there are p time stars and q pairs of circumpolars, using the same notation as before we have

$$\begin{aligned} \alpha - \alpha' &= (c - c') \left(C - \frac{\sum C_i}{p} \right) - (c - c') \left(A - \frac{\sum A_i}{p} \right) \frac{1}{q} \sum \frac{C_2 - C_3}{A_2 - A_3} \quad \text{or} \\ \alpha - \alpha' &= (c - c') \left(C - \frac{A}{q} \sum \frac{C_2 - C_3}{A_2 - A_3} \right) - \frac{c - c'}{p} \left(\sum C_i - \frac{\sum A_i}{q} \sum \frac{C_2 - C_3}{A_2 - A_3} \right) \end{aligned} \quad (III)$$

On substituting the known transit factors of the fundamental stars and the values of p and q for any one night's work formula (III) assumes the form

$$\alpha - \alpha' = (c - c')(C - AK_1) + K_2$$

K_1 and K_2 being constant quantities.

By exactly the same process we arrive at the following expression for $\alpha - \alpha'$ due to error of the level constant b

$$\alpha - \alpha' = (b - b') \left(B - B' - (A - A') \frac{B_2 - B_3}{A_2 - A_3} \right)$$

Now $A = \frac{\sin(\varphi - \delta)}{\cos \delta}$ and $B = \frac{\cos(\varphi - \delta)}{\cos \delta}$. Substituting these values of A and B and also the similar expressions for A_1, B_1, A_2, B_2, A_3 and B_3 we find, after reducing to the simplest form, that

$$\alpha - \alpha' = 0.$$

Therefore an error in the adopted level constant does not directly affect the observed Right Ascensions. But it

does affect the ΔT and therefore the determination of the clock rate.

When a wrong azimuth has been used let a be the correct value and a' the erroneous value.

$$\alpha = T + \Delta T + aA + bB + cC$$

$$\alpha' = T + \Delta T' + a'A + bB + cC$$

$$\alpha - \alpha' = \Delta T - \Delta T' + (a - a')A$$

For a time star at declination δ_1

$$\Delta T = \alpha_1 - T_1 - aA_1 - bB_1 - cC_1$$

$$\Delta T' = \alpha_1 - T_1 - a'A_1 - bB_1 - cC_1$$

$$\Delta T - \Delta T' = -(a - a')A_1$$

$$\alpha - \alpha' = (A - A_1)(a - a')$$

and when there are p time stars

$$\alpha - \alpha' = (a - a') \left(A - \frac{\sum A_i}{p} \right). \quad (IV)$$