A MODIFICATION OF THE FORMULA FOR THIN LENSES.

By CHARLES H. SKINNER,

*Assistant Professor of Physics, Ohio Wesleyan University,
Delaware, Ohio.*

The relation between the principal focal length, f , the index of refraction, n , and the radii of curvature, R_1 and R_2 , of the lens faces of a thin lens is usually expressed by the formula,

$$1/f = (n-1)(1/R_1 + 1/R_2).$$

Various methods may be followed in determining R_1 and R_2 . Perhaps the simplest method for measuring the radii of curvature in the case of double concave lenses is to use each face as a concave mirror, locating the center of curvature as the point where object and image coincide. If the faces of double convex lenses be used as concave mirrors, object and image will not coincide at the centers of curvature but at points which we may call the apparent centers of curvature.

In the figure, let ABC and ADC represent a cross-section of the faces of a double convex lens. Let O_1 be the true center of curvature of ABC, and O' the apparent center of curvature. In order that a ray leaving O' and incident at P may return over the same path, it is necessary that its path inside the lens be perpendicular to the face ABC; that is, along the line O_1P produced. Let us call the distance $O'B$ the apparent radius of curvature, R' , of ABC. Also, let O_2 be the true center of curvature of the face ADC, and O'' its apparent center of curvature. Then $O''D$ is its apparent radius of curvature, R'' . R_1 and R_2 are the true radii respectively of ABC and ADC.

Let us express the lens formula in terms of the apparent radii of curvature instead of the true radii. Draw the line O_2PN , and consider the ray $O'P$. Angle $O'PN$ is the angle of incidence, i ; and angle O_2PM is the angle of refraction, r . From the figure, it is easily seen that

$$i = \text{angle } PO'D + \text{angle } PO_2D = b + c,$$

and $r = \text{angle } PO_1D + \text{angle } PO_2D = a + c$.

Now, $n = \sin i / \sin r = \sin(b+c) / \sin(a+c) =$

$$(\sin b \cos c + \cos b \sin c) / (\sin a \cos c + \cos a \sin c).$$

For small angles (and the angles will be small for thin lenses) the cosines in the above expression are approximately equal to unity. Also, $\sin a = X/O_1P = X/R_1$ approximately (see figure),

$$\sin b = X/O'P = X/R' \text{ approximately,}$$

and $\sin c = X/O_2P = X/R_2$ approximately.

Therefore $\sin i = X/R' + X/R_2 = X(1/R' + 1/R_2)$,

$$\sin r = X/R_1 + X/R_2 = X(1/R_1 + 1/R_2).$$

$$\text{Hence, } n = (1/R' + 1/R_2) / (1/R_1 + 1/R_2) \quad (1)$$

By symmetry,

$$n = (1/R'' + 1/R_1) / (1/R_1 + 1/R_2) \quad (2)$$

Adding equations (1) and (2), and regrouping the numerator of the second member,

$$\begin{aligned} 2n &= (1/R' + 1/R'' + 1/R_1 + 1/R_2) / (1/R_1 + 1/R_2) \\ &= (1/R' + 1/R'') / (1/R_1 + 1/R_2) + 1. \end{aligned}$$

Hence, $(1/R_1 + 1/R_2) = (1/R' + 1/R'') / (2n - 1)$.

Substituting this in the original lens formula, we have

$$1/f = (n - 1)(1/R' + 1/R'') / (2n - 1).$$

Thus it is seen that the apparent radii of curvature may be substituted for the real radii of curvature, in the original lens formula, provided the right hand member of the formula be divided by $2n - 1$.

If we solve for n from the modified formula, we have

$$n = [f(1/R' + 1/R'') - 1] / [f(1/R' + 1/R'') - 2].$$

To test the above expression for n , a double convex lens was selected and the following data taken, using a spherometer for getting R_1 and R_2 . R' and R'' were found as explained above that is, the lens faces were used as concave mirrors, and an illuminated point adjusted till it and its image coincided, thus giving the apparent centers of curvature.

$f = 34.5$ cm.; $R_1 = 32.6$ cm.; $R_2 = 42.5$ cm.; $R' = 16.9$ cm.; $R'' = 19.2$ cm.

By original formula, $n = 1.537$,
and by modified formula, $n = 1.540$.

It is often desirable to investigate lenses where the ordinary means of determining the radii of curvature of the faces are not available, or applicable. In such cases, the modified formula will be found useful.

It is interesting to note that in either equation (1) or equation (2) we have a relation whereby the index of refraction may be easily determined without a knowledge of the principal focal length of the lens.