XXI. A comparison of various notations employed in "Theories of Crystal-structure," and a revision of the 230 groups of movements

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two obvious ways of explaining this peculiar radiation. One of these is to regard the effect as due to ultra-violet light produced by heating the excited salt, this light acting on the negatively-charged salt and thereby causing a leak from its surface.

To test this view, an amalgamated zinc sphere, Z (fig.1), was attached to the lower end of the measuring system. With this arrangement, though the zinc sphere was within four millimetres of the radioactive salt, no indication of a discharge of negative electricity from the gold leaf was observed.

It seems scarcely possible that an ultra-violet radiation, capable of producing the effects observed, could be absorbed completely in passing through air at 15 mms. pressure for a distance of only four millimetres.

Probably the simpler view to take is that the salts on being heated emit a stream of negatively-charged particles or corpuscles which are drawn to the positively-charged gold leaf by the electrostatic field, and thus discharge it. If this view be correct, these negatively-charged particles must be sent out from the salt with exceedingly small velocities, otherwise we would expect the gold leaf, when initially unelectrified, to acquire at least a small negative charge.

The experiments described in this paper were conducted in the Cavendish Laboratory, and my sincere thanks are due to Professor Thomson for many valuable suggestions and much encouragement given throughout the investigation.

August 1st, 1901.

XXI. *A Comparison of various Notations employed in "Theories of Crystal-structure," and a Revision of the 230 Groups of Movements.* By HAROLD HILTON, Magdalen College, Oxford*.

SINCE the determination of the possible groups of movements must hold an important position in any future theories of crystal structure; and since the mathematical theory of these groups has been practically brought to a completion; it seemed advisable to draw up tables comparing the different notations of the various workers on the subject, with the hope that at the same time a judgment might be formed on the correctness of the statement that the number of groups of movements which are applicable to crystallography is 230.

* Communicated by the Author.

Mr. W. Barlow and Mr. G. F. H. Smith have begun a similar piece of work together for the latter reason. When they heard that I had independently finished such a comparison, they placed my work side by side with their own to some extent, and found that we agreed. They also kindly gave me some useful hints. I take this opportunity of thanking them, and also Prof. H. A. Miers, who has very kindly given me his help throughout.

The chief workers on the subject have been C. Jordan (*Annali di Matematica pura ed applicata*, Serie 2, Band ii. 1869); L. Sohncke (*Entwicklung einer Theorie der Krystallstruktur*, Leipzig, 1879); E. von Fedorow (*Zeitschr. f. Kryst. u. Min.* xxiv. 1895); A. Schoenflies (*Mathematische Annalen*, xxviii., xxix. (1887), & xxxiv. (1889), and *Krystallsysteme und Krystallstruktur*, Leipzig, 1891); and W. Barlow (*Zeitschr. f. Kryst. u. Min.* xxiii. 1895). (Fedorow's work was published in Russian in 1890.)

The notations are taken from the periodicals or books just named, and compared in the tables given below.

I have myself carefully compared Barlow's notation with that in Schoenflies' book; and have also compared the two notations used by Schoenflies in his book and in the *Mathematische Annalen*. Fedorow has compared his own work with that of Barlow* and of Schoenflies†, and thus indirectly has compared Schoenflies' and Barlow's work. If there was any discrepancy between this and my own direct comparison, I have investigated the case carefully, and have given a note on the subject. The comparison of Schoenflies' work (and hence of Fedorow's and Barlow's) with that of Jordan and Sohncke is that given by Schoenflies in the *Mathematische Annalen*.

The diagrams given by Fedorow‡ are not always correct; thus *e. g.* the diagrams of 62 *s*, 63 *s*, 64 *s*, 49 *h*, 50 *h*, 91 *a*, 92 *a*, should be identical with those of 18 *s*, 20 *s*, 21 *s*, 19 *h*, 24 *h*, 29 *a*, 21 *a*, respectively, which is not always the case; in particular, as Mr. Smith has kindly pointed out to me, the diagrams of 64 *s* and 50 *h* are identical and both wrong. It would be beyond the scope of this paper to prepare fresh diagrams. I hope to do this on some future occasion.

It is as well to call attention to the fact that Fedorow altered his notation a little in *Zeitschr. Kryst. Min.* xxiv. pp. 233 & 237; that Barlow pointed out a few errors in his previous work in *Zeitschr. Kryst. Min.* xxv. pp. 86 & 87;

* *Zeitschr. Kryst. Min.* xxiv. pp. 242-244.

† *Ibid.* xx. pp. 48-61.

‡ Tafeln V., VI., *Zeitschr. Kryst. Min.* xxiv.

and that Schoenflies did the same in *Krystallsysteme und Krystallstruktur*, p. 622.

In some cases Schoenflies (in his paper in the *Math. Annalen*, xxxiv.) gave the same symbol to two groups. Where one of these groups is derived from the other by the substitution of the operation $S(\tau)$ for S , or $S(t+\tau)$ for $S(t)$, I have underlined the symbol in the case of the latter group. Similarly he gives the symbol V_3 as representing 3 distinct groups.

For convenience of printing, English capitals are substituted for German in the symbols of Schoenflies, as no error can thereby arise.

An asterisk against a symbol means that a reference is made to it in the notes following the tables.

TABLE I.

A. Schoenflies. Krystallsyst. u. Krystall- struktur, 1891.	A. Schoenflies. Mathem. An- nalen, xxviii. p. 319; xxix. p. 50 (1887); & xxxiv. p. 172 (1889).	C. Jordan. Annali di Matema- tica pura ed appl. Band ii. 1869.	L. Sohncke. Entwickel- ung einer Theorie der Krystall- struktur, 1879.	W. Barlow. Zeitschr. f. Krystall. u. Min. xxiii. pp. 1-63 (1894).	E. v. Fedorow. Zeitschr. f. Kryst. u. Min. xxiv. pp. 232-237 (1895).
C_1	no symbol given.	3	1	65	1 s
C_2^1	$C_1(2)$	29	2	63	3 s
C_2^2	$C_2(2)$	31	3	62	1 a
C_2^3	$C_3(2)$	33	4	64	4 s
V^1	V_4	91	5	56	9 s
V^2	V_7	abs.	6	53	4 a
V^3	V_4	93	12	55	7 a
V^4	V_9	abs.	14	54	8 a
V^5	V_6	abs.	* 9, 13	57	5 a
V^6	V_3	92	7	59	10 s
V^7	V_2	99	8	61	12 s
V^8	V_1	100	10	60	11 s
V^9	V_5	101	11	58	6 a
C_3^1	$C_1(3)$	61	17	48	38 s
C_3^2	$C_2(3)$	63	15	42	68 a
C_3^3	$C_2'(3)$	64	16	43	69 a
C_3^4	$C_3(3)$	62	18	51	39 s
D_3^1	$D_1(3)$	130	21	49	* 45 s
D_3^2	$D_3(3)$	abs.	25	50	* 44 s
D_3^3	$D_2(3)$	132	19	44	* 72 a
D_3^4	$D_4(3)$	abs.	23	46	* 70 a
D_3^5	$D_2'(3)$	133	20	45	* 73 a
D_3^6	$D_4'(3)$	abs.	24	47	* 71 a
D_3^7	$D_5(3)$	131	22	52	46 s

TABLE I. (*continued*).

<i>A. Schoenflies.</i> Krystallogest. u. Krystall- struktur, 1891.	<i>A. Schoenflies.</i> Mathem. An- nalen. xxviii. p. 319; xxix. p. 50 (1887); & xxxiv. p. 172 (1889).	<i>C. Jordan.</i> Annali di Matema- tica pura ed appl. Band ii. 1869.	<i>L. Sohncke.</i> Entwickel- ung einer Theorie der Krystall- struktur, 1879.	<i>W. Barlow.</i> Zeitschr. f. Krystall. u. Min. xxiii. pp. 1-63 (1894).	<i>E. v. Fedorow.</i> Zeitschr. f. Kryst. u. Min. xxiv. pp. 232-237 (1895).
C_1^1	$C_1(4)$	54	30	34	22 <i>s</i>
O_1^2	$C_2(4)$	55	26	26	30 <i>a</i>
C_2^3	$C_3(4)$	56	29	29	33 <i>a</i>
C_3^4	$C_2'(4)$	57	27	27	31 <i>a</i>
C_4^5	$C_3(4)$	58	31	38	23 <i>s</i>
C_4^6	$C_4(4)$	59	28	28	32 <i>a</i>
D_4^1	$D_1(4)$	116	36	39	30 <i>s</i>
D_4^2	$D_2(4)$	124	41	40	43 <i>a</i>
D_4^3	$D_3(4)$	117	32	30	* 44 <i>a</i>
D_4^4	$D_4(4)$	125	38	32	* 48 <i>a</i>
D_4^5	$D_3(4)$	118	35	36	47 <i>a</i>
D_4^6	$D_3(4)!$	126	40	37	50 <i>a</i>
D_4^7	$D_3'(4)$	119	33	31	* 45 <i>a</i>
D_4^8	$D_4'(4)$	127	39	33	* 49 <i>a</i>
D_4^9	$D_7(4)$	120	37	41	31 <i>s</i>
D_4^{10}	$D_5(4)$	121	34	35	46 <i>a</i>
C_6^1	$C_1(6)$	47	47	23	49 <i>s</i>
C_6^2	$C_2(6)$	48	42	14	74 <i>a</i>
C_6^3	$C_2'(6)$	52	43	15	75 <i>a</i>
C_6^4	$C_3(6)$	49	44	16	76 <i>a</i>
C_6^5	$C_3'(6)$	51	45	17	77 <i>a</i>
C_6^6	$C_4(6)$	50	46	20	78 <i>a</i>
D_6^1	$D_1(6)$	108	53	25	54 <i>s</i>
D_6^2	$D_2(6)$	109	48	18	82 <i>a</i>
D_6^3	$D_2'(6)$	113	49	19	83 <i>a</i>
D_6^4	$D_3(6)$	110	50	21	84 <i>a</i>
D_6^5	$D_3'(6)$	112	51	22	85 <i>a</i>
D_6^6	$D_4(6)$	111	52	24	86 <i>a</i>
T^1	T_3	150	54	7	59 <i>s</i>
T^2	T_2	164	55	6	61 <i>s</i>
T^3	T_1	151	56	10	60 <i>s</i>
T^4	T_5	155	58	1	89 <i>a</i>
T^5	T_4	156	57	2	90 <i>a</i>
O^1	O_4	166	59	12	68 <i>s</i>
O^2	O_5	169	64	11	98 <i>a</i>
O^3	O_2	168	60	8	70 <i>s</i>
O^4	O_3	172	63	9	97 <i>a</i>
O^5	O_1	167	61	13	69 <i>a</i>
O^6	O_7	170	65	3	94 <i>a</i>
O^7	O_7'	171	66	4	95 <i>a</i>
O^8	O_8	abs.	62	5	96 <i>a</i>

The work of drawing up the tables has led me to the conclusion that the number of possible groups of movements which contain 3 independent translations which are not infinitesimal, is really 230.

NOTES.—Sohncke's groups 9 and 13 are identical, as Schoenflies pointed out.

Fedorow gives* 44 $s = D_3^1$, 45 $s = D_3^2$, 70 $a = D_3^3$, 71 $a = D_3^5$, 72 $a = D_3^4$, 73 $a = D_3^6$, in his table of his own and Schoenflies' results. That this is wrong appears from his own words on p. 228 of the *Zeitschr. f. Kryst. u. Min.* xxiv. Similarly he has wrongly compared 55 $s = D_{3d}^1$, 56 $s = D_{3d}^3$, 45 $h = D_{3d}^2$, 46 $h = D_{3d}^4$, 47 $s = D_{3d}^1$, 48 $s = D_{3d}^3$, 42 $h = D_{3d}^2$, 43 $h = D_{3d}^4$.

Again, he gives in the same place 44 $a = D_4^4$, 45 $a = D_4^8$, 48 $a = D_4^3$, 49 $a = D_4^7$; which is incorrect as appears from p. 229 of the *Zeitschr. f. Kryst. u. Min.* xxiv., and from Table V. at the end of the same volume.

In Barlow's table† there is a small error; there he says that 58 B₁, 58 B₂, 58 B₃ are of type 62 (3 of Sohncke), so specialized that its axes lie as those of the type 58 or 60 (11 or 10 of Sohncke); now these two last groups, which are V⁹ and V⁸ of Schoenflies, belong to Schoenflies' Raumgitter Γ_v''' and therefore all their 3 series of binary axes are of type C₂³, not C₂² (3 of Sohncke). For "62 (3 of Sohncke)" should be read "64 (4 of Sohncke)."

V_h²⁷ appears as "V_h²⁴" on p. 51 of the *Zeitschr. f. Kryst. u. Min.* xx.—obviously a misprint. The corresponding group (58 a₂) of Barlow is not given quite correctly by him. He gives as the position of the centre "On a line midway between two neighbouring screw-axes of different sorts in its point of intersection with a rotation-axis." This would make his types 58 a₁ and 58 a₂ identical, and the same as Schoenflies' V_h²⁸; the last phrase should run "midway between its points of intersection with a rotation axis."

* *Zeitschr. Kryst. Min.* xx. (1892) p. 56.

† *Ibid.* xxiii. (1895) p. 56.

TABLE II.

<i>A. Schoenflies.</i>	<i>W. Barlow.</i>	<i>E. von Fedorow.</i>	<i>A. Schoenflies.</i>	<i>W. Barlow.</i>	<i>E. von Fedorow.</i>	<i>A. Schoenflies.</i>	<i>W. Barlow.</i>	<i>E. von Fedorow.</i>
C_i	No sym- bol given.	2 s	C_{2v}^{18}	61 B_1	17 s	$V_{\hbar}^{24}$	61 a_2	24 h
C_1^1	No sym- bol given.	5 s	C_{2v}^{19}	61 B_2	16 h	$V_{\hbar}^{25}$	60 a_1	20 s
C_2^2	Abs.	1 h	C_{2v}^{20}	* 58 B_1	16 s	$V_{\hbar}^{26}$	60 a_2	23 h
C_3^3	No sym- bol given.	6 s	C_{2v}^{21}	* 58 B_2	15 h	$V_{\hbar}^{27}$	* 58 a_2	21 a
C_4^4	Abs.	2 h	C_{2v}^{22}	* 58 B_3	14 h	$V_{\hbar}^{28}$	58 a_1	20 a
C_{2h}^1	$c_1^h(2)$	7 s	$V_{\hbar}^1$	56 a_1	18 s	C_{3i}^1	48 a_1	51 s
C_{2h}^2	$c_2^h(2)$	2 a	$V_{\hbar}^2$	56 a_2	19 h	C_{3i}^2	51 a_1	52 s
C_{2h}^3	$c_3^h(2)$	8 s	$V_{\hbar}^3$	56 a_2	17 h	C_{3v}^1	48 b_3	41 s
C_{2h}^4	$c_1^i(2)$	3 h	$V_{\hbar}^4$	56 a_4	18 h	C_{3v}^2	48 b_1	40 s
C_{2h}^5	$c_2^i(2)$	3 a	$V_{\hbar}^5$	53 a_1	14 a	C_{3v}^3	48 b_4	40 h
C_{2h}^6	$c_3^i(2)$	4 h	$V_{\hbar}^6$	53 a_4	17 a	C_{3v}^4	48 b_2	39 h
C_{2v}^1	$c_1^e(2)$	13 s	$V_{\hbar}^7$	53 a_2	15 a	C_{3v}^5	51 b_1	42 s
C_{2v}^2	$c_2^e(2)$	9 a	$V_{\hbar}^8$	53 a_3	16 a	C_{3v}^6	51 b_2	41 h

C_{2v}^3	$c_1^s(2)$	$56B_2$	$5h$	V_h^9	v_8^h	$55a_3$	$22a$	D_{3d}^1	$p_1^a(3)$	$49a_1$	$*56s$
C_{2v}^4	$c_1^m(2)$	$56B_3$	$6h$	V_h^{10}	$v_8^{i_1}$	$55a_6$	$27a$	D_{3d}^2	$p_1^i(3)$	$49a_2$	$*46h$
C_{2v}^5	$c_2^m(2)$	$53B_3$	$11a$	V_h^{11}	$v_8^{m_1}$	$55a_2$	$23a$	D_{3d}^3	$p_3^s(3)$	$50a_1$	$*55s$
C_{2v}^6	$c_1^m(2)$	$56B_4$	$7h$	V_h^{12}	v_8^m	$55a_1$	$25a$	D_{3d}^4	$p_3^i(3)$	$50a_2$	$*45h$
C_{2v}^7	$c_2^m(2)$	$53B_2$	$10a$	V_h^{13}	v_8^s	$55a_7$	$24a$	D_{3d}^5	$p_5^a(3)$	$52a_1$	$57s$
C_{2v}^8	$c_1^{m_1}(2)$	$56B_5$	$9h$	V_h^{14}	v_8^i	$55a_5$	$26a$	D_{3d}^6	$p_5^i(3)$	$52a_2$	$47h$
C_{2v}^9	$c_2^s(2)$	$53B_4$	$12a$	V_h^{15}	v_9^i	$54a_1$	$29a$	S_4^1	$c_1^q(2)$	$63c$	$26s$
C_{2v}^{10}	$c_1^{m_1}(2)$	$56B_6$	$8h$	V_h^{16}	v_9^h	$54a_2$	$28a$	S_4^2	$c_3^q(2)$	$64c$	$27s$
C_{2v}^{11}	$c_1^d(2)$	$59B_1$	$14s$	V_h^{17}	v_6^s	$57a_1$	$18a$	C_{4v}^1	$c_1^s(4)$	$34b_1$	$24s$
C_{2v}^{12}	$c_2^d(2)$	$57B_1$	$13a$	V_h^{18}	$v_6^{m_1}$	$57a_2$	$19a$	C_{4v}^2	$c_1^d(4)$	$34b_3$	$26h$
C_{2v}^{13}	$c_1^d(2)$	$59B_2$	$10h$	V_h^{19}	v_3^s	$59a_1$	$19s$	C_{4v}^3	$c_3^s(4)$	$29b_1$	$*37a$
C_{2v}^{14}	$c_3^s(2)$	$59B_3$	$15s$	V_h^{20}	v_3^m	$59a_2$	$20h$	C_{4v}^4	$c_3^d(4)$	$29b_3$	$*38a$
C_{2v}^{15}	$c_3^{s_1}(2)$	$59B_4$	$11h$	V_h^{21}	$v_3^{m_1}$	$59a_3$	$21h$	C_{4v}^5	$c_1^s(4)$	$34b_2$	$25h$
C_{2v}^{16}	$c_3^m(2)$	$59B_5$	$12h$	V_h^{22}	v_3^i	$59a_4$	$22h$	C_{4v}^6	$c_1^d(4)$	$34b_4$	$27h$
C_{2v}^{17}	$c_3^m(2)$	$59B_6$	$13h$	V_h^{23}	v_2^h	$61a_1$	$21s$	C_{4v}^7	$c_3^a(4)$	$29b_2$	$*36a$

TABLE II. (*continued*).

<i>A. Schoenflies.</i>	<i>W. Barlow.</i>	<i>E. von Fedorow.</i>	<i>A. Schoenflies.</i>		<i>W. Barlow.</i>	<i>E. von Fedorow.</i>	<i>A. Schoenflies.</i>		<i>W. Barlow.</i>	<i>E. von Fedorow.</i>
			<i>A. Schoenflies.</i>	<i>A. Schoenflies.</i>			<i>A. Schoenflies.</i>	<i>A. Schoenflies.</i>		
C_{4v}^8	$C_4^d(4)$	39 <i>a</i>	D_{4h}^5	$D_2^h(4)$	40 <i>a</i> ₁	54 <i>a</i>	D_{6h}^1	$D_1^h(6)$	25 <i>a</i> ₁	58 <i>s</i>
C_{3v}^9	$C_3^s(4)$	25 <i>s</i>	D_{4h}^6	$D_2^m(4)$	40 <i>a</i> ₂	56 <i>a</i>	D_{6h}^2	$D_1^m(6)$	25 <i>a</i> ₂	48 <i>h</i>
C_{3v}^{10}	$C_3^b(4)$	28 <i>h</i>	D_{4h}^7	$D_2^s(4)$	40 <i>a</i> ₃	* 55 <i>a</i>	D_{6h}^3	$D_4^h(6)$	24 <i>a</i> ₂	87 <i>a</i>
C_{4v}^{11}	$C_4^d(4)$	34 <i>a</i>	D_{4h}^8	$D_2^i(4)$	40 <i>a</i> ₄	* 57 <i>a</i>	D_{6h}^4	$D_4^m(6)$	24 <i>a</i> ₁	88 <i>a</i>
C_{4v}^{12}	$C_4^d(4)$	35 <i>a</i>	D_{4h}^9	$D_5^h(4)$	36 <i>a</i> ₂	60 <i>a</i>	T_h^1	T_3^h	7 <i>a</i> ₁	62 <i>s</i>
C_{4h}^1	$C_4^h(4)$	28 <i>s</i>	D_{4h}^{10}	$D_5^m(4)$	36 <i>a</i> ₁	61 <i>a</i>	T_h^2	T_3^i	7 <i>a</i> ₂	49 <i>h</i>
C_{4h}^2	$C_4^h(4)$	41 <i>a</i>	D_{4h}^{11}	$D_5^i(4)$	36 <i>a</i> ₄	63 <i>a</i>	T_h^3	T_2^h	6 <i>a</i> ₁	64 <i>s</i>
C_{4h}^3	$C_4^i(4)$	29 <i>h</i>	D_{4h}^{12}	$D_5^d(4)$	36 <i>a</i> ₃	62 <i>a</i>	T_h^4	T_2^i	6 <i>a</i> ₂	50 <i>h</i>
C_{4h}^4	$C_3^i(4)$	42 <i>a</i>	D_{4h}^{13}	$D_6^h(4)$	37 <i>a</i> ₂	* 66 <i>a</i>	T_h^5	T_1^h	10 <i>a</i> ₁	63 <i>s</i>
C_{4h}^5	$C_5^h(4)$	29 <i>s</i>	D_{4h}^{14}	$D_6^m(4)$	37 <i>a</i> ₁	* 64 <i>a</i>	T_h^6	T_5^i	1 <i>a</i> ₁	91 <i>a</i>
C_{4h}^6	$C_4^i(4)$	40 <i>a</i>	D_{4h}^{15}	$D_6^s(4)$	37 <i>a</i> ₄	67 <i>a</i>	T_h^7	T_4^i	2 <i>a</i> ₁	92 <i>a</i>
V_d^1	V_4^d	32 <i>s</i>	D_{4h}^{16}	$D_6^s(4)$	37 <i>a</i> ₃	* 65 <i>a</i>	T_d^1	T_3^d	7 <i>b</i> ₁	65 <i>s</i>
V_d^2	V_4^q	30 <i>h</i>	D_{4h}^{17}	$D_7^h(4)$	41 <i>a</i> ₁	37 <i>s</i>	T_d^2	T_2^d	6 <i>b</i> ₁	67 <i>s</i>

V_d^3	V_8^d	$55 \beta_1$	$52 a$	D_{4h}^{18}	$D_7^m(4)$	$41 a_2$	$38 h$	T_d^3	T_1^d	$10 b_1$	$66 s$
V_d^4	V_8^q	$55 \beta_2$	$53 a$	D_{4h}^{19}	$D_8^d(4)$	$35 a_1$	$58 a$	T_d^4	T_3^q	$7 b_2$	$51 h$
V_d^5	V_3^d	$59 \beta_1$	$33 s$	D_{4h}^{20}	$D_8^i(4)$	$35 a_2$	$59 a$	T_d^5	T_2^q	$6 b_2$	$52 h$
V_d^6	V_3^q	$59 \beta_2$	$31 h$	C_{3h}^1	$C_1^h(3)$	$48 b_3$	$43 s$	T_d^6	Abs.	$2 b_1$	$93 a$
$*V_d^7$	$V_3^q a$	$59 \beta_3$	$32 h$	D_{3h}^1	$D_1^h(3)$	$49 b_1$	$* 48 s$	O_h^1	O_4^h	$12 a_1$	$71 s$
$*V_d^8$	$V_3^q \beta$	$59 \beta_1$	$33 h$	D_{3h}^2	$D_1^m(3)$	$49 b_2$	$* 43 h$	O_h^2	O_4^i	$12 a_2$	$53 h$
V_d^9	V_2^d	$61 \beta_1$	$35 s$	D_{3h}^3	$D_3^h(3)$	$50 b_1$	$* 47 s$	O_h^3	O_5^h	$11 a_1$	$102 a$
V_d^{10}	V_2^q	$61 \beta_2$	$34 h$	D_{3h}^4	$D_3^m(3)$	$50 b_2$	$* 42 s$	O_h^4	O_6^d	$11 a_2$	$103 a$
V_d^{11}	V_1^d	$60 \beta_1$	$34 s$	C_{6v}^1	$C_1^a(6)$	$23 b_1$	$50 s$	O_h^5	O_2^h	$8 a_1$	$* 73 s$
V_d^{12}	V_5^q	$58 \beta_1$	$51 a$	C_{6v}^2	$C_1^a(6)$	$23 b_2$	$44 h$	O_h^6	$O_2^{m'}$	$8 a_2$	$54 h$
D_{4h}^1	$D_1^h(4)$	$39 a_1$	$36 s$	C_{6v}^3	$C_4^a(6)$	$20 b_1$	$80 a$	O_h^7	O_3^d	$* 9 a_1$	$100 a$
D_{4h}^2	$D_1^m(4)$	$39 a_2$	$35 h$	C_{6v}^4	$C_4^d(6)$	$20 b_2$	$79 a$	O_h^8	O_3^i	$9 a_2$	$101 a$
D_{4h}^3	$D_1^d(4)$	$39 a_3$	$* 36 h$	C_{6h}^1	$C_1^h(6)$	$23 a_1$	$53 s$	O_h^9	O_1^h	$13 a_1$	$* 72 s$
D_{4h}^4	$D_1^i(4)$	$39 a_4$	$* 37 h$	C_{6h}^2	$C_4^h(6)$	$20 a_1$	$81 a$	O_h^{10}	O_6^i	$5 a_1$	$99 a$

Schoenflies does not appear to give his groups V_d^7 and V_d^8 quite correctly*; he puts the plane of the operations $S_d(\tau_d)$, $S_d(\tau_d + \tau_z)$ in the same position as the operations S_d and $S_d(\tau_z)$ of V_d^5 and V_d^6 ; but in that case the operations referred to would not bring the axes of V^6 into self-coincidence, as is necessary. Barlow gives the position of the plane quite correctly on p. 54, *Zeitschr. Kryst. u. Min.* xxiii.; and this remark agrees with Schoenflies' own treatment of these groups in the *Math. Annalen*, xxxiv.

Fedorow's comparison of his groups 36 *h*, 37 *h*, 55 *a*, 57 *a*, 60 *a*, 61 *a*, 65 *a*, 66 *a* with Barlow's types seems to be incorrect, judging by the Tables V. and VI. given at the end of the 24th volume of the *Zeitschr. f. Kryst. u. Min.*; as is also his comparison of 36 *a*, 37 *a*, 38 *a*, 64 *a*, 65 *a*, 66 *a* with the corresponding groups of Schoenflies.

Fedorow compares 72 *s*, 73 *s* with O^9 and O^5 ; an obvious misprint for O_h^9 and O_h^5 .

Fedorow's remark on Barlow's type 9 a_1 (*Zeitschr. Kryst. u. Min.* xxiv. p. 244) seems not to be accurate. The position of the inversion centre given by Barlow agrees entirely with that given by Schoenflies, and involves symmetry planes parallel, not to the surface planes, but to the diagonal planes of the cube.

XXII. *On the Electrical Conductivities produced in Air by the Motion of Negative Ions.* By P. J. KIRKBY, M.A., Fellow of New College, Oxford†.

IN the February number of the Philosophical Magazine for 1901 Prof. Townsend described in detail a series of experiments by which he demonstrated the remarkable fact that negative ions produced in a gas are capable of disintegrating other molecules with which they collide, and thus generate other ions by collision. As this adds one more to the few known methods by which ions are produced, the discovery opens up a new field of research. A series of experiments were given (J. S. Townsend and P. J. Kirkby) in the June number of the Phil. Mag. 1901, which proved that the same results which apply to air apply also to hydrogen and carbonic acid gas. These researches exhibited the extraordinary difference in the behaviour of positive and negative ions. It seemed therefore desirable further to emphasize this difference,

* *Krystalsysteme und Krystalstruktur*, p. 497.

† Communicated by Prof. J. S. Townsend.