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Leveling and Surveying by Means of a Visual Angle and Rod.

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In the February number of this *Journal*, for 1865, will be found a paper written upon this subject.

My chief object was there to give a rigorous analysis of the problem of leveling, and of measuring distances by means of a visual angle and rod, and to derive therefrom formulæ, accurate and also approximate, suitable for practical use. I considered the question with reference to holding the rod normal, or perpendicular, to the central visual ray, and to the horizon; and also adapted the analysis to an unequal division of the visual angle, allowing the middle thread of the reticule of the telescope to be that centered upon the target when measuring the vertical angle,—the target being attached to the rod, or painted upon the stadia. I also considered in the analysis the correction due to a change of the focus of the telescope for the stadia at small and greater distances, and gave the maximum amount of error which would follow from neglecting it entirely.

In the January number of the same volume of the *Journal*, there appeared a valuable article upon a similar subject, by Mr. J. R. Mayer, C.E. His formulæ agree exactly with the approximate formulæ which I gave, though neither had any knowledge of the doings of the other in the matter, until the results appeared in the *Journal*. He did not give the amount of, nor even mention the fact that the adjustment of the focus of the telescope for a satisfactory reading of the stadia at small and great distances is a source of error unaccounted for in the formulæ. I found, however, that the maximum error from this cause never exceeds about one and a half times the focal length of the object-glass in the horizontal measure, and the same multiplied by the sine of the vertical angle in vertical measure.

This error, it is plain, arises from a change in the magnitude of the visual angle, on account of adjusting the reticule—which should be constantly in the focus of the eye-glass for the same eye—to a proper distance from the object-glass for viewing the stadia in various positions. And it is also plain that for the stadia at the same distance from the object-glass, the reticule must always be brought to the same distance from it also. This will follow whether the eye of the observer be short or long sighted, because the focal length of the object-glass is entirely independent of it. Thus, if a screen be placed behind a convex lens or an object-glass, and the image of any object brought out distinctly upon it, and, if for the screen a reticule of wires be substituted, then the reticule will be in the focus of the glass. And now if the reticule be viewed by an eye-glass, both the image and reticule will appear together. Now, if the reticule and image be viewed by a different eye of different focus, it is only necessary to extend or retract the *eye-glass** slightly, when they will both appear to the new ob-

* Not the eye-piece, unless regarded as distinct from the slide carrying the reticule. In the inverting telescope where the Ramsden's eye-piece is used, the slide-tube usually carries both the eye-glass, or eye-piece and the reticule, which are kept at the same constant distance from each other, except when adjusting for the focus of the eye.

server as perfectly as they did to the other, without having changed the relative positions of the reticule and object-glass. All this is true of the telescope of the stadia instrument.

From this fact it is plain that *the only necessary change to adapt any stadia instrument to the use of any observer, is simply a proper adjustment of the eye-glass upon the reticule in the beginning.* He will then obtain the same position of the reticule with respect to the object-glass for distances of one, five, or more hundred feet. Consequently, a stadia properly made for any particular object-glass or telescope, will never need any alteration whatever for different observers. These principles, as well as being in keeping with the principles of optics, I have entirely corroborated by actual experiment with a good stadia instrument.

Mr. Mayer states that the vertex of the visual angle is at the eye of the observer. Although this is sufficient for the approximate formulæ which he gives, yet strictly it is at the *object-glass* at an infinite number of points, but may be regarded as at its optical centre. The two planes of rays which proceed from the spider threads of the reticule, and intersect at the optical centre of the object-glass, exactly contain the visual angle on either side of the glass. Knowing this fact, and the focal length of the glass, it is easy to calculate the interval between the wires of the reticule for any given case, which is equal to the focal length multiplied by the tangent of the given visual angle.

In my previous article upon the stadia, I stated that in order to find the length of the divisions of the stadia, it was desirable to use a base of about 1000 feet, assuming 1500 as the maximum reading to be made in practice. But when it is the intention to use the approximate formulæ, the work will be more nearly correct by selecting a base line, the length of which shall be near a mean of the readings that may be taken in the field work. Since the reading of distances less than this base will be too small, and of distances greater than the base too large, it is probable that the plus and minus errors will nearly balance in long courses. But when it is designed to use the accurate formulæ of (21) and (22) of my article in the number referred to, the base may be taken at pleasure.

Methods for determining the heights and horizontal distances graphically from the field notes have already been explained. This diminishes greatly the time and trouble otherwise required without much injury to the accuracy of the work, if the diagrams are properly constructed.

The diagrams illustrated by Mr. Mayer for reduction to the horizon are very good, although I would suggest the use of vertical lines drawn to the horizontal scale, and both this and the vertical scale numbered with the proper numbers, divided by 10, 100, 1000, &c., if necessary; then the diagram will give directly the required quantity, without the use of a pair of dividers and a scale.

The quadrants which were described, answer a good purpose for getting the heights, but these require a pair of dividers and a scale. The plan of using triangles does not necessitate the use of any other

instrument. They were designed by myself for this purpose. Noting in the mind a reading of stadia, and the corresponding vertical angle, the height is at once traced out upon the triangle and given in the required numerical quantity. In my previous paper upon the stadia, in giving directions for the construction of these triangles, I made an error in stating that the lines drawn from the horizontal scale should intersect at the opposite angle of the triangle. They should be drawn parallel to each other. Specific directions for the construction of the triangles are given below.

Heretofore we have only considered the question of graphical computation as regarding the reduction of observations by these approximate formulas, viz :

$$x = R \cos^2 v, \text{ or } R - x = R \sin^2 v, \text{ and } y = \frac{1}{2} R \sin 2v.$$

But I now propose to show how diagrams can be obtained for graphical determination of the heights and distances in strict accordance with the accurate formulas (21) and (22) given in the February number. These diagrams once constructed, the labor required in their use is the same as that for diagrams made to the approximate formulæ, except adding a small quantity, nearly constant, depending upon $(c + f)$ attached to the diagram, a sacrifice which is small compared with the satisfaction of having the work reduced by correct principles. The correct formulæ (21) and (22) are

$$x = R \cos^2 v \left(1 - \frac{c + f}{x^1} \right) + (c + f) \cos v, \quad (21)$$

$$y = \frac{1}{2} R \sin 2v \left(1 - \frac{c + f}{x^1} \right) + (c + f) \sin v, \quad (22)$$

in which x is the horizontal distance required, y the required height, R the reading of the stadia, v the vertical angle, f the principal focal length of the object-glass, c the distance of the Y's from the object-glass, and x^1 the base line used in getting the dimensions of the stadia divisions.

These formulæ are correct for a stadia uniformly divided, unless the two parts of the visual angle, formed by the three spider threads of the reticule of the telescope are *very* unequal.

Now, let us suppose for a moment, the diagram scale ABC , Fig. 1, to be so constructed that the vertical scale, AC , gives the horizontal distances $= x = R \cos^2 v$, which corresponds with the approximate formula. This quantity may be computed and the number put in its proper place as at D in the vertical scale, which will be the required horizontal distance for all intersections on the line DF .

We notice that the first part of the second member of (21) is $R \cos^2 v$ $\left(1 - \frac{c+f}{x^1}\right)$, which is the same as the approximate formula excepting the parenthetical quantity, which is always a constant fraction for the same stadia and instrument. Consequently, instead of $R \cos^2 v$ we should have a certain fractional part of it ($\frac{9}{100}$ for example) for every intersection on the line D F. Hence, if we put at D, $\frac{9}{100}$ of the quantity we had computed, we will have a diagram giving the correct amount corresponding to $R \cos^2 v \left(1 - \frac{c+f}{x^1}\right)$ instead of $R \cos^2 v$.

What is true of one line, D F, of the vertical scale is true of all. The diagram thus far completed gives the first and most important part of the second member of (21). The second part, $(c+f) \cos v$, is small and varies but slightly for a change of a whole degree in v . Consequently, values of this may be calculated and put on the diagram, or in a table at one side of it. All that has been said of (21) is also true of (22).

Let us calculate some of the quantities and construct the skeleton of a diagram which shall give some of the values of y required by (22) to the first five degrees. Let us take $x^1 = 1000$; and as R may be any quantity let it be taken at 1000. Let $f = 1.3$ and $c = 0.7$. Then for $v = 5^\circ$, $\frac{R}{2} \sin 2v \left(1 - \frac{c+f}{x^1}\right) = 500 \sin 10^\circ \left(1 - \frac{2.0}{1000}\right) = 86.6 +$.

Similarly if

$$v = 1^\circ \quad \frac{1}{2} R \sin 2v \left(1 - \frac{c+f}{x^1}\right) = 17.4, \text{ and } (c+f) \sin v = .03^{.02}$$

$v = 2^\circ$	"	"	34.8	"	"	$.07^{.05}$
$v = 3^\circ$	"	"	52.2	"	"	$.10^{.08}$
$v = 4^\circ$	"	"	69.5	"	"	$.14^{.12}$
$v = 5^\circ$	"	"	86.6	"	"	$.17^{.16}$

Now, draw A B and A C, Fig. 1, and take A B a scale of 1000 equal parts. Make a scale of equal parts on A C, and designate the tens by the proper numbers, and draw lines parallel to A B: and as we have designed to make 5° the upper limit, take the point c at 86.6 of the vertical scale, and draw B C. Then on B C at a point which shall correspond with 69.5 of the vertical scale, draw a line to A. This will be the line of 4° . Thus proceed until all the lines designating degrees are drawn. Draw lines parallel to B C from equidistant points of the horizontal scale A B. The quantities depending on $(c+f)$ may be placed in the diagram as represented. The diagram is now complete except that subdivisions may be made to any extent desired.

To use this diagram, find on the scale, A B, the number equal the reading of the stadia; then follow the line till it intersects with that drawn from A, expressing the number of degrees of the vertical angle. Note the position of this point in the vertical scale. Its height after adding $(c+f) \sin v$ is the height required.

If one triangle will not contain a sufficient number of degrees, 25 for example, of the vertical angle without confusion of lines, the num-

ber of triangles can be increased. But in any triangle which does not begin with zero degrees, the lines of the vertical scale, A C, will not be parallel to A B. To show this more clearly, let us construct a diagram commencing with 10° and ending with 14° . Then for

$$v = 10^\circ \frac{1}{2} \text{ R } \sin 2v \left(1 - \frac{e+f}{x^1} \right) = 170.7, \text{ and } (e+f) \sin v = .35$$

$\psi = 11^\circ$	“	“	186.9	“	“	38
$\psi = 12^\circ$	“	“	205.9	“	“	42
$\psi = 13^\circ$	“	“	218.8	“	“	45
$\psi = 14^\circ$	“	“	234.2	“	“	48

Now, draw A B and A C, Fig. 2, and construct a scale of equal parts on each as before, A C containing 234.2 equal parts. Since the diagram must give 170.7 for $R=1000$ and $v=10^\circ$, we will draw a line through B and 170.7 of the scale A C, parallel to which draw all the other lines of this scale. Then from the point of intersection of B C with a line through 186.9 of the vertical scale, draw a line to A. This is the line of 11° . Thus proceed to obtain the lines for 12° , 13° , &c. The quantities $(c+f) \sin v$ may be treated as before. I have only computed the quantities for the even degrees. Quantities for subdividing to any extent desired may be calculated in a similar manner. The lines I have supposed drawn from A C to B C for designating points on B C are only temporary or imaginary. It is better to draw permanent lines only at regular intervals which can be numbered with integral numbers.

Diagrams for (21) can be constructed by following a course entirely similar to that pursued for (22).

These diagrams when once constructed for any particular instrument, can only be used with strict propriety for this or other instruments having the same distance intervening between the Y's and the object-glass, of which also the principal focal length is the same, or in which $(c+f)$ is the same, since this in the formulæ may be regarded as a single quantity. The errors in the use of these diagrams, however, will always prove to be less than if obtained from diagrams answering to the approximate formulæ $R \cos^2 v$ and $\frac{R}{2} \sin 2v$, except where $(c+f)$ for the instru-

ment used is less than about half the value given it in making the diagrams. From this, in connexion with the fact that $(e + f)$ in all the ordinary stadia instruments used will never differ one-half, it appears that these diagrams are preferable to those adapted to the approximate formulæ.

FIG. 2.

Diagrams answering to the approximate formulæ, can be constructed in a *form* similar to that for (21) and (22).

To give good results the triangles should have eight or ten inches on a side.

The accuracy of the work which can be performed by the use of the stadia instrument is surprising. A fair trial is sufficient to remove all prejudice, if any exists. If it does exist, it must be in the idea of getting distances by sight. But if we inquire into the facts of the case, we find this to be a species of triangulation, the stadia being the base of a triangle in which all the angles are known. The visual angle being very much magnified, the power of measuring distances by it is magnified also. The only fear of great and unaccountable errors entering into the work must be chiefly from an apprehension of changes in the visual angle. Experience, however, shows that if the reticule be properly constructed, it cannot undergo appreciable changes. The redetermination of the delicate intervals of the reticules of astronomical transits proves their practical permanency.

For all the formulæ and diagrams considered in this article, the stadia should be held in a vertical position while taking observations upon it.

Some of the many advantages in the use of this instrument are that the data will always give both the heights and horizontal distances; and the vertical distance, easily gained by a single setting, will exceed ten times the rise possible to obtain by a leveling instrument at a single setting. In meandering rivers, lakes, &c., labor is saved of cutting lines for the chain; and it matters not so much what irregularities of surface are encountered.

The stadia and instrument are a substitute for the transit, the level, and the chain, and fewer assistants and men are required to accomplish the same amount of work, and, instead of its being in the hands of two or three, it may all be subject to the control of one observer.

It will be found to be a most economical and expeditious method of running preliminary lines for railroads, canals, laying water pipes, and for making topography, to which it is peculiarly adapted. It has advantages for locating points of interest in mining regions where there has been no land survey, and for furnishing data for geological maps and charts.

MECHANICS, PHYSICS, AND CHEMISTRY.

Incrustation of Steam Boilers.

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We have recently noticed corrosion as one of the ills to which the boiler is heir, and we will now consider another of a less insidious but not less dangerous nature, viz: incrustation. This evil is too frequently regarded merely as a matter of inconvenience, or, at the most, as a nuisance of which the owners of boilers would be very glad to be