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THE ADVISABILITY OF INCLUDING SOME INSTRUCTION IN THE SCHOOL COURSE ON THE HISTORY OF MATHEMATICS.

I HAVE, during the last two years, in addition to my ordinary routine work of preparing candidates for examinations, been giving a series of lectures to the Students of the Training Department of Alexandra College, Dublin. While reading for these lectures, I was greatly struck by the stress laid by Benchara Branford * in his *Study of Mathematical Education* on the importance of what one might call the historical method. He emphasises the fact that the history of each individual development is a brief compendium of the history of the race, and that the sound method of instruction is to let the student travel, in his quest for knowledge, roughly over the same path by which his fathers arrived.—roughly, only, because life is short, and there were quagmires in which our fathers floundered for many centuries.

I thought it would be very good for the training-students to learn a little of how "Mathematics" grew, before they studied how to teach them, and so I sacrificed a certain amount of their very limited time to this object. And I was glad to find how much their interest was stimulated—especially among those who knew a little mathematics,—and though it was barely possible to do more than stimulate interest, one hopes that some of them will care enough to read more of the subject for themselves when the brief fever of training is at an end.

I reproduced and enlarged for them a scheme given in Benchara Branford's book illustrating the development of Mathematical Science, and showing on one side the periods of human history to which its different stages belong, and on the other the corresponding periods of the student's life. This table (Fig. 1) is, very likely, familiar to many here, but as it has been suggested to me that a figure broad at the base and tapering to a vertex does not give quite the right impression, I have ventured to turn it upside down, as being more suggestive of a tree whose beginning is no bigger than a grain of mustard seed, but the spread of whose branches no man shall foretell.

In other classes with girls of sixteen and seventeen, I have also been trying, more informally, to bring in here and there a little of the story of mathematical growth. It could only be done quite casually, as we are much tied down by examinations in Ireland, but it has always aroused very real interest, and called forth a great many questions,—some of them rather posers, but that must, I suppose, be looked upon as a blessing in disguise. One child, after inquiring of the regions that lay beyond her own beat, said, "And after that we can go on and discover more mathematics." I am afraid she will not, but her attitude was encouraging.

There can be no doubt that it is a great gain to the young student, when he can look upon Mathematics as living and growing, rather than as a crystallised thing from a text-book. Does not even a rock appeal more to our imagination when we realise that it has a story? The subject is humanised; it takes a place in the pageant of our race's history. The student begins to take up a right attitude towards it. He realises what it is that makes progress possible,—how the first impulse came from practical need; how ideas can be extended from the purely concrete to the abstract; how necessary it is to have, besides the thought, a compact and adequate means of expressing that thought

* *A Study of Mathematical Education.* Benchara Branford. Clarendon Press, Oxford.

FIG. 1.

—the necessity, I mean, of a suitable notation. The mathematician is like an explorer who must have his instruments and commissariat in a portable form if he is to travel far.

No modern schoolboy can appreciate the blessings which he enjoys in the way of notation till he has seen something of the difficulties with which his predecessors had to wrestle. The Egyptian method of writing 21,232,378 will probably give him to think. The Egyptian method (Fig. 2), too, of solving a simple equation by successive guesses, is also very instructive (Fig. 3).

Few things are more amazing than the immense time it has taken for some of the simplest ideas to dawn upon the human mind,—and with what complications people had to struggle, just because the simple idea was not there ! Then the flash of genius came, the epoch-making idea (a commonplace now to the First-form child), came into being, and progress became possible.

Is it not an astonishing thing that in the oldest deciphered work on Mathematics,* a papyrus preserved in the British Museum, copied by the scribe Ahmes from a MS. dating about 2400 B.C., fractions should have been employed with a unit numerator only ($\frac{1}{2}$ being written $\frac{1}{2} + \frac{1}{8} + \frac{1}{14}$), and that in a papyrus found in Upper Egypt, written by a Christian Greek between the fifth and ninth centuries A.D., the very same method of handling fractions should be found ? Nearly three thousand years to take the step from a numerator of one to a numerator of two or three ! That this extreme delay has been largely attributable to the crystallising effect of the examination system of Ancient Egypt only a little diminishes the marvel of it.

What I maintain then is that even a slight knowledge of the outline of the story of Mathematical Science not only stimulates the student's interest, but really gives him a better grasp of his subject. Far too many of our students leave us with far too little an idea of what mathematics are really driving at ; and though, largely thanks to the labours of the Mathematical Association, much of that is changed, it is wonderful how uninquiring our attitude is apt to be towards that which we find set down for us in text-books. The small child is naturally inquisitive, but the middle-aged child takes things for granted rather too readily when they are found in text-books, and does not always wonder quite enough how they came there.

I am continually surprised by the number of pupils I get, at sixteen or seventeen years of age, who have not a notion why the number ten has a special place of importance in our system of numeration. I have frequently been told it is because it can be represented by the symbol 10. It comes to them as quite a refreshing piece of news that it actually has something to do with the human body. And then it pleases them to find that such primitive methods of measuring as those still employed in the British Islands are based on the supposed length of parts of the human body, and that to have a measure one could always carry about with one, and could not mislay, such as a foot, a hand, or a nail, had practical advantages at a time when the need of a more rigid standard was not pressing.

A teacher who is interested in the subject can do much towards widening the horizon of his class, without adding greatly to the burden of things to be learnt. The children's imagination is stirred when they realise something of the long struggle to acquire what to them comes so easily,—the ages it took to learn to count even up to ten ; and that at

* See *The Teaching of Elementary Mathematics*, by David Eugene Smith. (Macmillan.) From which much of the information in this paper is derived.

EARLY EGYPTIAN NUMBER SYMBOLS.

FIG. 2.

AN EGYPTIAN EQUATION (ABOUT 2000 B.C.).

$x = 14 + \frac{1}{4} + \frac{1}{9} + \frac{1}{36} + \frac{1}{64} + \frac{1}{100} + \frac{1}{144} + \frac{1}{196} + \frac{1}{256} + \frac{1}{324} + \frac{1}{384} + \frac{1}{441} + \frac{1}{484} + \frac{1}{529} + \frac{1}{576} + \frac{1}{625} + \frac{1}{676} + \frac{1}{729} + \frac{1}{784} + \frac{1}{841} + \frac{1}{900} + \frac{1}{961} + \frac{1}{1024} + \frac{1}{1089} + \frac{1}{1156} + \frac{1}{1225} + \frac{1}{1296} + \frac{1}{1369} + \frac{1}{1444} + \frac{1}{1521} + \frac{1}{1600} + \frac{1}{1681} + \frac{1}{1764} + \frac{1}{1849} + \frac{1}{1936} + \frac{1}{2025} + \frac{1}{2116} + \frac{1}{2209} + \frac{1}{2304} + \frac{1}{2401} + \frac{1}{2500} + \frac{1}{2601} + \frac{1}{2704} + \frac{1}{2809} + \frac{1}{2916} + \frac{1}{3025} + \frac{1}{3136} + \frac{1}{3249} + \frac{1}{3364} + \frac{1}{3481} + \frac{1}{3600} + \frac{1}{3721} + \frac{1}{3844} + \frac{1}{3969} + \frac{1}{4096} + \frac{1}{4225} + \frac{1}{4356} + \frac{1}{4489} + \frac{1}{4624} + \frac{1}{4761} + \frac{1}{4900} + \frac{1}{5041} + \frac{1}{5184} + \frac{1}{5329} + \frac{1}{5476} + \frac{1}{5625} + \frac{1}{5776} + \frac{1}{5929} + \frac{1}{6084} + \frac{1}{6241} + \frac{1}{6400} + \frac{1}{6561} + \frac{1}{6724} + \frac{1}{6891} + \frac{1}{7064} + \frac{1}{7241} + \frac{1}{7424} + \frac{1}{7611} + \frac{1}{7804} + \frac{1}{7999} + \frac{1}{8200} + \frac{1}{8401} + \frac{1}{8604} + \frac{1}{8809} + \frac{1}{9016} + \frac{1}{9225} + \frac{1}{9436} + \frac{1}{9649} + \frac{1}{9864} + \frac{1}{10081} + \frac{1}{10300} + \frac{1}{10521} + \frac{1}{10744} + \frac{1}{10969} + \frac{1}{11196} + \frac{1}{11425} + \frac{1}{11656} + \frac{1}{11889} + \frac{1}{12124} + \frac{1}{12361} + \frac{1}{12600} + \frac{1}{12841} + \frac{1}{13084} + \frac{1}{13329} + \frac{1}{13576} + \frac{1}{13825} + \frac{1}{14076} + \frac{1}{14329} + \frac{1}{14584} + \frac{1}{14841} + \frac{1}{15100} + \frac{1}{15361} + \frac{1}{15624} + \frac{1}{15889} + \frac{1}{16156} + \frac{1}{16425} + \frac{1}{16696} + \frac{1}{16969} + \frac{1}{17244} + \frac{1}{17521} + \frac{1}{17800} + \frac{1}{18081} + \frac{1}{18364} + \frac{1}{18649} + \frac{1}{18936} + \frac{1}{19225} + \frac{1}{19516} + \frac{1}{19809} + \frac{1}{20104} + \frac{1}{20401} + \frac{1}{20700} + \frac{1}{21001} + \frac{1}{21304} + \frac{1}{21609} + \frac{1}{21916} + \frac{1}{22225} + \frac{1}{22536} + \frac{1}{22849} + \frac{1}{23164} + \frac{1}{23481} + \frac{1}{23800} + \frac{1}{24121} + \frac{1}{24444} + \frac{1}{24769} + \frac{1}{25096} + \frac{1}{25425} + \frac{1}{25756} + \frac{1}{26089} + \frac{1}{26424} + \frac{1}{26761} + \frac{1}{27100} + \frac{1}{27441} + \frac{1}{27784} + \frac{1}{28129} + \frac{1}{28476} + \frac{1}{28825} + \frac{1}{29176} + \frac{1}{29529} + \frac{1}{29884} + \frac{1}{30241} + \frac{1}{30600} + \frac{1}{30961} + \frac{1}{31324} + \frac{1}{31689} + \frac{1}{32056} + \frac{1}{32425} + \frac{1}{32796} + \frac{1}{33169} + \frac{1}{33544} + \frac{1}{33921} + \frac{1}{34300} + \frac{1}{34681} + \frac{1}{35064} + \frac{1}{35449} + \frac{1}{35836} + \frac{1}{36225} + \frac{1}{36616} + \frac{1}{37009} + \frac{1}{37404} + \frac{1}{37801} + \frac{1}{38200} + \frac{1}{38601} + \frac{1}{39004} + \frac{1}{39409} + \frac{1}{39816} + \frac{1}{40225} + \frac{1}{40636} + \frac{1}{41049} + \frac{1}{41464} + \frac{1}{41881} + \frac{1}{42300} + \frac{1}{42721} + \frac{1}{43144} + \frac{1}{43569} + \frac{1}{43996} + \frac{1}{44425} + \frac{1}{44856} + \frac{1}{45289} + \frac{1}{45724} + \frac{1}{46159} + \frac{1}{46596} + \frac{1}{47033} + \frac{1}{47472} + \frac{1}{47913} + \frac{1}{48356} + \frac{1}{48801} + \frac{1}{49248} + \frac{1}{49697} + \frac{1}{50148} + \frac{1}{50601} + \frac{1}{51056} + \frac{1}{51513} + \frac{1}{51972} + \frac{1}{52433} + \frac{1}{52896} + \frac{1}{53361} + \frac{1}{53828} + \frac{1}{54297} + \frac{1}{54768} + \frac{1}{55241} + \frac{1}{55716} + \frac{1}{56193} + \frac{1}{56672} + \frac{1}{57153} + \frac{1}{57636} + \frac{1}{58121} + \frac{1}{58608} + \frac{1}{59097} + \frac{1}{59588} + \frac{1}{60081} + \frac{1}{60576} + \frac{1}{61073} + \frac{1}{61572} + \frac{1}{62073} + \frac{1}{62576} + \frac{1}{63081} + \frac{1}{63588} + \frac{1}{64097} + \frac{1}{64608} + \frac{1}{65121} + \frac{1}{65636} + \frac{1}{66153} + \frac{1}{66672} + \frac{1}{67193} + \frac{1}{67716} + \frac{1}{68241} + \frac{1}{68768} + \frac{1}{69297} + \frac{1}{69828} + \frac{1}{70361} + \frac{1}{70896} + \frac{1}{71433} + \frac{1}{71972} + \frac{1}{72513} + \frac{1}{73056} + \frac{1}{73601} + \frac{1}{74148} + \frac{1}{74697} + \frac{1}{75248} + \frac{1}{75801} + \frac{1}{76356} + \frac{1}{76913} + \frac{1}{77472} + \frac{1}{78033} + \frac{1}{78596} + \frac{1}{79161} + \frac{1}{79728} + \frac{1}{80297} + \frac{1}{80868} + \frac{1}{81441} + \frac{1}{82016} + \frac{1}{82593} + \frac{1}{83172} + \frac{1}{83753} + \frac{1}{84336} + \frac{1}{84921} + \frac{1}{85508} + \frac{1}{86097} + \frac{1}{86688} + \frac{1}{87281} + \frac{1}{87876} + \frac{1}{88473} + \frac{1}{89072} + \frac{1}{89673} + \frac{1}{90276} + \frac{1}{90881} + \frac{1}{91488} + \frac{1}{92097} + \frac{1}{92708} + \frac{1}{93321} + \frac{1}{93936} + \frac{1}{94553} + \frac{1}{95172} + \frac{1}{95793} + \frac{1}{96416} + \frac{1}{97041} + \frac{1}{97668} + \frac{1}{98297} + \frac{1}{98928} + \frac{1}{99561} + \frac{1}{100200} + \frac{1}{100841} + \frac{1}{101484} + \frac{1}{102129} + \frac{1}{102776} + \frac{1}{103425} + \frac{1}{104076} + \frac{1}{104729} + \frac{1}{105384} + \frac{1}{106041} + \frac{1}{106700} + \frac{1}{107361} + \frac{1}{108024} + \frac{1}{108689} + \frac{1}{109356} + \frac{1}{110025} + \frac{1}{110696} + \frac{1}{111369} + \frac{1}{112044} + \frac{1}{112721} + \frac{1}{113400} + \frac{1}{114081} + \frac{1}{114764} + \frac{1}{115449} + \frac{1}{116136} + \frac{1}{116825} + \frac{1}{117516} + \frac{1}{118209} + \frac{1}{118904} + \frac{1}{119601} + \frac{1}{120300} + \frac{1}{121001} + \frac{1}{121704} + \frac{1}{122409} + \frac{1}{123116} + \frac{1}{123825} + 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the present day some savages, and some very small children, still count, "one, two, three—many."

Then comes the struggle to find a notation. The two systems with which the children are already familiar are of course the Roman and the so-called Arabic system; but the simplest is the notched stick. Bakers' men even now, or until quite recently, sometimes leave a tally with a Dublin maidservant, and a notch is cut for every loaf delivered.

The Egyptian system, as is shown in the diagram, had different symbols for different powers of ten. The Babylonians, who wrote on soft bricks, which they afterwards baked, had fewer characters; one for 1, one for 10, one for 100. The Greeks used the first nine letters of the alphabet for the numbers 1...9. After that $i=10$, $\kappa=20$, $\lambda=30$, and so on. Thus $\kappa\alpha$ stood for 21.

The difficulty of working these systems will be obvious to any child, and the importance of the discovery of "place-value" will be evident. It is interesting to learn that the so-called Arabic system is really of Hindu origin; that inscriptions were found not fifty years ago in Bombay Presidency (Nana Ghat), probably dating from the third century B.C., in which these numerals appear; that the numerals 4...9 were probably letters of an ancient alphabet. An interesting point is that it was many centuries before the symbol 0 was evolved, and that therefore the place-value element was wanting—its real claim, of course, to superiority. The zero perhaps appeared in the Hindu system about 300 A.D., and the first known use of the symbol occurs in a document of the eighth century A.D.—another instance of the amazing length of time which a simple idea can take to dawn upon humanity.

Whether the Hindus or the Arabs invented the zero symbol is uncertain. It was brought to Europe by an Italian traveller in 1200 A.D. (Leonardo Fibonacci of Pisa), and the Arabs, being traders, helped to disseminate the system; but it was not till about 1500 that it began to get a firm footing in the schools—owing to the invention of printing. The zero was a mere dot, but as so small a symbol could easily be removed by unscrupulous merchants, it was written in the form of a polygon, which easily degenerated into the present symbol.

The history of the rise of geometry emphasises the two facts that theoretical science is first called into being by practical need, and that many facts are known empirically long before they can be scientifically proved. Butcher mentions in his *Greek Studies* that the right-angled property of a triangle whose sides are 3, 4, and 5 was treated by a Chinese Emperor of 1100 B.C. as a piece of magical knowledge, but it was the Greek mind which found out the reason. How the same law was known to the Egyptians, whose "harpedon aptae" or "rope-stretchers" used it in the building of temples, just as the surveyors of the present do, or school-children marking out a tennis-ground; how the periodic rise of the Nile and the consequent obliteration of landmarks brought about an early need for geometrical skill; how in the occult mysteries of astrology the beginnings of a science germinated; how because the Babylonians represented by a circle the year which they computed at 360 days, we still divide a right angle into 90 degrees; how the Egyptian priests measured areas empirically, and kept the results recorded in the temples (making a sacred mystery of the area of a rectangle), and yet Ahmes made a good approximation to the value of π ;^{*} how Thales, having earned a fortune by trade, spent it in studying Mathematics from the Egyptian priests, and how his pupil Pythagoras founded the school at Croton and Geometry became a science,—these and many other points of history readily rouse the interest of the young student.

^{*} 3·1605.

Then another point of interest is the rise of Algebra. The Greeks of Euclid's time saw mathematics through the medium of geometry ; therefore they did not recognise any power of a variable higher than the third. Euclid, Book II., even when not formally taught, gives many valuable illustrations of algebraic facts ; and a class is generally pleased to see Euclid II. 11 worked side by side with a quadratic equation, and to find that the steps—completing the square, and so on—exactly correspond.

About 100 B.C. an advance was made, and Heron of Alexandria spoke boldly of the fourth power of lines, and in solving the quadratic equation “struck” imaginary roots, and five hundred years later the first known work on Algebra was written by Diophantus of Alexandria. He only uses one unknown quantity s' , the square is Δ^Y ($\delta\upsilon\nu\alpha\mu\iota\varsigma$) and the cube (K^Y). He goes as far as the sixth power. He solves problems like “Find two numbers whose sum is 20 and the difference of whose squares is 80.” He only gives one answer to a quadratic equation, and ignores negative quantities.

A student should surely know that it was the Orient that gave the next important impulse to the science, and that to Bagdad in the eighth century, among other learned men summoned by an enlightened Calif, came the father of modern Algebra, al-Khuwarazmi, whose work *Ilm al-jabr wa'l muqabalah* (the science of redintegration and equation) gives its name to Algebra.

His division of quadratic equations into six classes $ax^2=bx$; $ax^2=c$; $bx=c$; $x^2+bx=c$; $x^2+c=bx$; $x^2=bx+c^2$ is interesting as showing that the simple idea of a general type was not grasped.

I have ventured to take up your time over these scraps of history, familiar as they may be to almost everyone here, because I wanted to bring out some of the points which can hardly fail to interest young students, and which every school-child who has gone any way in Mathematics must surely be the wiser for knowing.

It can hardly be disputed that the child's view of Mathematics must become more intelligent, but the question naturally arises as to how the instruction should be given. Should the history of mathematics be allotted a definite place on the school curriculum, or should the thing be left to the teacher's discretion ? There is no doubt that a teacher interested in the subject can quite informally and as occasion arises give all the instruction that is needed ; and if all teachers of Mathematics would be at some pains to do so, it would probably be the most satisfactory method, and would save the subject from the risk of becoming that lean and barren thing, a skeleton outline to be got up for examination.

I was greatly interested to receive Mr. Van der Heydn's *Notes on Algebra*, which he kindly sent me after the subject of this paper had been announced, and to find that he has made a special feature of the historical notes,—in which he has introduced, I see, the very same Egyptian equation that I had shown to my training-students, and which is here reproduced. I should like to acknowledge also with gratitude a very kind communication from Professor Genese, in which he tells me that Professor Zeuthen of Copenhagen informed him in 1908 at the Congress at Rome, that he had introduced History of Mathematics as one of the subjects for Degree at Copenhagen.

In a reform of this sort may probably be found the true solution of the problem. If all those who taught Mathematics had had to pass some such examination as that at Copenhagen, the subject would find its way naturally into school work ; but until other Universities follow

suit, may one not plead for a place for it in the curriculum of the school ?

In concluding her remarks, **Miss Barwell** said she would like to mention that a great deal of the historical part of her paper had been derived from Smith's *Teaching of Elementary Mathematics*, an American publication which she thought would be of great interest to any teacher.

DISCUSSION ON MISS BARWELL'S PAPER.

The President : I am sure we have all listened with great interest to this paper. There are several things in it which I did not know much about, but I think that probably what Miss Barwell said towards the close of her paper would strike most people as meeting the case best ; that is to say, the history of mathematics in schools should be given in small doses interspersed throughout the teaching. In those circumstances it would really enliven and give an interest to the work. I think there is a great danger in any kind of systematic treatment of history at an early stage, when the amount of actual knowledge of mathematics must be very small. In my opinion, if the history goes further than the mathematics which pupils have to learn, they would have no correct understanding of it, and that would be a very barren form of training. Surely the history ought only to keep pace with those aspects of mathematics that are real to the children who are taught. Under those conditions I think it would be of great value in connection with the teaching of arithmetic and algebra and geometry, if occasional indications were given of the way in which the points have historically arisen.

I was very much interested with the long number beginning with millions and the Egyptian notation. It was not so much that I was amused with the quaint animals and so forth, but it struck me that the mathematicians of that time were in some respects a great deal nearer the real thing than the Greeks and Romans were afterwards. They had the symbols according to the powers of ten from left to right. They had no such barbarism as XL. In that way addition and subtraction were not mixed up, and all that they wanted was the one final step of writing one symbol for different things according to its place. But they had the right arrangement, and it struck me that they were just a trifle nearer than the Greeks and Romans were to our actual notation. I had never seen that before, and I must say it was very interesting to me. I will now ask for remarks on the paper.

Miss E. M. Hancock : I thought I should like to say one thing. I noticed that in her paper Miss Barwell made reference to a course which had been introduced into a university abroad. I should like to say that such a course is not unknown here. I do not know whether any students from other universities can give a case, but I know that in my own University of London, those of us who were fortunate enough to work under Professor Harding had an historical course and, though it was not included in the examination for a degree, we did have the advantage of the training.

Miss Margaret Frodsham (Cardiff) : I should like to say that I have been very much interested in hearing the paper. Such a course is of very great value to the student in training, for, later, as teacher in the schools, he is able to give some historical outline of the work being done. Even in the earlier stages, if he chooses he may interest the pupils in some particular facts in the historical development of mathematics, but the great point is that the teacher is enabled, through the possession of these facts, to form the school course in a line with

this development, and this is a great gain to the mathematical course in the school.

I know that the discussion on Mr. Carson's paper is already closed, but there is a point which seems to me to link on here. If we take this historical development, and if we consider a point that has not been referred to by its actual name (though Mr. Carson got very near to it in his answers), that is, if we study in this connection the *psychological* development of a child, we shall then find what work should naturally come first, and we shall, by means of this association, feel justified in dealing with the mathematical facts from a practical point of view, and *before* reasons are found; later, by making clear from the standpoint of "development" the necessity for those reasons, we should give the subject an interest which would not otherwise exist.

One particular point which the teacher can illustrate is the historical development of the use of the decimal point. This is given clearly in Mr. B. Branford's book, to which Miss Barwell has referred.

Mr. Plum : I should like to say that for some years I have used what knowledge of the history of mathematics I possess in forms generally, and that I have noticed it has been most useful in keeping up the interest of boys in mathematical work, and has often been the means of inciting an interest in mathematics in the case of boys who ordinarily could not be said to have any great interest in the subject.

I want to ask one question. I did not catch the name of the book from which Miss Barwell said she obtained some of her information. I shall be very glad to have it presently if I may. I am afraid I came in late, so that I heard only a portion of her paper, and in this I was very much interested. May I ask for the name of the book now? I think it was an American publication.

The Chairman : If nobody else wishes to make any remarks on Miss Barwell's paper, I will ask her to reply to those who have spoken.

Miss Barwell (replying): First of all, the names of the books I mentioned are: Benchara Branford's *Study of Mathematical Education* (Clarendon Press); D. E. Smith's *Teaching of Elementary Mathematics* (Macmillan). I think there were not really very many questions asked. I am very much pleased to learn that people are interested in the subject, and, as I am sure will be more and more the case, to know that many of the teachers have already introduced the teaching of this subject into their school work. With regard to the Chairman's remarks as to the introduction of the history of the subject so far as it applies to work which the children have not yet done, I do not think the history should be insisted upon and learnt in that case. I do not think it interests children to know that a thing has been discovered and is there, but visually to show them the growth of mathematics cannot fail to be a good thing. I quite agree that we cannot drill into them the study of things which, from the limits of their knowledge bearing upon them, they could not understand; but it is certainly an advantage to teach them those things that do come within their grasp.

The President : I am asked to call your attention to the fact that there will be at 11.15 to-morrow, in connection with the Association of Public School Science Masters, a discussion on the teaching of Statics, initiated by Mr. Siddons, and a discussion later in the day on the teaching of Dynamics, initiated by Mr. Ashford. All members of the Mathematical Association are cordially invited to come and take part in the discussion.

Dr W. P. Milne read a paper on "The Teaching of Scholarship Mathematics in Secondary Schools."