Plane Algebraic Curves by H. Hilton

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Plane Algebraic Curves. By H. HILTON, M.A., D.Sc. Pp. xvi, 388. 28s. net. 1920. (Oxford: Clarendon Press.)

Professor Hilton has an exceptional gift for writing mathematical text-books, and this volume illustrates the fact. Indeed, its merits are so many that it must suffice to emphasise a few of them. The scope of the work (considering its size) is remarkably wide, without leading to scrappiness on the one hand or obscurity on the other. Besides indispensable elements we have a discussion of both classes of cubics, which includes all their leading properties; three chapters on quartics; a most interesting and original chapter on derived curves (evolutes, pedals, etc.); and others of even greater novelty that will be referred to presently.

Like his other text-books, this treatise provides a large number of examples. Many of them are worked out in detail, and the real branches drawn approximately to scale. The amount of work that this has involved must be very great, and deserves the recognition of the student; for, whatever logicians may assert, the visible parts of curves are, and probably always will be, those which chiefly interest those who have a truly geometrical form of mind.

One of the most important things in connection with a given curve is the determination of its Plücker numbers ($\delta, \kappa, \tau, \iota$). Up to the present time this remains a troublesome problem, though its solution has been shown to be definite and unique. The general proof would be quite outside the range of a book like this; but it is remarkable how nearly the author reaches it. I think we may say that he gives, in different places, all the elements of the proof except a complete discussion of quadratic 1-1 transformations, and their effect in resolving multiple points. What he does prove is amply sufficient for the curves which he discusses.

Two definitions of *deficiency* are given (pp. 113, 127): the first is

$$\frac{1}{2}(n-1)(n-2) - \delta - \kappa.$$

The other depends on the notion of an adjoint curve. On p. 376 we have a proof that two curves with a 1-1 correspondence have the same deficiency.

Representation of points on a curve by means of a parameter is discussed, not only for rational curves, but also for elliptic cubics and quartics, and simple applications of Abel's theorem are made, though, of course, the theorem itself is not explicitly used.

One of the most interesting chapters is XX., called "Circuits." This deals with the number of real branches of a curve which has a real equation. Here we have Klein's theorem ($n+i+2t=m+k+2d$) with its application to the non-singular quartic; also fundamental theorems due to Harnack and Hilbert.

It is interesting to see how small an amount of invariant-theory the author has to use, and how much he makes it do for him. Here is a great contrast between this book and Salmon's classical treatise. It is a very good thing that Professor Hilton has made no attempt to imitate his great predecessor, but has followed independent lines of his own. With the latest edition of Salmon, the reprint of Frost's "Curve Tracing," and the present work, an English student has materials for studying a most fascinating subject under the best conditions; and it may be hoped that the theory of algebraic curves will now be more frequently taken up at the Universities. When I was an undergraduate, it was a current saying, not devoid of truth, that all a Tripos candidate need know about higher plane curves was that two cubics each passing through eight fixed points had a ninth fixed point in common. Things have changed since then, and probably will still improve as time goes on.

One word more in conclusion. A great deal of ingenious nonsense has been written about the circular points at infinity, even by people who ought to have known better. Professor Hilton does not introduce v. Staudt's theory at all; so the circular points are merely analytical conceptions; but he uses them with great effect, and in a perfectly unobjectional way, because it is clear throughout that any geometrical language that he uses is merely a paraphrase of algebraical results, where the distinction between real and imaginary is irrelevant.

It might have been an advantage if, besides the references in the text, Professor Hilton had given a short list of really first-rate books and memoirs.

G. B. MATHEWS.