THE

AMERICAN JOURNAL OF SCIENCE

[FOURTH SERIES.]

ART. XLII.—*On a Group of Visual Phenomena depending upon Optical Errors of the Human Eye*; by CHARLES S. HASTINGS.

IN two preceding articles* the writer has contributed measurements which serve to define the average human eye, both with respect to its color error and its error of collimation, to a closer degree of approximation than that of Helmholtz and his successors as embodied in the well known *schematic eye*. The aim of this paper is to discuss certain consequences which may be based upon these more exactly determined data and to describe, and explain as far as practicable, a number of visual phenomena not thoroughly studied heretofore, or, perhaps, wholly known. For this purpose we require, in addition to the elements established in the papers cited, a knowledge of the positions of the external and internal pupils of the model eye. By the former term is meant the virtual image of the real pupil as formed by refraction at the cornea; and by the latter, the virtual image of the real pupil as formed by successive refractions at the anterior and posterior surfaces of the lens. These are evidently so related to each other that a ray from an object outside to a point in the exterior pupil will correspond to a ray from the corresponding point in the interior pupil to the image of the object. If we take, as in the preceding articles, the vertex of the cornea as the origin of coördinates, together with the place of the center of the real pupil as established in the second paper, a simple calculation shows that the centers of the pupils are at 0.3046^{cm} and 0.3705^{cm} , respectively, both lying on the nasal side of the axis

* This Journal, vol. xix, p. 205 and p. 310.

of symmetry and approximately midway between these axes and the axes of vision. These values are for wave-lengths of mean refrangibility, the variations in the values for different wave-lengths not being of moment for our present purposes; the small difference in relative size of the two pupils is also ignored as immaterial.

The elements of the schematic eye, so far as they are necessary to our investigation, are:

	$a = 4^{\circ}2$	Dist. π_1 to $\pi_2 = 0.0659\text{cm}$
Dist. π_2 to $n_1 = 0.3264\text{cm}$		" n_1 to $n_2 = 0.0357\text{cm}$
" n_2 to retina = 1.55cm		" F_b to $F_r = 0.0566\text{cm}$

These elements are represented in fig. 1, in which, for the sake of perspicuity, the scale is greatly distorted by increasing those

of the first column in the ratio of 2.5 to 1 and those of the second column in the ratio of 25 to 1. With this qualification fig. 1 represents accurately the necessary elements of a pair of normal eyes, the lines n_1n_2 being the axes of the eyes, the lines $a n_1$ the external, and n_2r the internal lines of vision, and the points π_1 and π_2 the positions of the centers of the external and internal pupils, respectively. The points b and r are the positions of the images of a distant object on the line of vision for two wave-lengths of light corresponding to a difference of refrangibility in water of .0100, which corresponds to an interval in spectral colors nearly equivalent to that separating C and G.

I. *Utility of Error of Collimation.*

Suppose both eyes directed to a distant surface divided vertically into two portions of which the left side is red and the right blue, the point of fixation for each eye being on the line of demarcation. In this case the image of the red surface would be represented by the line rr' in each eye and that of the blue surface would be represented by the lines bb' . If the eyes were sharply focussed for red light, that is, if the retinas corresponded with the surfaces rr' , the observer would have a sharply defined image of the boundary in red and an ill-defined image in blue as the resultant effect of the diffusion circles due to the chromatic aberration of the eye. The diameter of these diffusion circles will bear the same ratio to the diameter of the interior pupil as the distance br does to π_2b , while the centers of these diffusion circles corresponding to points at the boundary between the colored fields will lie on a line on the retina which is represented in the diagram by the projection of this line, namely, by the point where the line π_2b extended intersects the retina. It is obvious from inspection that only from the line π_2b , towards the nasal side in the right eye, and towards the temporal side in the left eye, do we have a full illumination of the retina equal to that of the blue field itself. We see, therefore, that in the case supposed there is a narrow region, relatively dark, bordering the sharply defined edge of the red image in the right eye, while in the left eye there is a corresponding region where the contrast is reduced by a commingling of the two colored lights.

It is easy to see that if the eye is adjusted for distinct vision of the blue surface, that is, if the retina corresponds to $b'b'$ of the diagram, exactly the same conditions as regards sharpness of distinction hold as before. If the observer's attention were directed to this particular feature of the object, that is, to the dividing line in the field, he would inevitably accommodate either for the red or for the blue, and, ignoring the confused sensation of the left retina, recognize a sharpness of division which would be lacking if the right eye were symmetrical in construction. If the red and blue fields were interchanged the left eye would become the discriminating one.

Before we can attach great importance to this conclusion we must see if the relations quantitatively considered are such as to support it. To do this we must calculate the angular width of the darkened strip and of the radius of the diffusion circles; the first giving us a notion as to its conspicuousness as an interruption in the field of vision, and the second a notion as to its intensity. We read at once from the diagram that the true width of the region is—

$$\left\{ (n_2 - n_1)tg a + (\pi_2 - n_1)tg \frac{1}{2} a \right\} \frac{br}{\pi_2 b} :$$

If this quantity be divided by the distance from n_2 to the retina, we shall have its angular width. This is found to be equal to $1'$, a conspicuous magnitude, since alternate bright and dark lines of half this angular width can be distinguished under favorable circumstances.

The second part of the problem is not quite definite because the radius of the pupil is not a constant; but if we assume a diameter equal to 0.2^{cm} we shall be near the true value for ordinarily bright illumination. This assumption gives 6.8 for the angular value of the radius of the diffusion circles; in other words, this is the angular width of the strip within which the illumination of the retina by blue light falls from its maximum to zero. As experiment shows that a diminution of luminous intensity of less than one per cent is obvious at a sharply defined border, there is no room for doubt that the peculiarity of construction does possess a useful function in vision, inasmuch as that with the established collimation errors we are enabled to detect boundaries of colored fields with a degree of precision which would be wanting in eyes without such errors.

This is a highly interesting reason for the persistence of a systematic optical error in the human eyes which is otherwise extremely difficult to account for, since no other possible error admits of correction so easily. If we recognize the advantage which the peculiar relation of the collimation errors in the two eyes gives to one searching for colored fruits or colored animals, we may be led to the conclusion that in an earlier state of racial development the peculiarity would have been more important than at present, and then, perhaps, to speculation as to whether it may not now be regarded as largely vestigial. This last idea would doubtless find support in the extraordinary irregularity of the constant of collimation in the sixty or more cases recorded.

II. *Geranium Phenomenon.*

This is a name given provisionally to a peculiar visual phenomenon with which I have been familiar for an indefinite time, although it does not seem to have ever been noted, or at least recorded, by other observers. It is not unlikely that relatively few persons are able to see it, but it is not a personal peculiarity. Briefly described the phenomenon is this: When-

ever I look at red geranium petals, in the brilliant light of out of doors, projected against the more remote background of its green leaves, the petals seem—in portions of the visual field at any rate—to be bordered with an exquisitely fine line of intense blackness, much more intense than that of black velvet under the same illumination. My eyes cannot make these black lines point of fixation for they are singularly elusive, but they are too delicate to be seen if they did not fall very near the axis of vision. Red petals of other flowers exhibit quite the same phenomena, notably red nasturtiums, but only strikingly when the green background is sufficiently luminous.

The more important general conclusions from such observations seem to be embodied in the following list:

- (a) The phenomenon is monocular.
- (b) The dark lines are conspicuous only when the illumination is intense (equals, probably, when the pupillary aperture is small).
- (c) The contrasting colors must be well separated as regards refrangibility and of approximate equal brightness.
- (d) The difference of the distances of the two colored fields from the eye must not be small compared to the distance of the nearer one.

By attention to these precepts I have been able to observe the same phenomenon with a considerable range of pure spectral colors as well as to prove that the order of the colors, as measured from the eye, is not essential. The reason that I see the black lines as described above, and no black border to a green leaf projected upon a red petal, is to be ascribed, probably, to the fact that my slightly myopic eye can form a sharp retinal image of the red when not more than one or two meters distant, while the green leaf would be notably out of focus.

A highly probable explanation of this phenomenon can, I think, be found from an inspection of fig. 1 above. Suppose the parti-colored object be shifted so that the line of demarcation falls on the line n, π , extended; then, in the right eye, r and b , will have approached each other, but will still be distinct. Now imagine the blue portion of the object carried a considerable distance farther from the eye on the line n, π , extended, then light from points in the blue object very near its edge will send light to only the outer, or temporal, half of the pupil, the inner half being shaded by the nearer red object. It is, however, just this latter half of the pupil that transmits the light which diffuses blue light on the temporal side of the line π, b , hence there is a region of the retina between r and b , in the right eye, upon which, under the conditions considered, no light falls on any color whatever. No doubt such an

unilluminated portion of the retina, if it caused any visual perception, would be interpreted as a line of more than absolute external blackness.

In the case of the model eye the breadth of the dark band would be considerably less than one minute of arc and at a distance of about two degrees from the point of most distinct vision, hence it is perhaps doubtful whether it could be perceived as a black line. In the case of my own eyes the constant of collimation is about half that of the normal eye and the pupils are nearer the axis of vision. From these data we may estimate the angular value of $r b_1$ for my eyes, as something less than one half minute of arc at the *fovea centralis*. The value for the interval red-green may be regarded as half this value; and finally, at the eccentric position where the strip on the retina is wholly unilluminated this must be again reduced by an amount which could be determined accurately only when the constants of the eye are known with a greater precision than now attained. It is probable, however, that this value would not be less than ten seconds of arc. Whether a strip of the retina of a width only one sixth of the ultimate perceptual elements of vision would, when deprived of light, give rise to a conspicuous sensation, is a question which, as far as I can find, has not been considered by investigators in this field. It has been abundantly proved that the ultimate precision of optical definition, under the most favorable circumstances, is not less than $60''$; on the other hand, there is obviously no minimum limit for the angular width of a bright line for visibility, since this is only a question of the amount of light received by the eye, or of its brightness. But the question of how narrow a black line on a bright field can be seen is wholly different. My own experiments, although of great simplicity, seem to be perfectly adequate for our present purposes. I found that I could see a black hair of a measured diameter of 0.0025 inch as a black line, against a sky of favorable brightness, up to a distance of 300 inches. At the greatest distance the angular width is only $1''.72$. There is, therefore, no difficulty in adopting the above explanation of the phenomenon under discussion even if we were obliged to assume a much smaller value for the area of unilluminated retina.

A familiar example of this same phenomenon, if I am not mistaken, is presented to us by the appearance of a red billiard ball on a green table. Here the vertical contours of the ball are astonishingly sharp as compared to that of the upper surface, and also to what we might expect from the pronounced chromatic aberration of the eye.

III. *Phenomenon of "Fluttering Hearts."*

This phenomenon, upon which much has been written, is thus described by Helmholtz:—"A peculiar phenomenon, which perhaps belongs in the same domain as that of the flickering rotating disks, is that of the so-called fluttering hearts. On a colored sheet of stiff paper are placed figures of another vivid color; red and blue seem to yield the best results—the colors must be very vivid and saturated. If one looks at the sheet while it is moved to and fro with a certain quickness, the figures seem to move themselves and to shift forwards and backwards on their support. The cause of this appearance seems to depend upon the fact that the visual impression for different colors does not originate and die out with equal quickness, and consequently the blue appears left somewhat behind the red in the path described by the sheet."

Numerous experiments with a considerable number of observers made in accordance with this description proved wholly futile. Small disks of vividly colored paper scattered upon strongly contrasting grounds were tried under greatly varying circumstances of illumination and of observer without once succeeding. It was only after one of my colleagues brought me a particular book having green and black lettering printed upon a red cover, both colors being nearly saturated and of approximately equal luminosity, that the real phenomenon could be observed by me and exhibited to others. Even in this case it could not be recognized in a good illumination, either by day light or by artificial light, when the lighting was such that the texture of the surface and outlines of the green letters was well seen; but when illuminated by a single source of light, sufficiently remote so that the sharpness of the green letters was lost, the effect became absolutely startling—the green letters appearing to slip about among the black in a most unaccountable way. The astonishment shown by all to whom I have exhibited this optical illusion is a sufficient proof of its rarity in ordinary experience.

The indicated conditions of success in the experiment seem to be these:—

(a) The saturated colors must differ widely in refrangibility and not too widely in luminosity.

(b) The position of one of the colors in the visual field must be well determined—as by the sharply printed black letters in the object described—while the other must have its outlines ill defined.

With attention to these precepts I have been able to secure the illusion invariably with every one with whom I have experi-

* Helmholtz, *Physiol. Opt.*, 2te Aufl. p. 533.

mented. By isolating small pieces of colored paper by a black border from the red background I have been able to show the peculiar shifting in question for various hues of green, blue, and ultramarine blue. Sometimes when not obvious to direct vision it becomes striking to averted vision.

The explanation, it seems to me, may be found in the differing angular velocity with which the images of different colors move over the retina. This difference can be readily calculated in the following method.

Let the object move so that its image (fig. 2) in red light shifts its position from r to r' ; the corresponding image in blue light will shift from b to b' , while the center of the diffusion circle on the retina, supposed to be at r for convenience, shifts from r to b' . It is at once obvious that $rr'/bb' = n_2r/n_2b$; also $bb'/rb'_1 = \pi_2b/\pi_2r$; whence $rr'/rb'_1 = n_2r \cdot \pi_2b/n_2b \cdot \pi_2r$. From the table of dimensions given above this ratio is found to equal 1.0072; that is to say, when an object having red points is shifted in the visual field, the red points seem to have an angular velocity and an angular acceleration about three-fourths of one per cent greater than that of accompanying blue points. Thus, when the red surface described in the experiment is suddenly set in motion, the eye, for an instant fixed, judges of its acceleration and of the less acceleration of the green; then, following it in its motion until it is arrested, concludes that the more refrangible color is left behind—a false judgment which is corrected by the sense of a gradual approach of the green to its true position of rest, just as when the eyes, observing a procession of points passing fixed points, corrects a false judgment as to the place of the former, when the motion suddenly ceases, by an apparent temporary motion in the opposite direction. Of course, if we gain our notion of the angular velocity of the green areas from sharp visual perceptions of their boundaries, the illusion ought not to appear; it is only when the position of the colored area is recognized by its color alone that the effect becomes striking; hence the significance of faint illumination.

IV. *Binocular Color Relief.*

This is an optical illusion not noted in Helmholtz but described and studied at length by W. Einthoven, who was incited to the investigation by Donders.* According to this writer, Professor Donders was the first one to record the fact that red and blue objects in the same plane appear to most observers, when looked at with both eyes, to lie in different planes, especially when on a black background. To him, as well as to Einthoven, the red appears nearer than the blue. The explanation offered by Einthoven can be described by reference to fig. 1 above. Imagine both axes of the vision converged towards a red point which is accompanied by a blue point just above it: the red images would fall on the points of distinct vision, while the centers of the diffusion circles for blue would fall on disparate points of the retinas nearer the median plane, just where sharp images of a more distant red point would fall; hence the interpretation of the resulting sensation is that the blue point is in fact more remote. He finds evidence in favor of this view by the ingenious experiment of cutting off, by means of movable diaphragms in front of the eyes, first the inner halves of the pupils and then the outer; by this means he enhances or inverts the illusion. His experiments are easily repeated and are highly interesting.†

There are, however, serious objections to this explanation, not only as accounting for the observations, but also from theoretical considerations. Of about thirty individuals investigated Einthoven found that nearly one-third were not confident at first that there was any sensation of relief, while ultimately about half recognized the relief as did Donders, and the other half inferred an inverted order. This general conclusion is fairly well supported by my own more limited experiments. According to the theory, we are obliged to conclude that half of those investigated had the centers of their pupils lying on the nasal side of the axes of vision, and half on the temporal. It is, however, quite certain that a displacement of the pupil towards the nasal side of the axis of vision

* Stereoskopie durch Farbendifferenz. Graefe's Arch., xxxi (3).

† A convenient and interesting modification of this experiment may be made by observing a flat surface divided into separated areas of saturated colors through a binocular telescope of which the separation of the axes can be varied at will. The familiar prismatic binoculars are best adapted to the purpose, and any mosaic glass window in which the colors are vivid forms a far better object than colored pigments. If the axial separation is a little greater than that of the eyes, a striking relief will be observed in the sense described above, while a smaller separation will invert the relief.

is extremely rare ; consequently there seems to be no escaping the conclusion that similar visual impressions receive, not merely unlike, but opposite interpretations in different individuals. Any acceptable theory must adapt itself to this fundamental fact.

It seems to me that we can find a probable explanation in the following considerations. Imagine a number of red points distributed in two parallel planes which lie at right angles to their direction from the observer. If the eyes be fixed upon one of the points in the nearer plane, the images of all the other points in that plane will fall on pairs of congruent points of the two retinas, but the images of the points on the more remote plane, which will be quite sharp if the separation of the planes is moderate, will fall on disparate points of the two retinas. Now consider the case of red and blue points in a single plane. Here, if the eyes be fixed upon one of the red points, all the other red points will have their images on congruent points of the right and of the left retina, while the blue points, relatively very diffuse as compared to the images of the more distant red points in the former case, will fall on disparate points. So far there is a formal similarity between the two cases, but it cannot be carried further : in the first case a simple change of convergence of the axes of vision will change the disparate points to congruent, and *vice versa*, while in the second case no such change can produce such an effect ; but a change of accommodation proper to blue light will do so at once. The resemblances and differences are such, therefore, that it should hardly surprise us that certain persons recognize the second case as only a confusion of visual images, when others interpret the effect as a sort of chromatic relief ; nor is it astonishing that of the latter group, since there is really no fundamental relation between the two phenomena, some should imagine the red to be nearer than the blue points and others invert the order. It is singular, however, that Einthoven found those whom he observed to be distributed so nearly equally among the three classes.

The experiments of restricting the pupils by screens in front of the eyes is by no means conclusive ; indeed, it is questionable whether it has any bearing whatever upon the phenomenon. The screens will, primarily, increase very greatly the separation of the centers of the red and blue areas on the retinas which represent the images of the points ; moreover, the areas themselves will be much decreased. This will become evident from an inspection of fig. 1. But, as is easy to prove by a diagram, any lateral change of the common point of fixation will produce a relative change in the position of the colored areas on

the retinas, which can be compensated by an alteration in the convergences of the axes, and thus the phenomenon is reduced to one which admits of a simple stereoscopic interpretation in complete accordance with the observations. This is because such a shifting—whether produced by a change in the direction of vision or by a change of place in the object itself—partially uncovers one pupil while increasing the obstructed portion of the other, in short, virtually moves one pupil inwards and the other outwards. The resultant effect is like that produced by placing a thin wedge of glass before one eye, when the red appears in advance of the blue if the thicker edge of the wedge is on the nasal side; a reversal of the wedge inverts the apparent relief.

There is, however, one phenomenon which often gives a determining impulse to the interpretation in accordance with the experience of Donders, namely, the relative angular displacement of different colors upon the retina. Thus, in my own case, although quite unable to recognize anything like stereoscopic relief among a series of strongly colored figures on a black background when the eyes are fixed pretty steadily upon them, the relief appears very striking when I walk past it, or when the object as a whole is moved to and fro. So too, in the experiment of the fluttering hearts described above, although there is no chromatic relief under ordinary illuminations, such relief is an invariable accompaniment of the fluttering when produced. As has been shown above, the angular accelerations and angular velocities of moving objects in the visual field would vary with their color, so that colors of greater refrangibility would appear to change their directions from the observer more slowly, exactly imitating in this particular the effect of greater distance.

The illusion described would appear of rather abstract scientific interest were I not convinced that the incomparable French artists of the thirteenth century had recognized it and employed it for the purpose of artistic expression. Indeed, it was a casual inspection of the marvelous medieval windows in the great cathedral at Bourges which first turned my attention to the studies embodied in these papers, and which persuaded me that the one essential distinction between these antique windows and their unsatisfactory modern imitations lies in the knowledge, possessed by the old artists, of the effect gained by an ordering of their vivid colors so that the resulting chromatic binocular relief should fit the composition of their pictures. As far as known to me, the most beautiful surviving examples of this lovely art, as well as the most convincing support for the views here presented, are contained in that unapproached

collection; and not alone in the wonderful achievements of those forgotten artists, but also in the instructive failure which has attended modern restorations.*

This is not the place to suggest reasons why so charming an art should hardly have survived the thirteenth century, nor to discuss certain peculiar restrictions to which the artists subjected themselves; but were such an extension of this paper desirable, it seems to me that it would add material weight to the explanation, founded upon principles of physiological optics, of the acknowledged superiority of antique mosaic windows over their modern imitations.

*In two quadrifoils in the window given by the Guild of Tanners the artist has chosen a red background in place of the almost universal blue; but he has reversed the order of his colors throughout the composition so that the effect, to my eyes at least, was that of two charming little intaglios. It was this which first suggested to me the distinction between ancient and modern mosaic windows described above and which I thought abundantly verified by subsequent observations. Certain very puzzling contradictions to this theory—I have no means now of determining how many—were eliminated by a subsequent discovery that considerable areas of some of the windows are nineteenth century substitutions for the original designs which had been lost. There was no suggestion of this significant fact in my hands at the time of my visit.

Yale University, May, 1905.