Polarisation and Energy Loss in Dielectrics

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IV. *Polarisation and Energy Loss in Dielectrics.* By A. W. ASHTON, D.Sc., M.I.E.E., Battersea Polytechnic, Battersea, S.W.

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§ 1. *Introduction.*—The object of this Paper is to discuss the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D.

Pellat* showed that (at least qualitatively) the displacement in a dielectric can be represented by

$$D_t = KE_0 + \varepsilon KE_0(1 - e^{-at}),$$

where D_t = displacement at time t since P.D. E_0 was applied, a , K and ε are constants, K being the instantaneous capacity of the condenser and a the reciprocal of the time constant of the viscous displacement, while ε is a coefficient such that displacement after infinite time of charge is equal to $KE_0(1 + \varepsilon)$.

Pellat recognised that it would be necessary in some cases to use several terms having different values for a in order to represent quantitatively the results obtained from dielectrics.

It was pointed out by Schweidler† that by modifying Pellat's equation into the form

$$D_t = KE_0 + KE_0 \Sigma \varepsilon (1 - e^{-at})$$

the charging current for constant voltage becomes for $t > 0$

$$i_t = KE_0 \Sigma a \varepsilon e^{-at},$$

and in the series of exponential terms a and ε can be so chosen as to make

$$\Sigma a \varepsilon e^{-at} = \beta t^{-n}.$$

This is necessitated by the results of experiments in which condensers have been charged for a considerable time at a constant voltage. The charging current for $t > 30$ seconds has been shown in a large number of cases to be of the form

$$i_t = i_a + \beta KE_0 t^{-n},$$

where $n < 1$, and it has generally been considered that i_a is a true conduction current whilst the recoverable energy is due to the term $\beta KE_0 t^{-n}$.

* "Annales de Chimie et de Physique," Ser. 7. Vol. XVIII., p. 150, 1899.

† "Annalen der Physik," Vol. XXIV., p. 711, 1907.

§ 2. *Relations between α and ε for any given Value of the Exponent n in the Equation for Polarisation Current.*—If we consider the above equation for polarisation current it can be shown that, in order that the current may be proportional to t^{-n} , the variations of α and ε must be such that the product $\alpha\varepsilon$ is proportional to α^n .

In a physical sense $K\varepsilon$ may be considered to be the viscous displacement due to unit P.D. (*i.e.*, the viscous capacity) corresponding to all molecules of the dielectric having the same displacement time constant.

$$\begin{aligned} \text{Let} \quad C_1 &= \Sigma \alpha \varepsilon e^{-\alpha t_1} = \beta t_1^{-n} \\ \text{and let} \quad C_2 &= \Sigma \alpha \varepsilon e^{-\alpha t_2} = \beta t_2^{-n}. \end{aligned}$$

In order to make $C_1 = kC_2$, where k is any positive number, we must have $t_1^{-n} = kt_2^{-n}$, *i.e.*, $t_1 = k^{-1/n}t_2$.

Although α varies continuously we can choose the values of α in the series so that the values of $\alpha\varepsilon$ are in geometrical progression and write

$$C_1 = \alpha_1 \varepsilon_1 e^{-\alpha_1 t_1} + \alpha_2 \varepsilon_2 e^{-\alpha_2 t_1} + \dots + \alpha_r \varepsilon_r e^{-\alpha_r t_1},$$

where $\alpha_2 \varepsilon_2 = k\alpha_1 \varepsilon_1$, $\alpha_3 \varepsilon_3 = k\alpha_2 \varepsilon_2$, &c.

In this case $k\varepsilon_m$ is the total capacity of that group of molecules whose time constants have a mean value $1/\alpha_m$, and which fall within certain limits as determined by the values of α in the terms on either side.

$$\text{Now } C_2 = \frac{1}{k} \{ \alpha_2 \varepsilon_2 e^{-\alpha_1 t_2} + \alpha_3 \varepsilon_3 e^{-\alpha_2 t_2} + \dots + \alpha_{r+1} \varepsilon_{r+1} e^{-\alpha_r t_2} \},$$

and since, when $1/\alpha$ is either very small or very large compared with t , $\alpha \varepsilon e^{-\alpha t}$ becomes negligible, we can write $C_2 = \frac{C_1}{k}$, provided $\alpha_2 t_1 = \alpha_1 t_2$, $\alpha_3 t_1 = \alpha_2 t_2$, $\alpha_4 t_1 = \alpha_3 t_2$, &c., &c., and the first and last terms of the series are negligibly small.

Since $t_1 = k^{-1/n}t_2$, in order to make $\alpha_2 t_1 = \alpha_1 t_2$, $\alpha_3 t_1 = \alpha_2 t_2$, &c., we must have $\alpha_2 = k^{1/n}\alpha_1$, $\alpha_3 = k^{1/n}\alpha_2$, &c., and therefore $\alpha_2^n = k\alpha_1^n$, $\alpha_3^n = k\alpha_2^n$, &c., from which it follows that since $\alpha_2 \varepsilon_2 = k\alpha_1 \varepsilon_1$, $\alpha_3 \varepsilon_3 = k\alpha_2 \varepsilon_2$, &c., therefore the values of α and ε must be so chosen that $\alpha\varepsilon$ shall be proportional to α^n .

From this it follows further that ε is proportional to $(1/\alpha)^{1-n}$ and we must have the following law:—

Where the polarisation current of a dielectric is proportional to t^{-n} the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$ th power of the time constant.

§ 3. *Energy Loss and Variation of Capacity with an Alternating E.M.F. as determined from the Equations of Hopkinson and Wilson.*—Equations for the displacement in a viscous dielectric under the action of an alternating E.M.F. have been given by Hopkinson and Wilson* on the assumption that the charging current is proportional to t^{-n} , where $n < 1$. From these equations the apparent capacity is given by

$$C = K + p^{n-1} \beta K \Gamma(1-n) \cos (1-n) \frac{\pi}{2}$$

and the alternating-current conductivity by

$$S = S_0 + p^n \beta K \Gamma(1-n) \sin (1-n) \frac{\pi}{2},$$

where S_0 is the true conductivity under a steady E.M.F. and $p = 2\pi f$, where f = frequency of the applied E.M.F.

For all frequencies greater than about one per second S_0 is negligible compared with S , the energy loss being therefore proportional to f^n and the viscous capacity proportional to f^{n-1} .

Now the apparent conductivity for a steady E.M.F. is

$$S'' = \beta K t^{-n}$$

and the ratio of the alternating-current conductivity to the apparent conductivity under a steady E.M.F. is

$$\begin{aligned} r &= p^n t^n \sin (1-n) \frac{\pi}{2} \Gamma(1-n) \\ &= \frac{p^n t^n \pi}{2 \Gamma(n) \cos (1-n) \frac{\pi}{2}}. \end{aligned}$$

This ratio is of considerable importance in connection with cable testing in which it has been the practice to measure the apparent insulation resistance after one minute's electrification in order to detect faults in manufacture.

The above expression for alternating-current conductivity becomes negative when $n > 2$ and this is also the case with regard to the viscous capacity when $n > 1$. The variations of energy loss and capacity with frequency can be shown to be true within the above limits for n by investigating the series obtained on the assumption that the value of ε and α are such that $\alpha \varepsilon$ is proportional to α^n .

* "Phil. Trans.," Vol. 189A., 1897.

§ 4. *Variation of Energy Loss with Frequency as determined from Pellat's Theory.*—The total alternating-current conductivity of the dielectric due to viscous polarisation is

$$S = K \sum \frac{p^2 \epsilon \alpha}{p^2 + \alpha^2} \quad \bullet$$

If we consider two terms of the series, α_1, ϵ_1 and α_2, ϵ_2 such that $\alpha_2 = k\alpha_1$, then $\alpha_2 \epsilon_2 = k^n \alpha_1 \epsilon_1$ and the conductivity at frequency $p_2/2\pi$ for the group α_2, ϵ_2 is

$$S'_2 = \frac{K p_2^2 \alpha_2 \epsilon_2}{p_2^2 + \alpha_2^2}$$

Now, if S'_1 = conductivity at frequency $\frac{p_1}{2\pi}$ such that $p_2 = k p_1$ for the group α_1, ϵ_1 , then it follows that

$$S'_2 = \frac{K p_1^2 \alpha_1 \epsilon_1 k^{n+2}}{k^2(p_1^2 + \alpha_1^2)} = k^n S'_1.$$

Therefore in the series for the total conductivity of the dielectric at frequency $p_2/2\pi$, the terms are each k^n times the corresponding terms in the series for conductivity at k^{-1} times the frequency, and provided the terms of the two series vanish in the limit when α is indefinitely increased or diminished we can write S proportional to p^n .

Now it can be shown that S' is a maximum for a given frequency in the case of that term of the series for which

$$\alpha = p \sqrt{\frac{n}{2-n}}.$$

For values of n lying between 0.5 and 1.5 the maximum energy loss occurs in those groups for which α does not greatly differ from p , while for values of α much greater than this $S' = \frac{K p^2 \epsilon}{\alpha}$. Further, since $\frac{\epsilon}{\alpha}$ is proportional to α^{n-2} the terms in the series must vanish in the limit as α increases provided n is < 2 . For values of α much smaller than p , $S' = K \epsilon \alpha$, where $\epsilon \alpha$ is proportional to α^n and the terms in the series vanish in the limit as α is diminished provided n is positive. It appears therefore that for all positive values of $n < 2$ the alternating-current conductivity is proportional to the n th power of the frequency.

§5. *Variation of Viscous Capacity with Frequency as determined from Pellat's Theory.*—The capacity under the application of an alternating E.M.F. is given by $C = K \left(1 + \Sigma \frac{\alpha^2 \varepsilon}{p^2 + \alpha^2} \right)$. If, as before, α_1, ε_1 and α_2, ε_2 are values of α and ε such that $\alpha_2 = k\alpha_1$, then it follows that $\alpha_2 \varepsilon_2 = k^n \alpha_1 \varepsilon_1$. The viscous capacity for the group α_2, ε_2 at a frequency $p_2/2\pi$ is

$$\begin{aligned} C'_2 &= \frac{K \alpha_2^2 \varepsilon_2}{p_2^2 + \alpha_2^2} \\ &= \frac{K k^{n+1} \alpha_1^2 \varepsilon_1}{k^2 (p_1^2 + \alpha_1^2)} \\ &= k^{n-1} C'_1, \end{aligned}$$

where C'_1 is the viscous capacity for the group α_1, ε_1 , at frequency $\frac{p_1}{2\pi} = \frac{p_2}{2\pi k}$. Provided the terms of the two series vanish in the limit as α either increases or decreases the viscous capacity will be proportional to the $(n-1)$ th power of the frequency.

Now it can be shown that $\frac{\alpha^2 \varepsilon}{p^2 + \alpha^2}$ is a maximum for $\alpha = p \sqrt{\frac{1+n}{1-n}}$, and, provided n is not approaching unity, there is a well-defined maximum value for which α is in the neighbourhood of p .

As α diminishes so that α^2 becomes negligible compared with p^2 the capacity of each group $= \frac{K \alpha^2 \varepsilon}{p^2}$, which is proportional to α^{n+1} , and the terms of the series vanish in the limit for all positive values of n . On the other hand, as α increases so that p^2 is negligible compared with α^2 the capacity of each group becomes $= K \varepsilon$, which is proportional to α^{n-1} , and the terms of the series only vanish in the limit as α increases provided $n < 1$. The viscous capacity will therefore be proportional to f^{n-1} provided $n < 1$.

It will be noticed that for a given frequency the groups of molecules having the greatest influence on the capacity and conductivity may have very different time constants. Thus the ratio of the value of α for maximum conductivity to that

for maximum viscous capacity is given by $d = \sqrt{\frac{n(1-n)}{(2-n)(1+n)}}$.

The value of d is approximately one-third when n lies between 0.1 and 0.9 but increases rapidly as n approaches unity.

§ 6. *Variation in Capacity when n is Unity.*—The variation in capacity with change of frequency can easily be determined for the case in which $n=1$. In this case ϵ is constant and α alone varies. The terms in the series for capacity are all equal until α becomes comparable with p . From this point the terms in the series diminish to zero, and this part of the series is the same for all values of p . The number of equal terms becomes greater as p diminishes. If α_1, α_2 be values of α and p_1, p_2 values of p such that $\frac{\alpha_1}{p_1} = \frac{\alpha_2}{p_2}$, then the viscous capacity C_1 for frequency $p_1/2\pi$ differs from the capacity C_2 for frequency $p_2/2\pi$ by $K\Sigma \alpha_1^2 \epsilon$.

We may therefore write, if $p_2 > p_1$,

$$C_1 = C_2 + C_0 \log_{10} \frac{p_2}{p_1},$$

where

$$C_0 = K \Sigma \alpha_a^{10} \epsilon.$$

From this it follows that for frequencies increasing in geometrical progression the capacity decreases in arithmetical progression.

A similar formula can be shown to represent the variation of capacity with time under the action of a steady E.M.F.

The above properties of the dielectrics should be true whether n is a constant over the whole range or not, since, except in cases where n is approaching its limiting value for the particular property considered (such as $n=1$ for capacity variation), the range of values of α having most effect in any given case will be restricted within limits closely corresponding to the frequency or the time for which the variation is being investigated.

§ 7. *Experimental Evidence of Theory.*—A large amount of valuable experimental research on dielectrics has been recently carried out by various investigators and it is proposed to examine some of the results obtained in order to test the above conclusions.

Addenbrooke* has investigated the energy loss in gutta-percha for comparatively slow cycles and has expressed his results for each frequency as a multiple of the apparent loss with a steady E.M.F. after two minutes' application. Except for frequencies of less than 2 cycles per second the ratio of losses can be shown to be fairly accurately represented by

* See "The Electrician," Vol. LXX., p. 673.

$r=93f^{0.79}$, where f =periods per second. Table I. gives observed and calculated values of r .

TABLE I.—*Ratio of Loss in Gutta Percha under an Alternating E.M.F. to that under a Steady E.M.F. as given by Addenbrooke.*

f .	r (observed).	r (calculated).	Error %.
0.2	20	25.5	+27.0
0.4	38	44.0	+16.0
1.0	89	93.0	+ 4.5
2.0	160	160.0	0
4.0	276	281.0	+ 1.8
8.0	480	482.4	+ 0.5
16.0	830	829.0	— 0.1
32.0	1,440	1,440.0	0
42.0	1,830	1,785.0	— 2.5

A large number of dielectrics have been tested by Fleming and Dyke* over a wide range of temperature for alternating-current conductivity and capacity at telephonic frequencies. From these experiments they conclude that for most dielectrics the alternating-current conductivity can be represented by a formula $S=A+B/f$, where f is frequency and A and B are constants, of which, however, A is very small for most good insulators. Their results can also be shown to be closely represented by an equation of the form $S=Q/f^n$, where Q is a constant, the case in which $A=0$ corresponding to $n=1$. The value of n varies from 0.4 to 1.4 but is nearly unity for a considerable number of the dielectrics tested. Table II. shows the values of n for celluloid at temperatures varying from -184°C . to 80°C . taken from the same Paper, and, except at 15°C . and 60°C ., the calculated values do not differ more than 2 per cent. from the observed values.

TABLE II.—*Values of n determined from Fleming and Dyke's Tests on Celluloid.*

n .	$^{\circ}\text{C}$.	n .	$^{\circ}\text{C}$.	n .	$^{\circ}\text{C}$.
0.85	-184°	1.22	-11°	0.56	60°
0.90	-140°	1.15	2.5°	0.49	68°
1.02	-120°	1.05	15°	0.44	80°
0.99	-75°	0.71	30°	...	...
1.20	-38°	0.59	51°	...	...

Curtis† in connection with a series of tests for dielectric loss and capacity variation in mica condensers, has discussed the question as to the agreement between his results and Schweidler's

* See "Journal" Inst. Elec. Eng., Vol. XLIX., p. 323, 1912.

† See "Bulletin" of Washington Bureau of Standards, Vol. VI., p. 431.

modification of Pellat's theory and appears to have come to the conclusion that the agreement is not satisfactory. In at least one case, however, agreement between his results and the theory can be shown to be remarkably close.

Columns 1, 2 and 5, Table III., show the results Curtis obtained where $\phi = 90 - \theta$ for power factor $\cos \theta$. ΔC is variation in capacity (parts in 100,000). S is the conductivity for alternating currents for a condenser of 1 microfarad capacity. The fourth column gives the conductivity calculated from $S = 5.334 \times 10^{-9} p^n$, where $n = 0.654$, and the sixth column the viscous capacity calculated from $\Delta C = 853.3 p^{n-1}$.

TABLE III.—*Conductivity and Capacity of Mica Condenser (Curtis).*

f .	ϕ .	$S \times 10^6$ (observed).	$S \times 10^6$ (calculated).	ΔC (parts in 100,000 observed).	ΔC (parts in 100,000 calculated).
1,200	50"	1.827	1.8317	7	38.86
100	2' 0"	0.3654	0.3606	60	91.83
50	2' 30"	0.2284	0.2291	85	116.7

The increase in capacity from $f = 1,200$ to $f = 100$ is 53 observed and 52.97 calculated, while from $f = 100$ to $f = 50$ the corresponding values are 25 and 24.87.

The agreement is even more remarkable with regard to the values of βK calculated from the conductivity and capacity measurements. For the former we have

$$S = \frac{\beta K \pi p^n}{2\Gamma(n) \cos(1-n)\frac{\pi}{2}} = 5.334 \times 10^{-9} p^n,$$

where $n = 0.654$.

From this $\beta K = 4.005 \times 10^{-9}$.

From the capacity variation

$$\Delta C = 8.533 \times 10^{-9} p^{n-1}$$

in practical electromagnetic units, and from this $\beta K = 3.87 \times 10^{-9}$, which agrees with the former value within $3\frac{1}{2}$ per cent.

H. A. Wilson* has carried out an investigation of the change in capacity due to absorption under the action of a steady E.M.F., and has shown that the capacity after T minutes bears a ratio $1 + B \log(1 + p'T)$ to the capacity after 1 minute's charge.

* See "Proc." Roy. Soc., Vol. 82A., p. 409, 1909.

In the case of the ebonite condenser it can be shown that the capacity at 30°C. is very closely represented by

$$C=0.4545+0.3618 T^{0.098},$$

the maximum deviation of the calculated values from the values observed being 0.15 per cent. From the above equation $n=0.902$, which is in agreement with values of 0.9 to 1.0 given by Fleming and Dyke's results on ebonite.*

It is interesting to note that equations for conductivity and capacity variation under an alternating E.M.F. can be readily obtained from the above equation.

For t in seconds we have

$$C=0.4545 \times 10^{-6} + 0.2421 \times 10^{-6} t^{0.098}$$

in electromagnetic units, from which it follows that

$$\beta K = 2.373 \times 10^{-8}.$$

The alternating-current conductivity is given by

$$S = 3.534 \times 10^{-8} p^{0.902}.$$

The capacity under alternating E.M.F. is given by

$$C = 0.4545 \times 10^{-6} + 2.278 \times 10^{-7} p^{-0.098}.$$

The above discussion of results obtained on various dielectrics shows the possibility of correlating the various absorption phenomena of a dielectric. Before coming to any very definite conclusion, however, it is proposed to carry out a series of tests on various dielectrics over a wide range of frequency, as well as with steady E.M.F.s, in order to determine to what extent n is constant for a given dielectric at a definite temperature.

PELLAT'S THEORY.†

The following brief résumé of Pellat's theory of "true polarisation" differs mainly from the original treatment in that the condenser is assumed to have only one layer of dielectric between the plates instead of also having two layers of air.

Let λ = specific inductive capacity of the dielectric,

ϕ = electric intensity,

σ = surface density on the condenser plates.

$$\text{Then} \quad \phi = \frac{4\pi}{\lambda} (\sigma - j), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where j is called by Pellat the true polarisation. At the end of

* See "Journal" Inst. Elec. Eng., *loc. cit.*

† See Pellat, "Annales de Chimie et de Physique," Vol. XVIII., p. 150. Also Pellat and Beaulard, "Comptes Rendus," Vol. CXXX., p. 1457.

infinite time j takes a final value, J . It is assumed that the rate of variation of j is proportional to the difference between its present and final values—that is,

$$\frac{dj}{dt} = \alpha(J - j) \dots \dots \dots (2)$$

From which it follows that

$$j = J(1 - e^{-\alpha t}) \dots \dots \dots (3)$$

Pellat justifies the assumption made in equation (2) by proving from it Curie's Principle of Superposition.

The final polarisation J is a function of the final electric intensity Φ , and we may write $J = h\Phi$, where h is a constant for weak fields but a function of Φ for strong fields.

If a steady E.M.F., E_0 , be applied to the dielectric we have

$$J = h \frac{E_0}{c}, \dots \dots \dots (4)$$

where c = thickness of dielectric, and from (1) and (3)

$$\sigma = \frac{\lambda E_0}{4\pi c} + cJ(1 - e^{-\alpha t}) \dots \dots \dots (5)$$

Put $K = \frac{\lambda A}{4\pi c}$, where A = area of each condenser plate, then the displacement (D_t) at time t is given by

$$D_t = KE_0 + AhE_0(1 - e^{-\alpha t}) \dots \dots \dots (6)$$

and

$$D = KE_0 + \epsilon KE_0(1 - e^{-\alpha t}), \dots \dots \dots (7)$$

where

$$\epsilon = \frac{Ah}{K}.$$

The current is therefore given by

$$i = KE_0 \epsilon \alpha e^{-\alpha t} \dots \dots \dots (8)$$

The capacity and the energy loss under the application of an alternating E.M.F. were deduced by Pellat and Beaulard on the assumption that h is a constant.

Let $e = E_m \sin pt$ be the applied E.M.F. From equation (2)

$$\frac{dj}{dt} + \alpha j = \frac{\alpha h}{c} E_m \sin pt \dots \dots \dots (9)$$

The solution of equation (9) is $j = B \sin (pt + \theta)$, where

$$B = \frac{E_m h \alpha}{c \sqrt{p^2 + \alpha^2}}$$

and

$$\cos \theta = \frac{\alpha}{\sqrt{p^2 + \alpha^2}}, \quad \sin \theta = \frac{-p}{\sqrt{p^2 + \alpha^2}} \dots \dots \dots (10)$$

From (1), (4) and (10) we have

$$\sigma = \frac{\lambda E_m \sin pt}{4\pi c} + B \sin (pt + \theta).$$

$$\text{Therefore } i = A \frac{d\sigma}{dt} = pKE_m \cos pt + \frac{pKE_m \alpha}{\sqrt{p^2 + \alpha^2}} \cos (pt + \theta);$$

$$\text{that is, } i = pKE_m \left\{ 1 + \frac{\epsilon \alpha^2}{p^2 + \alpha^2} \right\} \cos pt + \frac{p^2 KE_m \epsilon \alpha}{p^2 + \alpha^2} \sin pt.$$

The capacity under an alternating E.M.F. is

$$C = K \left(1 + \frac{\epsilon \alpha^2}{p^2 + \alpha^2} \right)$$

and the rate at which energy is lost is given by

$$w = \frac{p^2 KE_m^2 \epsilon \alpha}{2(p^2 + \alpha^2)}.$$

ABSTRACT.

The Paper discusses the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D. It is proved that where the polarisation current of a dielectric under the action of a steady E.M.F. is proportional to t^{-n} (t =time), the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$ th power of the time constant.

Starting with this hypothesis it is proved that under an alternating E.M.F. the energy loss varies as the n th power of the frequency, and the viscous capacity varies as the $(n-1)$ th power. An equation is also obtained giving the variation in capacity for the particular case in which $n=1$.

Experimental results published by various investigators are given, and are shown to be not inconsistent with Pellat's modified theory. A brief résumé of Pellat's Theory is also given.

DISCUSSION.

Dr. RUSSELL said that the author's method was ingenious and that the formula deduced for the dielectric current agreed well with many experimental results. He thought, however, that Pellat's theory was not sound and that von Schweidler's modification of it was too elaborate to be of much practical use. He pointed out that Dr. Grover had proved that, from the mathematical point of view, Hopkinson's and von Schweidler's theories were identical. He considered that Fleming and Dyke's experimental results could not be ignored, and that none of the theories advanced up to the present explained them satisfactorily. Very different formulæ gave results which were practically identical over wide ranges, and so they were of doubtful help in enabling us to discover the causes of the phenomena. He considered that the nomenclature of dielectric theory stood badly in need of standardising. For instance, what the

author called the displacement was practically the same as Maxwell's electric displacement, but Sir J. J. Thomson and several other writers called it the polarization.

Dr. ECCLES stated that the Paper compelled remark on the artificial nature of Pellat's hypothesis. This hypothesis led to an expression for the displacement current as a sum of exponential terms. Experiment required it to be a power of t , or the sum of several power terms. This required that the exponential terms of Pellat's hypothesis should be infinite in number—which deprived the hypothesis of physical meaning. It would be more logical, therefore, to begin with the power terms as fundamental, and to discard the false start given by Pellat.

Mr. ADDENBROOKE stated that in order to get at the proper theory of experiments the best dielectrics should not be used, as the effects were so minute that the chances of error were great. With poorer dielectrics, the actions were on a greater scale and easier to observe. Using an ellipsoid suspended at 45 deg. to the electric field, he had been able to carry his experiments down to a frequency of 1 cycle in two hours, taking, of course, time readings of current and voltage. Dr. Ashton had not mentioned a Paper by Wagner, which seemed to cover much the same ground.

Prof. C. H. LEES mentioned a later Paper on the subject by Curtis, who had found that Schweidler's equations held if several terms were taken.

The AUTHOR, in reply, agreed with Dr. Russell as to the need for a standard nomenclature with regard to dielectrics, and called attention to the numerous methods of expressing the quantity known as the electric force. With regard to Dr. Russell's objection that the exponent n has not been shown to be constant, except over a limited period of time, some of the author's published results show that n has remained constant for over 24 hours, but it appeared impossible to carry out satisfactory experiments over long periods without adopting stringent precautions for keeping the temperature and the applied voltage absolutely constant. The author did not wish to give the impression that Fleming's and Dyke's results were not of the highest accuracy, but he believed their results were equally consistent with the equations deduced from Pellat's theory as with the equation of Fleming and Dyke. With regard to the criticism of Dr. Eccles and Prof. Lees that a theory involving the representation of results by an infinite series was of little use, the author contended that the theory did not necessarily involve an infinite series. The upper limit of α is fixed by the point at which the capacity ceases to be affected by an increase in frequency. The lower limit is fixed by the time required to fully polarise the dielectric by a steady E.M.F. An infinite series would lead to the displacement being infinite for $t = \text{infinity}$. The great advantage of the theory is the fact that it correlates all the so-called "absorption" phenomena, and if established by experiment it could be used to extend our knowledge of the properties of dielectrics. For instance, it can be shown that if the theory is true the hysteresis loop must be elliptical, and that a third harmonic cannot be introduced by dielectric hysteresis as it can be by magnetic hysteresis. With regard to Mr. Addenbrooke's work on slow cycles, it should be pointed out that at frequencies of the order of one per second the rate of absorption of energy becomes comparable with the rate of dissipation in the dielectric, and according to Pellat's theory the former will depend on the particular point in the cycle at which the E.M.F. was first applied to the condenser.