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ART. I.—*On the Energy of the Cathode Rays*; by W. G. CADY.

[An investigation carried out at the Physical Institute of the University at Berlin.]

It has long been known that the cathode rays possess a considerable amount of energy; yet the relation between this and the other discharge phenomena has hitherto received but little attention. Ebert and Wiedemann,* E. Wiedemann,† and Ewers‡ measured the energy of cathode rays by means of calorimeters. For exact work, however, the thermo-element is to be greatly preferred. Among the recent investigations based upon the emission hypothesis, this method has been employed by J. J. Thomson§; O. Berg has also used a thermo-element in his work with the cathode rays.||

According to the emission hypothesis there exists the following relation between the energy of the cathode rays, potential of discharge, and amount of electricity transported by the rays. Suppose that we have a bundle of rays given off from a cathode of potential V , and striking a metallic conductor which is led to earth through a galvanometer; let the conductor be struck by N particles per second. We will at first assume that the entire charge of the particles, as well as all their energy in the form of heat, is given up to the conductor. Then if v [cm. sec⁻¹] be the velocity of the particles, and m

* H. Ebert and E. Wiedemann, Sitzungsber. der phys. med. Soc. zu Erlangen, Dec. 14, 1891.

† E. Wiedemann, Wied. Ann., lxvi, p. 61, 1898.

‡ P. Ewers, Wied. Ann., lxix, p. 167, 1899.

§ J. J. Thomson, Phil. Mag., xliv, p. 293, 1897.

|| O. Berg, Ber. d. naturforsch. Gesellsch. zu Freiburg i. Br. XI, vol. ii, p. 73, July, 1899.

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[gr.] their mass, we have for the amount of heat given up to the conductor

$$Q = N \cdot \frac{1}{2}mv^2 \quad (1)$$

(ergs per second).

If the charge on each particle be ϵ , we shall have flowing to earth through the galvanometer a "cathode-current" of intensity

$$i = N \cdot \epsilon \quad (2)$$

Now the kinetic energy of each particle is

$$V \cdot \epsilon = \frac{1}{2}mv^2. \quad (3)$$

Hence

$$Q = N \cdot V \cdot \epsilon = i \cdot V,$$

or

$$\frac{iV}{Q} = 1. \quad (4)$$

According to Starke's investigations,* a part of the cathode rays are reflected. This can have no effect upon the above, provided that after reflection the velocity of the rays remains unchanged; and indeed the measurements made by Merritt† upon the deflection of reflected cathode rays seem to show this to be the case. We will, however, assume for the present that upon reflection the energy of each particle is diminished in the ratio $r':1$, while the charge remains unchanged. Then if the fraction r of the rays be reflected, we have

$$Q = N(1-r) \cdot \frac{1}{2}mv^2 + Nr(1-r') \cdot \frac{1}{2}mv^2 = \frac{1}{2}Nmv^2(1-rr') \quad (1a)$$

$$i = N\epsilon(1-r) \quad (2a)$$

$$V\epsilon = \frac{1}{2}mv^2 \quad (3a)$$

$$\frac{iV}{Q} = \frac{1-r}{1-rr'}. \quad (4a)$$

If $r' = 1$ (Merritt), we have still $iV/Q = 1$. If in the limiting case $r' = 0$, then $iV/Q = 1-r$.

These relations were investigated experimentally as described in the following paragraphs.

Apparatus.

Fig. 1 shows the arrangement of apparatus at first used. For measuring the energy Q a thermo-element was here employed; it was later displaced by a bolometer (see below.) The thermo-element, S , was of the Melloni type, consisting of 49 bismuth-antimony couples each 25^{mm} long; the exposed surface of the junctions was 196^{mm}². It was placed inside the

* H. Starke, Wied. Ann., lxvi, p. 49, 1899.

† E. Merritt, Phys. Rev., vii, p. 217, 1898.

Faraday cylinder C , which was connected to earth. The rays entered through the hole o , 6^{mm} in diameter. R is the glass discharge-tube, 25^{cm} long, cemented to the glass plate P .

The copper bottom of the brass tube K served as cathode; in order to reduce the heating as far as possible, a stream of water was kept constantly flowing through the tube. The anode A was a zinc diaphragm.

The discharge current was furnished by a secondary battery B of 2288 cells, supplemented at times by a rotary transformer, so that a maximum E. M. F. of 5400 volts could be reached. The current was regulated by the Hittorf cadmium-iodide resistance I , and the potential of the cathode measured by means of the quadrant-electrometer E . G is a Thomson galvanometer used in measuring the thermal and cathode-currents.

A serious source of error in observations lay in the evolution of gas from the cathode and other parts of the apparatus during discharge. This difficulty, so common in all gas-discharge work, was nearly always present, even after the apparatus had been in use for many hours. A partial remedy was found in running the air-pump slowly during observations, but even then the potential usually changed somewhat during the

minute or so necessary to take a set of readings. This is the chief cause of the irregularities in the numerical values given below.

Before the apparatus was put together, the thermo-element was calibrated as follows. The radiation from the blackened surface of a Leslie cube, filled with boiling water, passed through a series of diaphragms and fell upon the thermo-element. The constant of radiation for lampblack at 100° was taken according to Kurlbaum* as 0.0176 gr.-cal/sec. From the dimensions of the diaphragms could then be calculated, by use of the Stefan-Boltzmann law, the amount of heat Q_0 radiated from the hot surface at temperature T to the thermo-element at temperature t . Like the Leslie cube, the thermo-element was also covered with a layer of lampblack.

The galvanometer deflection s_0 was observed after the thermo-element had been exposed for one minute to radiation from the cube. The amount of heat per scale division is then $e = Q_0/s_0$, and the amount corresponding to the deflection s is $Q = s.e$.

During calibration the thermo-element was in air at atmospheric pressure; when in use with the cathode rays, it was in a partial vacuum, and moreover the layer of lampblack had then been removed. This would tend to make the rise in temperature of the junctions in the latter case greater than in the former, assuming the quantity of heat supplied to be the same: for the loss of heat by radiation and conduction to the air must have been greater during calibration. The effect of this error would be to make the observed energy of the cathode rays too great; but the error is smaller, the more the loss of heat by conduction and radiation to the air can be neglected in comparison to the loss by conduction through the metal of the thermo-element itself.

In measuring the cathode-current, the contact ac (fig. 1) was closed, thereby connecting the thermo-element to earth through the galvanometer. For the thermal circuit, ab was closed, and at the same time either ce or cd , since it was necessary to connect some point to earth. But when for example cd was closed, it was found that a small fraction of the cathode-current also flowed to earth through the galvanometer, along the path $SbaGdc$. Closing ce caused approximately the same deflection in the opposite direction. Hence the mean of the two readings could be taken as the deflection for the thermal current alone.

In order to eliminate the effect of heat radiated from the cathode, the rays were deflected before each observation by

* F. Kurlbaum, Wied. Ann., lxv, p. 759, 1898.

means of the coils MM (fig. 1); the small remaining galvanometer deflection was then subtracted from the total deflection with rays falling.

Observations with Thermo-element.

The value $i \cdot V$ (eq. 4) in gram-calories is cathode-current in amperes $\times$ difference of potential in volts $\times 0.24$. Pressures are always given in mm. of mercury.

The first measurements yielded values of $i \cdot V/Q$ much greater than 1, as shown in the following table:

TABLE I.

Pressure.	$i \cdot 10^7$ Amp.	V	$Q \cdot 10^5$ gr.-cal.	$i \cdot V/Q$.
0.067	152	2280	90.3	9.18
0.046	138	2470	141	5.80
0.044	113	2490	123	5.50
0.042	99.4	2600	127	4.89

The junctions of the thermo-element were here, as at the calibration, covered with a layer of lampblack. It may, however, be assumed that the long-wave heat radiations used in calibrating penetrate quite deeply into the interior of the lampblack, while the cathode rays are absorbed close to its surface. If the heat conductivity of the lampblack is small compared with that of the metal of the thermo-element, it is not impossible that for the same amount of heat, the rise in temperature of the junctions under the influence of the cathode rays may be smaller than at the calibration. The lampblack was therefore removed, and in fact the value of $i \cdot V/Q$ was thereby diminished.*

The following measurements were next made after removal of the lampblack:

TABLE II.

Pressure.	$i \cdot 10^7$ Amp.	V	$Q \cdot 10^5$ gr.-cal.	$i \cdot V/Q$.
0.046	180	3100	412	3.25
0.050	175	3100	397	3.28

TABLE III.

Pressure.	$i \cdot 10^7$ Amp.	V	$Q \cdot 10^5$ gr.-cal.	$i \cdot V/Q$.
0.043	122	3150	331	2.80
0.029	116	3500	416	2.34
0.026	100	3620	416	2.09
0.025	65.4	3820	309	1.94
0.022	61.7	4070	344	1.75

* This diminution no doubt comes in part as described above from the change in constant of the thermo-element after removal of the lampblack, also perhaps partly from the altered reflection of the cathode rays.

It is to be noted that in Table III as also in Table I, *the measured value of iV/Q decreases with decreasing pressure and cathode-current.*

In order to investigate this relation more closely, a series of observations was made, varying pressure and cathode-current within as wide limits as possible. The cathode-current could be altered either by changing the number of accumulator cells, or by varying the cadmium-iodide resistance. Owing to the evolution of occluded gases from the cathode and its gradual discoloration, causing sudden irregularities in potential, it was impossible to get a perfectly consistent set of readings. Still in general it was evident that the ratio iV/Q diminished with decreasing i , at first rapidly, then more slowly. At high pressures ($>0.05^{\text{mm}}$), the ratio seemed to depend also upon the pressure; but when the pressure had sunk below 0.03^{mm} , the drift was hardly perceptible.

These results led to the supposition that beside the cathode rays, a fraction of the discharge current also was conducted from the cathode to the thermo-element; for as J. J. Thomson* has shown, the portion of gas transversed by the rays becomes a relatively good conductor, except at very low pressures. This supposition was confirmed by inserting a battery between c (fig. 1) and the earth, thereby charging thermo-element and Faraday cylinder to a positive or negative potential v .† This cannot affect the true value of the ratio iV/Q as long as V denotes the difference of potential between cathode and thermo-element; but the supposed disturbing current must grow stronger or weaker according as v is made positive or negative.

The following table shows how the apparent value of iV/Q was correspondingly increased or diminished according to the sign of v .

Pressure.	v	V	$i \cdot 10^7$ Amp.	$Q \cdot 10^5$ g-cal.	iV/Q .
0.050	+70	4050	3.54	34.1	1.01
0.050	0	3980	2.64	30.2	0.84
0.050	-70	3910	2.29	30.5	0.71

The potential of the cathode as measured on the electrometer is here $V-v$; as in all measurements, i denotes the observed sum of cathode- and disturbing currents.

The variations in the value of Q result from the fact that the path of the rays was visibly altered while the thermo-element was charged.

* J. J. Thomson, Phil. Mag., xliv, p. 293, 1897.

† The cylinder was here connected with the point c . Owing to incomplete insulation between thermo-element and cylinder it was not practicable to connect the latter directly to earth.

The cathode-current is here only about $1/30$ as intense as in Tables II and III, hence the value of iV/Q is also much smaller. The extremely small value of iV/Q when the charge is negative is explained as follows: owing to the conductivity of the path of the rays, a weak current finds its way from the battery used in charging the thermo-element, along this path to the anode. This current is opposed to the cathode-current, thereby reducing the apparent value of the ratio iV/Q . When the thermo-element is positively charged, the ratio is on the other hand greatly increased.

Measurements similar to the above, made at a pressure of 0.012^{mm} , showed that charging the thermo-element no longer had any influence upon the value of the ratio. These observations on the conductivity of the cathode rays agree well with those of Starke* and Thomson.†

The attempt was also made to intercept the disturbing current by means of a grating of several fine wires laid over the opening of the Faraday cylinder. This was, however, of no practical assistance, as even at high pressures the effect was very slight.

The results that have been given show that the observed value of iV/Q was in general too large. The disturbing current was at times greater than the cathode-current itself, but it was impossible to measure directly its absolute value. It therefore remained to establish the lower limit of the ratio. For this purpose a series of 46 observations was selected, having a range of pressure from 0.01 to 0.05^{mm} , of potential from 2400 to 5400 volts, and of cathode-current from $40 \cdot 10^{-7}$ to $0.23 \cdot 10^{-7}$ amp. When arranged in order of decreasing cathode-current, the values of iV/Q showed great irregularities, since the order was without regard to pressure and other conditions. Still a distinct drift in the ratio was evident, especially when the observations were divided into five groups and the average values taken:

Pressure.	$i \cdot 10^7$ Amp.	iV/Q
0.041	19.6	1.32
0.035	7.41	0.97
0.026	4.29	0.85
0.033	2.02	0.84
0.023	0.49	0.83

From this it is evident that when the cathode-current decreases to the fortieth part of its original value, the ratio iV/Q is changed by only 38 per cent. This at least makes it seem

* H. Starke, Wied. Ann., lxvi, p. 52, 1899.

† J. J. Thomson, Phil. Mag., xlv, p. 293, 1897.

probable that eq. (4a) is approximately correct, although the limiting value of the ratio is considerably smaller than unity. Still owing to the sources of error in the use of the thermo-element, no great dependence can be placed upon absolute values. It was therefore determined to measure the energy Q in a surer manner, by Kurlbaum's bolometric method.*

Observations with Bolometer.

A surface-bolometer forms one arm of a Wheatstone bridge; suppose the bridge balanced, and a current of intensity I_1 flowing through the bolometer. Let the resistance of the bolometer under these conditions be w . If now an amount of heat Q per second be supplied by the cathode rays to the bolometer, the galvanometer in the bridge will suffer a certain deflection. Suppose the same deflection caused by increasing I_1 to I_2 , the cathode rays no longer acting. Then from Kurlbaum's formula,

$$Q = w \cdot \frac{I_2^2}{I_1} (I_2^2 - I_1^2) \text{ watts,}$$

where I and w are expressed in amperes and ohms respectively.

If the four bridge resistances are equal, then $I = \frac{1}{2}c$, where c is the current in the battery branch, and

$$Q = \frac{1}{4}w \cdot \frac{c_2^2}{c_1} (c_2^2 - c_1^2) \text{ watts.}$$

As it was not practicable to make both deflections exactly equal, the following method was adopted:

1. galv. defl. a_1 with current c_1 , rays falling.
2. " " a_2 " " c_2 , " deflected.

We thus get the following equation:

$$Q = \frac{1}{4}w \frac{a_1}{a_2} \frac{c_2}{c_1} (c_2^2 - c_1^2)$$

The thermo-element formerly used (S , fig. 1) was replaced by a bolometer, whose terminals were led out through the plate P . The bolometer was made after Lummer and Kurlbaum's process,† with a few modifications. In the first observations it consisted of a platinum foil about 0.001^{mm} thick, mounted with mica insulation upon a brass frame. The form is shown in B (fig. 2). Care was taken that the entire bolometric resistance should be reached by the rays; also that all cathode rays entering the Faraday cylinder that did not strike

* F. Kurlbaum, Wied. Ann., lxx, p. 746, 1898.

† O. Lummer and F. Kurlbaum, Wied. Ann., xlv, p. 204, 1892.

the bolometer, should give up their charge to earth through the metallic frame. Although the density of the rays is not perfectly uniform throughout the bundle, still it can be assumed that the distribution of temperature in the thin bolometric foil under the influence of the rays and of the electric current is practically identical.* The silver was not dissolved away from the platinum-silver foil until the bolometer had been mounted.

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Fig. 2 shows the arrangement of the bridge. The current-intensity i in the battery branch was measured by means of the ampere-meter M . Shunting the resistance r parallel to R increased the current from i_1 to i_2 and gave an accurate means for measuring the increase. A Thomson galvanometer of du Bois-Rubens type was employed. For the thermal measurements two 20-ohm coils mm were used; in order to get rid of the cathode-current, a point between the two coils was led to earth (contact a , fig. 2) and the resistance T so chosen that with battery disconnected the cathode-current caused no deflection of the galvanometer. Two 2000-ohm coils n served to measure the cathode-current, by closing the contact b , while a remained open.

S is a shunt for reducing the sensitiveness of the galvanometer when necessary. The bridge resistances were balanced by means of the sliding contacts at D . The manganin wire resistances W were as nearly equal to the bolometric resistance

* F. Kurlbaum, Wied. Ann., lxx, p. 755, 1898.

as possible, but small discrepancies were always taken into account. It was necessary to use great care in insulating the entire circuit from earth.

The bolometer was found to be much more sensitive than the thermo-element, although its effective surface was smaller. The source of error from gas-evolution was also less serious, owing to the rapidity with which observations could be made.

The process of etching and washing such small bolometers is somewhat complicated when a metallic frame is used, as the latter has to be protected by a coating of wax, which is afterwards dissolved away. Thus it is difficult to avoid straining the delicate platinum strips. This may become a very serious source of error, as it is impossible to know the extent of the injury. Moreover, a brownish deposit was nearly always formed upon the strips, probably from some action of the acid used to dissolve the silver, upon impurities in the wax. This may in some way have caused a further source of error.

Four different bolometers with brass frames were used. The results are briefly given below, though in at least two cases great irregularities occur. It was not until a frame of slate had been made that reliable observations were obtained (see below).

Bolometer I. Eleven strips, $11.5 \times 0.66^{\text{mm}}$. Resistance w at room temperature = 26.21 ohms.

The observations were all made at high pressures; the ratio iV/Q is therefore chiefly dependent upon the pressure and not upon the cathode-current.

Pressure.	$i \cdot 10^7$ Amp.	V	$Q \cdot 10^5$ Watt.	iV/Q .
0.116	18.0	2590	213	2.21
0.113	12.3	2610	140	2.28
0.064	22.3	4090	541	1.69
0.055	13.7	4210	376	1.53

Bolometer II. Nine strips, $11.5 \times 1.0^{\text{mm}}$. $w = 15.74$ ohms.

An influence machine was used instead of the battery as source of discharge. As the range of values of i is not great, only the average values are here given: $i = 1.40 \cdot 10^{-7}$ amp.; pres. = 0.020^{mm} ; $iV/Q = 0.81$.

Bolometer III. Seven strips, $11.5 \times 1.3^{\text{mm}}$. $w = 8.75$ ohms.

A battery of 30 cells was so arranged that it could be connected at will between P (fig. 2) and the earth. This raised the bolometer and of course the entire bridge to the potential of the battery. The Faraday cylinder was kept in contact with P . Readings were taken with the bolometer alternately at 0 and +60 volts.

From the table it is evident that as in the case of the thermo-element, charging the bolometer to a certain potential increases the ratio iV/Q to a greater extent, the stronger the cathode-current, that is, the greater the conductivity of the gas.

Pressure.	v	$i \cdot 10^7$	V	$Q \cdot 10^5$	iV/Q .
0.033	0	5.30	3130	106	1.57
	+60	7.20	3090	119	1.91
0.028	0	2.10	4320	80.8	1.12
	+60	3.23	4200	113	1.20
0.029	0	1.96	5150	75.5	1.33
	+60	4.91	4960	154	1.58
0.030	0	1.90	4230	67.5	1.19
	+60	2.80	4130	88.5	1.31
0.030	0	0.54	3910	20.8	1.02
	+60	0.67	3880	25.3	1.02
0.031	0	0.28	4220	11.9	0.98
	+60	0.56	3900	21.9	1.01

Bolometer IV. Seven strips, $11.5 \times 1.3^{\text{mm}}$. $w = 9.20$ ohms.
Values of the ratio were irregular, but the averages from three groups of observations are here given :

Pressure.	$i \cdot 10^7$ Amp.	iV/Q .
0.070	9.43	0.81
0.040	11.48	0.76
0.023	0.18	0.72

As with Bol. I, the ratio at high pressures seems dependent upon the pressure alone. In the third group the cathode-current was diminished by increasing the cadmium iodide resistance.

These results are of only relative importance except as they indicate that the absolute value of iV/Q is somewhat less than 1.

A fifth bolometer was therefore made and mounted after the Kurlbaum process upon a slate frame. After the silver had been dissolved from the foil, the strips looked perfectly clean and seemed free from strains. In order to keep the cathode rays from the slate, a small brass diaphragm was placed close over the bolometer, with a square opening of the same size as the bolometer surface.

Bolometer V. Seven strips, $7 \times 0.73^{\text{mm}}$. $w = 8.72$ ohms.
The following observations were taken :

Pressure.	$i \cdot 10^7$	V	$Q \cdot 10^5$	iV/Q .
0.038	6.10	2270	76.9	1.80
0.061	5.00	2690	82.3	1.58
0.035	4.80	3480	149	1.12
0.024	2.26	3770	85.5	1.00
0.024	2.09	3560	81.2	0.92
0.030	1.69	4460	92.6	0.81
0.022	1.36	3730	58.0	0.87
0.032	1.09	3740	50.4	0.81
0.032	1.06	3750	47.9	0.83
0.028	0.50	3900	23.6	0.83
0.026	0.255	4740	14.5	0.83
0.023	0.040	4800	2.29	0.84

This bolometer was apparently free from the faults of its predecessors: therefore only the usual errors of observation need be considered, especially that arising from the evolution of gas. This error can hardly have amounted to more than 3 per cent with weak cathode-current; hence it follows from the last table that the true value of the ratio iV/Q lies between 0.80 and 0.86.

Since the bolometer foil was only 0.001^{mm} thick, the question arises whether an appreciable part of the rays can have penetrated through to the other side. In this case it might easily happen that the particles on passing through would lose a part of their energy while retaining their charge. McClelland* and Wien† have shown that cathode rays after traversing an aluminium window carry a negative charge; but Lenard did not succeed by means of a thermo-element in detecting any heating effect.‡ Still if we assume according to Lenard that for solids the quotient of absorptive-power divided by density = 3200,§ we find from his formula that not more than 1 per cent of the rays can have passed through the bolometric foil used. This can have had practically no effect upon the ratio under discussion.

Conclusion.

It only remains to compare the limiting value of the ratio as experimentally determined, with the equation (4a)

$$\frac{iV}{Q} = \frac{i-r}{1-r'r'},$$

where r denotes the reflected part of the rays, while upon reflection the rays lose their energy in the ratio $r':1$.

Starke|| has found that platinum reflects about 40 per cent of the rays. Assuming, therefore, that for the bolometer, $r=0.4$, we find that when $iV/Q=0.83$, $r':0.7$.

This result is in contradiction with the observations of Merritt already referred to. The latter are unfortunately not above criticism, as the author himself admits. Merritt compared the deflectibility of direct and reflected rays by observing the phosphorescence on the walls of the tube. It is hardly to be supposed that the spot from the reflected ray can have been even approximately as distinct as that from the

* J. A. McClelland, Proc. Roy. Soc., lxi, p. 227, 1897.

† W. Wien, Verh. Phys. Ges. Berlin, xvi, p. 165, 1897.

‡ Ph. Lenard, Wied. Ann., li, p. 239, 1894. This experiment was, however, performed in air at ordinary pressure, which rapidly absorbs the energy of the cathode rays.

§ Ph. Lenard, Wied. Ann., lvi, p. 274, 1895.

|| H. Starke, Wied., lvi, p. 58, 1899.

direct; moreover, since an induction coil was used as source of discharge, one would expect the formation of a "magnetic spectrum,"* which would greatly complicate the appearance of both spots. Thus it is easy to see that a systematic error may have crept into the results.

Whether such an error can have been great enough to give the value $r' = 1$ instead of $r' = 0.7$ is, of course, not certain; but it may be added that the energy of the cathode rays is proportional to the square of their deflectibility, hence an error of n per cent in the measurement of the deflection would correspond to an error of $2n$ per cent in the energy. In any case it is desirable to determine both the deflectibility and the energy of reflected cathode rays by as accurate a method as possible.

My thanks are due to Prof. Warburg for his kind advice and assistance during this investigation.

* K. Birkeland, *Comptes Rendus*, cxxiii, p. 492. 1896.

Providence, R. I., May 5, 1900.